The Political Economy of Municipal Pension Funding*

Jeffrey Brinkman
Federal Reserve Bank of Philadelphia

Daniele Coen-Pirani
University of Pittsburgh

Holger Sieg
University of Pennsylvania and NBER

August 3, 2017

*We would like to thank the editor of the journal, three anonymous referees, David Albouy, Dan Bernhardt, Steve Coate, Victor Ríos-Rull, and seminar participants at various institutions and conferences for helpful comments. Sieg acknowledges financial support from the National Science Foundation (SES-0958705). The views expressed here are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia, the Federal Reserve System, or the National Science Foundation.

Abstract

Many U.S. municipalities have committed to pay retirement benefits to public sector employees but have not saved enough to fulfill these obligations. This paper studies the determinants of municipal pension funding and its implications for intergenerational redistribution using an overlapping generations model. Under perfect capital markets, pension funding choices are fully capitalized into land prices. This neutrality result fails if agents face a binding downpayment constraint in the land market: old agents prefer a pay-as-you go system while young agents find a fully funded system optimal. Empirical evidence based on cross-city comparisons of pension liabilities is consistent with these predictions.

JEL Classifications: E6, H3, H7, R5

Keywords: unfunded liabilities, land prices, capitalization, downpayment.

1 Introduction

U.S. cities face stringent requirements to balance their operating budgets each year (Epple and Spatt (1986)). Reducing funding for city employees' pension plans is one of the few viable options to effectively take on debt that is not linked to capital expenditures. In fact, a large number of cities and local governments in the U.S. have less than fully funded their employees' pension plans, allowing them to potentially shift the tax burden across generations of residents. According to the Pew Charitable Trusts (2013, Exhibit 1), the aggregate unfunded pension and retiree health-care liabilities of a sample of 61 large U.S. cities add up to more than two hundred billion dollars, with considerable variation across cities.[1] In this sample, the City of Chicago stands out with unfunded pension liabilities in excess of $11,000 per household in 2009.[2] When these liabilities come due, a local government will either need to raise taxes or try to renege on some of its promises. The latter option appears more difficult to implement than, for example, changing the parameters of the Social Security system because local pensions are usually protected by state constitutions.[3]

In this paper, we investigate the politico-economic origins of local pension funding and its implications for the welfare of different cohorts in the context of an overlapping generations (OLG) model. We make four contributions. First, we develop an analytically tractable model that delivers transparent intuitions about the main forces at play. To the best of our knowledge, only a few papers work out an analytical solution to Markov perfect equilibria of these types of dynamic political economy models. One prominent example is the work by Hassler, Mora, Storesletten, and Zilibotti (2003).[4] Second, we clarify the extent to which

[1] This phenomenon is not confined to large urban central cities. For example, according to the 2012 Status Report on Local Government Pension Plans released by the Public Employee Retirement Commission of Pennsylvania, 630 of 3,161 local pension plans in Pennsylvania were less than 80 percent funded. These estimates of unfunded liabilities are probably a lower bound as the latter are typically computed using an 8 percent discount rate following government accounting standards (Novy-Marx and Rauh, 2009 and 2011).

[2] See Pew Charitable Trusts (2013, Exhibit 2) for data on unfunded pensions at the city level and U.S. Census for number of households.

[3] The pensions of public employees of the City of Detroit were affected by this city's bankruptcy proceeding. Recent attempts by the Illinois legislature to change the negotiated pensions of public employees were, however, blocked by the state's Supreme Court (Davey (2015)).

[4] Other papers that analytically characterize the equilibrium of dynamic political economy models are Grossman and Helpman (1998) and Battaglini and Coate (2008).

land price capitalization effects neutralize the impact of debt financing on agents' utility.[5] Third, we show that a binding downpayment constraint leads to an intergenerational conflict over pension funding policies. Last, we provide empirical evidence based on a cross-section of cities that is consistent with the key predictions of the version of the model with binding constraints.

In our model, agents live for two periods, as young and old. Young agents purchase land from old agents, and consume land services, private consumption goods, and a public good. Public goods are produced by municipal workers who are compensated through a combination of wages and promised future pension benefits. The current period's policymaker in a locality chooses how much to save to finance future pension benefits, taking into account the effect of her choices on land values, and, potentially, on the policy followed by next period's policymaker. The characterization of a politico-economic equilibrium in our model follows the pioneering work of Krusell, Quadrini and Ríos-Rull (1997), Krusell and Ríos-Rull (1999), and Klein, Krusell, and Ríos-Rull (2008). An important aspect of our analysis is to study the implications for pension funding policy of a downpayment constraint that limits the amount a young agent may borrow when purchasing land (housing). A growing literature in macroeconomics has argued that such constraints play an important role in accounting for fluctuations in housing prices and macroeconomic aggregates.[6] In the context of our model, downpayment constraints have significant implications for the nature of politico-economic equilibria and the implied welfare of agents of different generations.

In OLG models without altruism, Ricardian equivalence typically does not hold so that taxation and debt are not equivalent ways to finance public goods from the perspective of different generations. However, in an economy with endogenous land prices, public debt and

[5] The importance of land price capitalization for many issues such as debt, school quality, taxation, etc. was first emphasized by Oates (1969) and has received a considerable amount of attention in the local public finance literature. Recent contributions to this literature include Schultz and Sjostrom (2001); Conley and Rangel (2001); and Conley, Driskill and Wang (2013). The key difference between these papers and ours is the fact that young agents in our model face a downpayment constraint.

[6] See, for example, Iacoviello and Pavan (2013); Campbell and Hercowitz (2005); Favilukis, Ludvigson, and Van Nieuwerburgh (2008); Landvoigt, Piazzesi, Schneider (2015) for macroeconomic models with housing and downpayment constraints.

taxes are capitalized into the equilibrium value of land. This capitalization effect reduces, or even completely eliminates, the scope for intergenerational redistribution associated with the government's debt policy. Two cases arise in our model economy.

First, we consider a frictionless asset market and establish that, if young agents can freely borrow and lend at the same rate as the local government, both young and old agents are indifferent about the locality's pension funding policy. A property tax cut today must be met by an increase in future taxes to finance promised pensions. Higher future taxes, in turn, reduce the resale value of land tomorrow. These two effects cancel out exactly, leaving the current price of land unaffected by the locality's pension funding policy. Similarly, young agents either pay for promised pensions directly through higher current taxes or, indirectly, in the future through a lower resale price of land. Given their unrestricted ability to borrow, they are indifferent between these two options.

Second, we introduce in our model an imperfection in the capital market. Young agents are subject to a downpayment constraint when purchasing land and can only borrow up to a fraction of their housing wealth next period. In this case, they are not indifferent about the timing of taxes. A policy that reduces current property taxes in exchange for higher future taxes increases their willingness to pay for land today, benefitting the old generation of land owners. The key intuition is that, since young agents are constrained, they discount changes in future land prices at a rate higher than the interest rate. Differently from old agents, young agents would rather follow a policy of full funding of pensions because it maximizes the resale value of the land they buy. The intergenerational conflict about pension funding that ensues is one of our main theoretical findings and represents the foundation of our empirical analysis.

We turn to empirical analysis to test the key predictions of the model. We combine data on unfunded liabilities collected by Munnell and Aubry (2016) with demographic data from the U.S. Census for a sample of 173 large U.S. cities. We find that, on average, municipalities with younger populations of homeowners have lower levels of unfunded liabilities. These results are robust to the inclusion of controls for population growth, median income, and aggregate liabilities, which might otherwise confound the relationship of interest.

This paper contributes to the literature on the political economy of debt, and in particular to the literature on intergenerational conflict in the political economy of debt, going back to the work of Cukierman and Meltzer (1989) and more recently developed by Song, Storesletten, and Zilibotti (2012). In both of these papers, a dynamic emerges regarding intergenerational conflict whereby the old are prone to immiserate the young by voting for higher levels of debt today.[7] A distinctive feature of our paper is the presence of a land market and the related issue of capitalization of unfunded liabilities into land prices. Land market capitalization can, in principle, provide an answer to the question asked by Song, Storesletten, and Zilibotti (2012, p. 2785): "What then prevents the current generations from passing the entire bill for current spending to future generations?"[8] Our paper is also related to the small, but growing literature on dynamic political-economy models in public finance. In a related paper, Barseghyan and Coate (2015) develop a dynamic Tiebout model and use it to study the efficiency of zoning regulations.[9] Last, our paper is related, although less directly, to the macroeconomic literature on asset prices and portfolio choices in OLG models (see, e.g., Glover et al. (2014)).

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 illustrates our argument and main results using an example based on logarithmic utility. Section 4 extends our results to the case of a general utility function. Section 5 presents some empirical evidence consistent with our mechanism. Finally, Section 6 concludes. The Appendix contains the proofs of all propositions, extensions of the benchmark model, and robustness checks of our empirical analysis.

[7] In addition to these two references, a non-exhaustive list of dynamic political-economy papers on debt, taxes, and government spending includes Bassetto and Sargent (2006); Bassetto (2008); Battaglini and Coate (2008); Yared (2010), Azzimonti (2011); Azzimonti, Battaglini and Coate (2015); and Bouton, Lizzeri, and Persico (2016), among others.

[8] In their OLG model Ricardian equivalence does not hold. However, young agents have a disciplining effect on debt because they anticipate that increasing debt today results in lower public good expenditures when they are old. We view our answer to the question in the text as different from, and complementary to, theirs.

[9] See also Glomm and Lagunoff (1999). The recent literature on local pensions includes Bohn (2011), Albrecht (2012), Bagchi (2013), and Glaeser and Ponzetto (2014). Earlier contributions to the literature on pension funding include Inman (1982), Mumy (1978), and Epple and Schipper (1981).

2 A Model of Pension Funding and Land Price Capitalization

In this section, we introduce a model of public employees' pension funding. Ex-ante identical agents live in a locality for two periods, as young and old. As young, agents purchase land from the old, pay property taxes, and consume goods and housing services. As old, agents sell their land and consume the proceedings.[10] The municipality is characterized by a fixed mass of land and offers a certain exogenous amount of public goods to its young residents. Public goods are produced by absentee municipal employees who receive a compensation package composed of current wages and future pension benefits.[11] Municipal services are financed through property taxation.[12] While current wages of municipal employees have to be financed out of current taxes, promises of future pensions may be financed when they come due. The problem of the policymaker in each municipality is to fund the municipal pension system.

Agents' preferences are represented by the following utility function:

$$U(c_{yt}, l_t, c_{ot+1}) = u(c_{yt}, l_t) + v(c_{ot+1}), \tag{2.1}$$

where c_{yt} denotes consumption of the numeraire good when young, l_t denotes the services of the land purchased by the agent, and c_{ot+1} denotes consumption when old. We make the following assumptions concerning utility.

Assumption 1 *The functions $u(c_y, l)$ and $v(c_o)$ are twice differentiable and such that i)*

[10] According to Poterba, Venti and Wise (2011), in 2008 equity in real estate represents roughly half of non-annuitized wealth for households ages 65–69.

[11] By the word "absentee", we mean that public sector employees do not live in the locality and so do not contribute to the demand for land there. We have experimented with versions of the model in which this is not the case, but concluded that such extension complicates the algebra without adding insight, so we chose to abstract from it.

[12] According to the 2012 Census of Governments, property taxes account for 72 percent of local own-source tax revenues. Lutz et al (2011) study the dynamics of property tax rates during the recent housing crisis and show that local governments responded to budget shortfalls and declines in assessed property values associated with the recession by increasing property tax rates.

$u_1(c_y, l) > 0$, $u_2(c_y, l) > 0$, $v'(c_o) > 0$; ii) $u_{11}(c_y, l) \leq 0$, $u_{22}(c_y, l) \leq 0$, $v''(c_o) \leq 0$, with at least one of these inequalities being strict; and iii) $u_{12}(c_y, l) \geq 0$.

The first two sets of assumptions are standard. Higher consumption of each good increases utility, and the marginal utility of consumption of each good is weakly decreasing. Condition (iii) is sufficient to guarantee that the second-order condition of the agent's optimization problem under a binding downpayment constraint is satisfied.

Each agent is endowed with w units of the consumption good when young and has to decide how much to consume when young and old and how much land (housing) to purchase when young.[13] An agent's budget constraint is:

$$w = c_{yt} + (1 + \tau_t) q_t l_t + \frac{b_{t+1}}{R}, \quad (2.2)$$

$$c_{ot+1} = q_{t+1} l_t + b_{t+1}, \quad (2.3)$$

where q_t denotes the price of land in the municipality in period t and τ_t is the property tax rate. There are two assets in this economy. In addition to land, there is also a risk-less bond. The quantity of bonds purchased (or issued) by the agent is denoted by b_{t+1}, and $R > 1$ is the exogenous gross interest rate paid by a bond.

An important feature of our analysis is a downpayment constraint on land purchases. The importance of downpayment requirements in constraining households' housing purchases has been documented by Linneman and Wachter (1989), Zorn (1989), Jones (1989), and Haurin, Hendershott and Wachter (1996), among others. We assume that borrowing is constrained to a fraction of the value of land next period:

$$-b_{t+1} \leq \kappa q_{t+1} l_t, \quad (2.4)$$

[13]We assume that in the benchmark economy there is no rental market for land. With a frictionless rental market, the downpayment constraint we consider would have no effect on policies if the owner of the housing stock is unconstrained. Notice that many models (see, e.g., Bajari et al. (2013), and Iacoviello and Pavan (2013)) with a downpayment constraint and tenure choice restrict the scope for renting. In Section 3.4.1 we extend the benchmark model to consider the case in which old agents can rent housing.

where $0 < \kappa \leq 1$ is a parameter that indexes the size of the loan relative to the future value of the land.[14] An equivalent way to express the constraint (2.4) is to use equation (2.2) and replace b_{t+1}:

$$w - c_{yt} \geq d_t l_t, \tag{2.5}$$

where the downpayment per unit of land is defined as:

$$d_t \equiv (1 + \tau_t) q_t - \frac{\kappa}{R} q_{t+1}. \tag{2.6}$$

According to (2.5), agents need to self-finance the downpayment d_t, where the latter is equal to the gross-of-tax price of land in the current period minus the maximum amount that a young agent is able to borrow per unit of land purchased. Notice that when $\kappa = 1$, the natural borrowing limit applies, and the required downpayment coincides with the user cost of land.[15] When $\kappa = 0$, the agent needs to pay for her land acquisition entirely out of her endowment when young.

The supply of land in the municipality is fixed at an exogenous level normalized to one. In the benchmark economy we consider, the measure of young workers living in the municipality in each period is fixed exogenously and also normalized to one.[16] With a fixed population, land market equilibrium requires that:

$$l_t = 1. \tag{2.7}$$

[14] This specification of the downpayment constraint presents at least two advantages. First, in terms of interpretation, when $\kappa = 1$, equation (2.4) coincides with the natural borrowing limit (i.e. non negativity of consumption when old), which must prevail to prevent default on the debt. Second, if the downpayment constraint depended on the current rather than the future price of land, we would not be able to derive an analytical solution to the model for the case of a general utility function. In the case of logarithmic utility all of our results would go through, as shown in Appendix C.1. In this Appendix we also discuss alternative specifications of the credit market friction.

[15] The fact that the downpayment coincides with the user cost of land is due to the specification of the borrowing constraint in equation (2.4). See Kiyotaki and Moore (1997, p. 221) for a discussion of this point in the context of a model with a downpayment constraint similar to ours.

[16] In Appendix C.2 we consider the case in which young agents are perfectly mobile across locations and show that our results are robust to this extension.

The assumption that the supply of land is fixed exogenously should be viewed as a convenient approximation that we defend on three grounds. First, land price capitalization operates more strongly when the supply of land is fixed, otherwise the adjustment in the quantity of land would absorb some of the effect of various policies. In fact, we show that in our model, absent the downpayment constraint, pension funding policies are fully capitalized into land prices. Thus, any departure from this benchmark can be ascribed exclusively to binding downpayment constraints. Second, while housing supply can, in principle, be expanded through construction, the quantity of land available in many cities is significantly less price elastic. Third, allowing for endogenous land supply in a general equilibrium setting is difficult from a modeling perspective, because one needs to both specify a technology for transforming "raw" land into "habitable" land, and to assign ownership of the former to some agent within the model. For these three reasons, we focus on the case in which the supply of land is fixed exogenously.

The government of the municipality finances the provision of a local public good. The quantity of the public good consumed is an exogenous constant, and, for simplicity of notation, we do not include it in the utility function.[17] The local government has committed in each period to current wage payments w^g and future pension benefits b^g. We take the vector (w^g, b^g) as given and focus on the decision to fund promised benefits.[18] The government collects revenue $\tau_t q_t$ by taxing property values and uses it to pay the wage w^g of current public sector workers, to fund some (or none) of their promised retirement benefits b^g and to pay for the unfunded portion of the pension benefits of last period's public sector workers. Thus, in period t, a municipality's budget constraint is:

$$\tau_t q_t = w^g + \frac{f_{t+1} b^g}{R} + b^g (1 - f_t), \tag{2.8}$$

[17]Formally, consumption of the public good can be ignored if it enters additively in utility.

[18]In reality, the compensation package of public sector employees might emerge from the interaction between the city and the labor unions representing public sector workers. For simplicity, we abstract from modeling such interaction.

where f_t is the fraction of pensions due in period t that is funded.[19] Notice that in equation (2.8) pensions due in the next period are discounted at the rate R because this is the rate at which the local government can save. We assume that f_t is constrained to be between some lower bound $f_{\min} \geq 0$ and one, in which case, the municipality fully funds the future pensions of its employees. We interpret the lower bound $f_{\min}$ as a policy parameter that can, in principle, be manipulated by a higher level of government. The latter might force the locality under consideration to fund its public sector pensions more than it would otherwise wish to do.

The policy decision in each period t is the mix (τ_t, f_{t+1}) of current taxes and funding of future public sector pensions. We assume that (τ_t, f_{t+1}) is chosen in each period by either the current young or the current old generation in the municipality. In Section 3.4.2 we also consider a version of the model with probabilistic voting. The timing of events within each period is as follows. Policy is set at the beginning of the period. Then, young agents make consumption and land demand choices. Last, the land market clears. Thus, the policymaker takes into account the effect of her choices on land values within the period. She also understands the effect of these policies on the policies chosen by future policymakers.

3 Analytical Solution with Logarithmic Utility

We begin by considering a specific utility function that delivers closed-form solutions and, therefore, allows us to fully illustrate the main mechanisms at work in our model. The utility function we consider is:

$$U\left(c_{yt}, l_t, c_{ot+1}\right) = \left(1 - \psi - \beta\right) \ln c_{yt} + \psi \ln l_t + \beta \ln c_{ot+1}, \tag{3.1}$$

[19] The initial funding level $f_0 \in [f_{\min}, 1]$ is given exogenously. Notice that our model could be interpreted as a general model of local debt. To see this, let B_t denote debt due in t, with $B_t \equiv b^g\left(1 - f_t\right)$, and define local public expenditures $g_t \equiv w^g + b^g/R$. Then, the government budget constraint reads: $\tau_t q_t + B_{t+1}/R = g_t + B_t$. In this paper, we focus on unfunded pension liabilities because we take the expenditure term g_t as given, while a comprehensive analysis of local debt might require endogenizing it.

where the parameters ψ and β are positive and such that $\psi + \beta < 1$. In Section 4 we show that our results also apply for more general utility functions that satisfy Assumption 1. We distinguish between two cases. In one case the capital market is frictionless, and young agents are free to borrow as much as they wish against the future value of the land they buy. In the second case, we introduce the downpayment constraint. Our main point is that the binding downpayment constraint has important implications for the locality's equilibrium pension funding policy and the implied welfare of the agents.

3.1 Frictionless Asset Market

Agents' Choices and Land Market Equilibrium In order to establish a useful benchmark, we start by considering a financial market in which young agents are able to borrow freely against the future value of land. In this case, young agents maximize utility subject only to the budget constraints (2.2)-(2.3). It is straightforward to verify that in this case the optimal choices are:

$$\begin{aligned} l_t &= \frac{\psi w}{(1+\tau_t)\, q_t - q_{t+1}/R}, & (3.2) \\ c_{yt} &= (1-\psi-\beta)\, w, & (3.3) \\ c_{ot+1} &= \beta R w. & (3.4) \end{aligned}$$

In what follows we show that the equilibrium utilities of both young and old agents are independent of the locality's pension funding policy. Consider a young agent first, and notice that her consumption choices c_{yt} and c_{ot+1} are independent of the current or future price of land due to the logarithmic utility specification. Land market equilibrium requires that $l_t = 1$, implying that, in equilibrium, the utility level of a young agent is equal to the following constant:

$$V^{\text{young}} = (1-\psi-\beta) \ln (1-\psi-\beta)\, w + \beta \ln \beta R w. \tag{3.5}$$

The latter, of course, does not depend on the locality's pension funding policy.

Consider now an old agent. An old agent wishes to sell a unit of land and needs to pay back the debt b_t she issued when young. Formally, in period t an old agent's utility is given by:

$$V_t^{\text{old}} = \ln\left(q_t + b_t\right). \tag{3.6}$$

Given that b_t is pre-determined, the objective of an old agent is simply to maximize the *current* land price q_t.

In what follows we analyze how pension funding policies affect q_t. Replace equation (3.2) into the land market clearing condition $l_t = 1$ to uniquely pin down the user cost of land in the locality:

$$\left(1+\tau_t\right) q_t - q_{t+1}/R = \psi w. \tag{3.7}$$

Property taxes are given by the government's budget constraint in equation (2.8). Replacing the latter into equation (3.7), and solving for q_t, yields the equilibrium price of land as a function of the locality's pension funding policy and its future land price:

$$q_t = \psi w - w^g - \frac{f_{t+1} b^g}{R} - b^g\left(1 - f_t\right) + \frac{q_{t+1}}{R}. \tag{3.8}$$

Intuitively, for given price of land tomorrow, the price of land today increases with f_t, i.e., the extent to which pensions due today are funded, and decreases with f_{t+1}, i.e., the extent to which the locality has to tax land today in order to fund the pension promises due tomorrow.

Recursive Formulation In order to solve for equilibrium land prices and determine their dependence on pension funding policies, it is convenient to recast the equilibrium equation (3.8) in recursive form and then define a Markov perfect equilibrium, following Krusell, Quadrini, and Ríos-Rull (1997), Krusell and Ríos-Rull (2000), and Persson and Tabellini (2002). The recursive representation is fairly straightforward. Let f denote the funding ratio for pensions due in period t (i.e. the equivalent of f_t) and f' denote the funding ratio for pensions promised in t but due in $t+1$ (i.e. the equivalent of f_{t+1}). The state variable in this model is f which is predetermined at the beginning of period t. We are interested in determining the equilibrium funding policy rule $f' = F(f)$. With this notation in hand we can re-write equation (3.8) as:[20]

$$Q(f;F) = \psi w - w^g - \frac{F(f)\, b^g}{R} - b^g(1-f) + \frac{Q(F(f);F)}{R}, \tag{3.9}$$

where we acknowledge the dependence of the current price of land, $Q(f;F)$, on pension funding f at the beginning of the period and on the funding policy rule F which determines the funding policy from today onward.

Following the terminology of Persson and Tabellini (2002, chapter 11), we first define an *economic equilibrium under a given policy rule* F for pension funding. We then consider a one-period deviation from this rule and define an *economic equilibrium after a deviation*. Last, we define an *equilibrium without commitment* - the Markov perfect equilibrium - by imposing that the one-period deviation preferred by the policymaker coincides with the original policy rule.

Definition 1 ***Economic equilibrium under a policy rule F****.*
Fix the funding rule $f' = F(f)$*. An equilibrium under this policy rule is given by a function* $Q(f;F)$ *such that the equilibrium equation (3.9) holds.*

The Price of Land after a One Period Deviation from Equilibrium Funding In order to characterize the equilibrium $F(f)$ and $Q(f;F)$, it is necessary to determine the impact of a one-period deviation of pension funding policy from the equilibrium rule $F(f)$. Let this deviation be denoted by $\widetilde{f}'$ and let $\widetilde{Q}\left(f, \widetilde{f}'; F\right)$ denote the current price of land following a deviation $\widetilde{f}'$. Since the policy deviation lasts only one period, the pension funding policy reverts back to the rule F, starting in period $t+1$. Thus, the economy will begin period $t+1$ with a pension funding ratio given by $\widetilde{f}'$ and choose pension funding for

[20] It is worth to point out that by focusing on recursive equilibria, we are ruling out non-stationary solutions of equation (3.8).

pensions due in $t+2$ according to the rule $F(\widetilde{f}')$. It follows that the equilibrium price of land after a policy deviation $\widetilde{f}'$ is given by:

$$\widetilde{Q}\left(f,\widetilde{f}';F\right)=\psi w-w^{g}-\frac{\widetilde{f}'b^{g}}{R}-b^{g}\left(1-f\right)+\frac{Q\left(\widetilde{f}';F\right)}{R}, \tag{3.10}$$

where the price of land in $t+1$ is given by the equilibrium pricing function Q evaluated at the funding ratio $\widetilde{f}'$ chosen in period t. Again, following Persson and Tabellini (2002, chapter 11), we define an economic equilibrium after a deviation as follows.

Definition 2 ***Equilibrium after a one-period deviation*** $\widetilde{f}'$ ***from the policy rule*** F. *An equilibrium after a one-period deviation* $\widetilde{f}'$ *is given by the functions* $\widetilde{Q}\left(f,\widetilde{f}';F\right)$ *and* $Q\left(f;F\right)$ *such that for all* $\left(f,\widetilde{f}'\right)$*, the land market equilibrium equation (3.10) holds and* $Q\left(f;F\right)$ *satisfies (3.9).*

As already mentioned, old agents would like to set $\widetilde{f}'$ in order to maximize the current price of land $\widetilde{Q}\left(f,\widetilde{f}';F\right)$ while young agents are indifferent among alternative values of $\widetilde{f}'$. It is important to notice that the optimal policy deviation $\widetilde{f}'$ is independent of the state variable f, and is therefore a constant, because there is no interaction between the current state f and the policy deviation $\widetilde{f}'$ in equation (3.10). In other terms, the level of pension funding f with which a locality enters the period is capitalized into the current price of land and has no impact on the costs and benefits of varying the future pension funding $\widetilde{f}'$. This observation implies that we can focus on candidate equilibrium policy rules such that the optimal funding ratio for next period is a constant, or $f^{*}=F(f)$ for all f. Replacing $f^{*}=F(f)$ into the equilibrium equation (3.9) yields the equilibrium pricing function in terms of the state variable f:

$$Q\left(f;f^{*}\right)=\psi w-w^{g}-\frac{f^{*}b^{g}}{R}-b^{g}\left(1-f\right)+\frac{Q\left(f^{*};f^{*}\right)}{R}. \tag{3.11}$$

This relationship allows us to replace $Q\left(\widetilde{f}';f^{*}\right)$ into the right-hand side of equation (3.10) and, therefore, to evaluate the effect of a deviation $\widetilde{f}'$ on the current price of land:

$$\widetilde{Q}\left(f,\widetilde{f}';f^*\right) = \psi w - w^g - \frac{\widetilde{f}' b^g}{R} - b^g(1-f) + \\ +\frac{1}{R}\left[\underbrace{\psi w - w^g - \frac{f^* b^g}{R} - b^g\left(1-\widetilde{f}'\right) + \frac{Q(f^*;f^*)}{R}}_{=Q\left(\widetilde{f}';f^*\right)}\right]. \quad (3.12)$$

It is immediate to notice that, in equation (3.12), the two terms in $\widetilde{f}'$ cancel out. Thus, the price of land $\widetilde{Q}\left(f,\widetilde{f}';f^*\right)$ is independent of the deviation $\widetilde{f}'$. As a result old agents' are also indifferent about alternative funding ratios $\widetilde{f}'$. In other words, *with a frictionless asset market, both young and old agents' utilities are independent of the locality's pension funding policy* $\widetilde{f}'$. The intuition is straightforward. A marginal reduction in pension funding $\widetilde{f}'$ leads to a reduction in *current* property taxes by b^g/R, and leads to an increase in the current price of land by the same amount. It also leads to an increase in *future* property taxes by b^g and a decline in the *future* price of land by the same amount, b^g. What effect does this reduction in the future price of land have on its current price? Under frictionless capital markets young agents discount the reduction in the future price of land using the interest rate R, so their willingness to pay for land today drops exactly by b^g/R, fully offsetting the land price increase generated by lower current taxes. As a consequence, old agents cannot benefit from reducing pension funding.

Equilibrium The requirement of a politico-economic equilibrium is that the optimal one-period deviation from the policy rule $F(f)$ coincides with $F(f)$ itself. In the case of frictionless capital markets, both the indirect utility function of a young agent and the current price of land are independent of a policy deviation $\widetilde{f}'$. As a result, any (constant) funding rule is an equilibrium funding rule because agents do not care about whether pensions are financed through current or future taxes.

Proposition 1 (Frictionless asset market) *Under a frictionless asset market the equi-*

librium funding rule is indeterminate and irrelevant for agent' utilities.

In the next section we consider the case in which the downpayment constraint binds, in which case a locality's pension funding policy has implications for the agents' utility. We postpone a formal definition of the politico-economic equilibrium to that section.

3.2 Binding Downpayment Constraint

Agents' Choices, Utility, and Land Market Equilibrium We now turn to characterize the politico-economic equilibrium of the economy under a binding downpayment constraint. Maximizing utility subject to the budget constraints (2.2)-(2.3) and the downpayment constraint (2.5) allows us to calculate the optimal choices, assuming that a young agent is constrained:

$$l_t = \frac{(\psi+\beta)\,w}{(1+\tau_t)\,q_t - \kappa q_{t+1}/R}, \tag{3.13}$$

$$c_{yt} = w\,(1-\psi-\beta)\,, \tag{3.14}$$

$$c_{ot+1} = \frac{(1-\kappa)\,(\psi+\beta)\,wq_{t+1}}{(1+\tau_t)\,q_t - \kappa q_{t+1}/R}, \tag{3.15}$$

where the denominator of equations (3.13) and (3.15) is the downpayment d_t, defined in equation (2.6). A young agent is constrained if and only if the future price of land is sufficiently large relative to the downpayment d_t:[21]

$$\frac{q_{t+1}}{d_t} > \frac{\beta R}{(1-\kappa)\,(\psi+\beta)}. \tag{3.16}$$

In what follows, we proceed under the assumption that this condition holds and later check that this is indeed the case under suitable restrictions on the model's parameters. Land market equilibrium requires that $l_t = 1$, which, in conjunction with the demand for land in

[21] The intuition is that a higher ratio q_{t+1}/d_t increases consumption when old (equation 3.15) and thus lowers its marginal utility relative to the marginal utility of consumption when young. Notice that the latter is a constant.

equation (3.13), pins down the equilibrium downpayment:

$$d = (\psi + \beta)\, w, \tag{3.17}$$

a constant. Notice that, after imposing equilibrium in the land market, the lifetime utility of a young agent in period t is:

$$V_t^{\text{young}} = (1 - \psi - \beta) \ln (1 - \psi - \beta)\, w + \beta \ln (1 - \kappa) + \beta \ln q_{t+1}. \tag{3.18}$$

As this equation shows, in the version of the model with a binding downpayment constraint, a young agents' utility is increasing in the *future* price of land, q_{t+1}.[22] By contrast, and as before, the objective of an old agent is to maximize the *current* price of land q_t.

The analysis follows similar steps as in the case of frictionless capital markets. Replace the equilibrium downpayment d and the government's budget constraint in equation (2.6) to obtain the analog of equation (3.8) for the economy with binding constraints:

$$q_t = (\psi + \beta)\, w - w^g - \frac{f_{t+1} b^g}{R} - b^g (1 - f_t) + \frac{\kappa}{R} q_{t+1}. \tag{3.19}$$

This equation characterizes the equilibrium current price of land as a function of the locality's pension funding choices and of the future price of land. Notice that an important difference between equation (3.19) and its frictionless analog (3.8) is that with a binding downpayment constraint, the current price of land q_t is less sensitive to variation in the future price q_{t+1}, as $\kappa < 1$. This is the case because with a binding downpayment constraint a young agent can borrow only a fraction κ of the value of land in $t + 1$.

[22]Notice that a young agent's utility is independent of the current price q_t. The reason why the current price of land has no independent impact on a young agent's utility is that a young agent cares about q_t only to the extent that this price affects the downpayment that she has to make to buy land. Notice, however, that land market equilibrium requires that this downpayment is the constant in equation (3.17). Only at this downpayment, young agents are willing to purchase the land sold by old agents.

Recursive Representation The recursive version of equation (3.19) is:

$$Q(f;F) = (\psi+\beta)w - w^g - \frac{F(f)b^g}{R} - b^g(1-f) + \frac{\kappa Q(F(f);F)}{R}, \tag{3.20}$$

with the same notation as in the frictionless case. Also, following the same steps as in the frictionless case, we obtain the current price of land $\widetilde{Q}\left(f,\widetilde{f}';F\right)$ following a one-period deviation $\widetilde{f}'$ from equilibrium:

$$\begin{aligned}\widetilde{Q}\left(f,\widetilde{f}';f^*\right) &= (\psi+\beta)w - w^g - \frac{\widetilde{f}'b^g}{R} - b^g(1-f) + \\ &+\frac{\kappa}{R}\left[\underbrace{(\psi+\beta)w - w^g - \frac{f^*b^g}{R} - b^g\left(1-\widetilde{f}'\right) + \frac{\kappa}{R}Q(f^*;f^*)}_{=Q(\widetilde{f}';f^*)}\right],\end{aligned} \tag{3.21}$$

where we have taken into account the fact that, as under frictionless markets, the equilibrium policy must be a constant $f^* = F(f)$ for all f. Differently from equation (3.12), $\widetilde{f}'$ does not drop out of the expression in (3.21). Specifically, with a binding downpayment constraint, the current price of land *falls* with $\widetilde{f}'$:

$$\frac{\partial\widetilde{Q}\left(f,\widetilde{f}';f^*\right)}{\partial\widetilde{f}'} = -\frac{b^g}{R}(1-\kappa) < 0. \tag{3.22}$$

The intuition is that, since a young agent is constrained, the increase in future property taxes by b^g is discounted at the rate R/κ, which is higher than under frictionless capital markets. As a consequence, since reducing pension funding brings about a reallocation of property taxes towards the future, it leads to an increase in the demand for land by young agents. In equilibrium, this increase gives rise to a *higher* land price. It follows that, if old agents are in charge of setting the pension funding policy, the optimal one-period deviation for them is to set $\widetilde{f}' = f_{\min}$.

Consider now the alternative scenario in which young agents set the pension funding policy $\widetilde{f}'$. They do so in order to maximize the *future* price of land $Q\left(\widetilde{f}';f^*\right)$. The latter is monotonically increasing in $\widetilde{f}'$ because starting the period with a better-funded pension system lowers the need for current taxes and thus increases young agents' willingness to pay for land in period $t+1$. Formally, from equation (3.20), it is straightforward to show that

$$\frac{\partial Q\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = b^g > 0.$$

Therefore, young agents' optimal one-period deviation is to fully fund the pension system by selecting $\widetilde{f}' = 1$. We are now ready to define a politico-economic equilibrium.

Definition 3 ***Equilibrium without commitment***.

An equilibrium without commitment for the municipality is given by a policy rule F and land pricing functions $Q(f;F)$ and $\widetilde{Q}\left(f, \widetilde{f}'; F\right)$ such that:

1. *The function $Q(f;F)$ constitutes an economic equilibrium under F according to Definition 1.*

2. *The functions $\widetilde{Q}\left(f, \widetilde{f}'; F\right)$, $Q(f;F)$ constitute an economic equilibrium after a one-period deviation from F according to Definition 2.*

3. *The policymaker has no incentive to deviate from F in any period and for any state, taking into account the economic equilibrium after a one-period deviation. Thus, if the policymaker belongs to the old generation, the consistency requirement is:*

$$F(f) = \arg\max_{\widetilde{f}'} \widetilde{Q}\left(f, \widetilde{f}'; F\right) \tag{3.23}$$

for all f. Alternatively, if the policymaker belongs to the young generation, the consistency requirement is:

$$F(f) = \arg\max_{\widetilde{f}'} Q\left(\widetilde{f}'; F\right) \tag{3.24}$$

for all f.

Based on the previous discussion we can therefore summarize our main result in the following proposition.

Proposition 2 (Binding downpayment constraint) *In a politico-economic equilibrium in which a young agent sets the policy, the equilibrium policy rule is $F(f) = 1$ for all f, while, if the policymaker is an old agent, the policy rule is $F(f) = f_{\min}$ for all f.*

Binding Downpayment Constraint The analysis of this section presumes that a young agent is always constrained, both in equilibrium under a funding policy f^*, as well as out of equilibrium, following a policy deviation $\widetilde{f'} \in [f_{\min}, 1]$. Replacing equation (3.21) into equation (3.16), it can be verified that this is indeed the case under the following necessary and sufficient condition on the model's parameters:[23]

$$\psi > \beta \frac{R-1}{1-\kappa} + \frac{w^g}{w} + \frac{b^g}{w}\left[1 + \frac{1-\kappa}{R}\left(1 - \frac{\kappa(1-f^*)}{R}\right) - \left(\frac{R-\kappa}{R}\right) f_{\min}\right], \tag{3.25}$$

with $f^* = f_{\min}$ or $f^* = 1$, according to whether the policymaker is old or young, respectively. Intuitively, this condition requires that the weight of land in utility, ψ, is sufficiently large to increase the demand for land and its price to the point where a young agent would like to borrow more than the downpayment constraint allows him to do. A large government, as proxied by w^g and b^g, tends to reduce the value of land and makes it less likely that the agent is constrained.

Summary of Results The analysis thus far illustrates two main points that will hold in the more general environment we consider in Section 4:

1. Under perfect capital markets, the welfare of both old and young agents is independent of a locality's pension funding policy.

2. With a binding downpayment constraint, the welfare of both generations depends on the pension funding policy. Specifically, a pay-as-you-go system maximizes the old generation's utility by increasing the *current* price of land, while a policy of full funding is optimal from the perspective of the young generation because it leads to a higher *future* price of land.

[23] We show how to derive this condition in Appendix A.1.

3.3 Magnitude of Land Price Effects

The analysis of Section 3.2 makes it clear that, all else equal, localities where the old generation controls the pension funding policy are characterized by *higher current* land prices and *lower future* land prices than localities in which the young generation controls the pension funding policy. Formally, consider two localities that enter a period with the same funding ratio f and that differ in their pension funding policy going forward. In one locality the pension funding policy going forward is $f^* = 0$ (no funding), while in the other it is $f^* = 1$ (full funding). Then, the *current* price of land is larger in the locality that does not fund pensions going forward:

$$Q(f;0) > Q(f;1), \tag{3.26}$$

while the *future* price of land is higher in the locality that fully funds pensions going forward:[24]

$$Q(0;0) < Q(1;1). \tag{3.27}$$

The closed-form expressions for land prices can be used to provide a quantitative estimate of the impact of pension funding policies on land prices. Manipulation of equation (3.20) allows us to write:

$$Q(f;0) - Q(f;1) = \frac{b^g}{R} + \frac{\kappa}{R}\left(Q(0;0) - Q(1;1)\right), \tag{3.28}$$

while the gap in future land prices is:

$$Q(0;0) - Q(1;1) = -\left(\frac{R-1}{1-\kappa/R}\right)\frac{b^g}{R}. \tag{3.29}$$

Notice that these price differences depend only on three parameters in our model, R, b^g, and κ. In order to obtain an idea of the magnitudes involved, consider a city like Chicago, where pension and retiree health care promises per household (b^g/R in the model) are about

[24] Notice that if the downpayment constraint does not bind, it is always the case that the funding policy going forward has no effect on current land prices, $Q(f;0) = Q(f;1)$. Even in this case, of course, a locality that enters the period with a higher funding ratio will have a higher price of land $Q(1;1) > Q(0;0)$.

$\$24,000$.[25] Assume that the risk-less interest rate at which the city can save is about one percent per year and that a model period corresponds to 20 years, so that the compound interest rate is $R = 1.22$. Last, consider a scenario in which a constrained individual can only borrow up to 80 percent of the future value of housing, so $\kappa = 0.8$.[26]

The model then implies that the gap in current land prices between a locality that pursues a policy of no pension funding forever onward and one that fully funds pensions forever onward is:

$$Q(f;0) - Q(f;1) \approx \$13,878.$$

This figure is the net effect of the two opposing forces in equation (3.28). On the one hand, property taxes per household in the locality that does not fund its pensions ($f^* = 0$) are $b^g/R = \$24,000$ lower than in the locality that fully funds its pensions ($f^* = 1$). This difference in property taxes is fully reflected in the difference in current land prices (first term on the right-hand side of equation (3.28)). On the other hand, the future land price in the $f^* = 0$ locality is smaller than in the $f^* = 1$ locality by an amount given by equation (3.29):

$$Q(0;0) - Q(1;1) \approx -\$15,337.$$

Given that in this example a dollar decline in the future land price reduces the current price of land by $\kappa/R \approx 0.66$ dollars, the lower future price of land in the locality that never funds pensions contributes to reduce its current land price by about \$10,122 (obtained as $0.66\,(\$15,337)$). Therefore, the difference in the current price of land between the two localities - equal to $\$13,878$ - in equation (3.28) is obtained by subtracting \$10,122 from \$24,000. To put this number in perspective, average housing values in Chicago in 2009 (the year in which unfunded liabilities are measured by the Pew Charitable Trust) were about \$260,000, with land accounting for about 23 percent of home values (Davis and Palumbo,

[25]This figure comes from the Pew Charitable Trusts (2013, Exhibits 2 and 3) data on unfunded liabilities (about \$26,000 million) and Census estimates of the number of households in the City of Chicago in 2009 (about 1.06 million households).

[26]Notice that, for the formulas above to apply, condition (3.25) has to be verified so that the downpayment constraint binds. In Appendix A.2 we provide a calibration of the remaining parameters that guarantees that this is indeed the case.

2007). We conclude this section by pointing out that these numbers are mostly illustrative back-of-the-envelope calculations as our model is highly stylized.

3.4 Extensions

The benchmark model is admittedly stylized in order to allow us to illustrate the key insight of the paper as clearly as possible. In this section we consider two important generalizations of the model and show that our main results apply in more general environments.[27] In the first one, we allow old agents to consume housing services. In the second one we assume that policy is set in order to maximize a weighted average of the utilities of young and old agents, as implied by a probabilistic voting model. A complete analysis of both cases would require us to take into account bond holdings of old agents as a new state variable of the model. In fact, both the old agent's choice of renting land as well as the utility comparison between young and old agents depend on the bond position of the old. In order to avoid increasing the number of state variables and significantly complicate the algebra, in the versions of the model with a downpayment constraint, we rule out borrowing all together and assume that $\kappa = 0$.

3.4.1 Old Agents Value Housing

In the benchmark version of the model, old agents sell the land in their possession and use the proceedings to pay back their debt and consume. Differently from young agents, they do not consume housing services. In this section we extend the benchmark model and assume that preferences take the more general form:

$$U\left(c_{yt}, l_{yt}, c_{ot+1}, l_{ot+1}\right) = \left(1 - \psi - \beta - \beta\theta\right) \ln c_{yt} + \psi \ln l_{yt} + \beta \left(\ln c_{ot+1} + \theta \ln l_{ot+1}\right),$$

so that, relative to equation (3.1), old agents care about consumption of land services l_{ot+1}. The timing of this version of the model is such that after land is sold by the old to the young,

[27] In Appendix C.1 we discuss alternative specifications of the credit constraint.

a rental market for land opens in which old agents may rent land at the rental rate r_t. In the agents' budget constraints, it is now necessary to distinguish between land ownership l_t and land consumption when young and old, l_{yt} and l_{ot+1} :

$$w + r_t (l_t - l_{yt}) = c_{yt} + (1 + \tau_t) q_t l_t + \frac{b_{t+1}}{R}, \tag{3.30}$$

$$c_{ot+1} + r_{t+1} l_{ot+1} = q_{t+1} l_t + b_{t+1}. \tag{3.31}$$

Constraint (3.30) reflects the fact that a young agent receives rental income corresponding to the land she owns but does not consume, or $l_t - l_{yt}$. The downpayment constraint depends on the quantity of land l_t that a young agent purchases and therefore remains the same as in equation (2.4). Equilibrium in the market for ownership of land requires that $l_t = 1$, while equilibrium in the rental market for land requires that total consumption of land services at a point in time does not exceed the available stock:

$$l_{yt} + l_{ot} = 1. \tag{3.32}$$

If the downpayment constraint (2.4) does not bind, the optimal choice of l_t implies that the user cost of land and its rental rate have to be equal to each other in equilibrium. The equilibrium condition (3.32) pins down the equilibrium user cost, which is a constant, independent of the locality pension funding policy. In other words, with a frictionless asset market, the results in Proposition 1 apply also to the version of the economy in which old agents consume land services.

If the downpayment constraint (2.4) binds, instead, the decision to purchase land cannot be considered independently of the decision to consume it. The expressions for the demand for land as an asset and for land services are:

$$
\begin{aligned}
l_t &= \frac{\beta(1+\theta)w}{(1+\tau_t)q_t - r_t}, &(3.33)\\
l_{yt} &= \frac{\psi w}{r_t}, &(3.34)\\
l_{ot+1} &= \frac{\theta q_{t+1}}{(1+\theta)r_{t+1}}, &(3.35)
\end{aligned}
$$

where we have already taken into account the fact that $\kappa = 0$. Replacing equations (3.33)–(3.35) into the market clearing conditions (2.7) and (3.32), and taking into account the government's budget constraint, yields the equilibrium price of land as a function of the locality's pension funding policy:

$$
q_t = (1+\theta)\left\{(\psi+\beta+\theta\beta)w - w^g - \frac{f_{t+1}b^g}{R} - b^g(1-f_t)\right\}. \tag{3.36}
$$

Notice that, in the same vein as in Section 3.2, a higher funding level for next period reduces the current price of land, while a higher current funding level increases it.[28] The market clearing condition (3.32) gives the expression for the rental rate of land as a function of q_t:

$$
r_t = w\psi + \frac{\theta q_t}{1+\theta}. \tag{3.37}
$$

A higher asset price of land is associated, in equilibrium, with a higher rental rate because it increases the wealth of old agents and their demand for land services. A higher rental rate is therefore needed to re-establish equilibrium in the rental market.

In this version of the model, both old and young agents care about the rental price of land, in addition to its asset price. Specifically, the utilities of old and young agents in period t are (up to some irrelevant constants):

$$
\begin{aligned}
V_t^{\text{old}} &\approx (1+\theta)\ln q_t - \theta \ln r_t, &(3.38)\\
V_t^{\text{young}} &\approx -\psi \ln r_t + \beta V_{t+1}^{\text{old}}. &(3.39)
\end{aligned}
$$

[28]Notice that the simplification $\kappa = 0$ makes the expression for q_t independent of q_{t+1}.

From equation (3.38), it appears that old agents might not necessarily want to choose the minimum funding policy because, due to $\theta > 0$, they benefit from reducing the current rental price of land. However, in equilibrium the asset price of land and its rental price are positively related - see equation (3.37) - so old agents face a trade-off. Replacing equation (3.37) into (3.38) and differentiating V_t^{old} with respect to q_t it can be shown that the utility of the old is in fact monotonically increasing in q_t. What's going on is that the positive wealth effect associated with a higher q_t allows an old agent to increase her consumption of both goods and land services, while reducing r_t allows her to only consume more land services.

As far as the young are concerned, instead, the inclusion of a rental market for land provides an additional incentive for young agents to want to fully fund the pension system. This policy lowers the current price of land and, through equation (3.37), reduces the current rental price. It also leads to higher future land prices, as was the case in the benchmark ($\theta = 0$) version of the model. Both effects work in the direction of increasing the young's utility, as it is clear from equation (3.39). We conclude this section by summarizing our results in the following proposition.

Proposition 3 (Old agents consume land services) *Consider the case $\kappa = 0$. In the version of the model in which old agents care about consumption of land, the results of Proposition 2 apply.*

3.4.2 Probabilistic Voting

In the benchmark version of the model, political power is concentrated in either the hands of young or old agents. A potential drawback of concentrating all political power in the hands of one group is that it makes it impossible to conduct smooth comparative statics, such as marginally increasing the power of one group at the expense of another. In order to overcome this drawback, in this section we consider a generalization of our politico-economic model that features probabilistic voting (Persson and Tabellini, 2002, Chapter 2). Specifically, we assume that in each period two candidates, denoted by A and B, compete for office by announcing pension funding policies $\widetilde{f}'_A$ and $\widetilde{f}'_B$. Candidates are able to commit to these policy platforms and select them in order to maximize the chance of being elected. In order to allow a candidate's vote share to vary continuously with its announced funding policy we also assume that each voter i, young or old, has idiosyncratic preferences ε_i for candidate B. We assume that ε_i is distributed as a uniform random variable with support $[-\eta^{\text{young}}, \eta^{\text{young}}]$, if i is a young voter, and with support $[-\eta^{\text{old}}, \eta^{\text{old}}]$, if i is an old voter. Recall from equations (3.6) and (3.18) that in the version of the model with binding downpayment constraints, young voters' indirect utility depends positively on the logarithm of *future* land prices, while old voters' indirect utility depends positively on the logarithm of *current* land prices. It follows that if voter i is young, she prefers candidate A to candidate B if and only if:

$$\beta \ln Q\left(\widetilde{f}'_A; F\right) > \beta \ln Q\left(\widetilde{f}'_B; F\right) + \varepsilon_i,$$

while if voter i is old she prefers candidate A to candidate B if and only if:[29]

$$\ln \widetilde{Q}\left(f, \widetilde{f}'_A; F\right) > \ln \widetilde{Q}\left(f, \widetilde{f}'_B; F\right) + \varepsilon_i.$$

The measure of voters favoring candidate A is therefore:

$$\frac{1}{2} + \frac{1}{4\eta^{\text{old}}}\left(\ln \widetilde{Q}\left(f, \widetilde{f}'_A; F\right) - \ln \widetilde{Q}\left(f, \widetilde{f}'_B; F\right)\right) + \frac{1}{4}\frac{\beta}{\eta^{\text{young}}}\left(\ln Q\left(\widetilde{f}'_A; F\right) - \ln Q\left(\widetilde{f}'_B; F\right)\right),$$

while support for candidate B is simply one minus the expression above. This specification implies that, in order to maximize their vote share, each candidate A and B will independently select a policy $\widetilde{f}'$ in such as way as to maximize the following weighted average of the utilities of old and young agents:

$$\alpha \ln \widetilde{Q}\left(f, \widetilde{f}'; F\right) + (1-\alpha)\, \beta \ln Q\left(\widetilde{f}'; F\right),$$

[29]Notice that the assumption of no borrowing guarantees that an old agent's utility depends only on the price of land and not on the amount she had borrowed when young. Therefore, we do not need to keep track of debt as a state variable of the problem.

where α is the weight received by an old agent.[30] The definition of politico-economic equilibrium (Definition 3) can be generalized in a straightforward manner by replacing the consistency requirement (point 3) with the following one:

$$F(f) = \arg\max_{\widetilde{f}'} \left\{ \alpha \ln \widetilde{Q}\left(f, \widetilde{f}'; F\right) + (1-\alpha)\beta \ln Q\left(\widetilde{f}'; F\right) \right\} \tag{3.40}$$

for all f. In the next proposition we show that this economy admits a politico-economic equilibrium in which both the equilibrium policy rule and the equilibrium land pricing function are affine functions of the state variable f.

Proposition 4 (Probabilistic voting) *Consider the case $\kappa = 0$. Then, the probabilistic voting version of the model admits a politico-economic equilibrium characterized by the following policy rule:*

$$F(f) = \frac{R\delta}{R-1}(\phi - 1) + \phi f, \tag{3.41}$$

where δ and ϕ are composite functions of the model's parameters:

$$\begin{aligned} \delta &\equiv (\psi + \beta)\frac{w}{b^g} - \frac{w^g}{b^g} - 1, \\ \phi &\equiv \frac{(1-\alpha)\beta R}{\alpha(1-\kappa) + (1-\alpha)\beta}. \end{aligned} \tag{3.42}$$

The equilibrium policy rule in equation (3.41) has a number of interesting properties. First, the pension funding policy selected in a period now depends on the beginning-of-the-period level of funding f. The reason for this dependence is that if the economy enters a period with a better-funded pension system (i.e. a higher f), the current land price is higher and therefore the marginal utility of old voters is lower. As a response, both candidates tilt the funding policy in the direction preferred by young agents. By selecting a higher $\widetilde{f}'$, in

[30]Notice that α is defined as:

$$\alpha \equiv \frac{\left(\eta^{\text{old}}\right)^{-1}}{\left(\eta^{\text{old}}\right)^{-1} + \left(\eta^{\text{young}}\right)^{-1}}.$$

Thus, a group of voters with a more dispersed idiosyncratic taste will receive a smaller weight in the policymaker's objective function.

fact, the marginal utility of young voters falls and the marginal utility of old voters increases until their original ratio is re-established. Second, it is straightforward to notice from the definition of ϕ in (3.42) that, given f, increasing the political power of old voters through a higher α leads to a smaller equilibrium funding level $F(f)$ for the following period. Thus, this version of the model allows us to make comparative statics predictions regarding the effect of varying the political power of young and old voters on equilibrium funding levels. These predictions conform well with the intuition gathered from the benchmark version of the model. They will form the basis of our empirical analysis in Section 5. Last, the equilibrium policy rule in equation (3.41) is such that, over time, the funding policy converges to either a situation of full funding or to the minimum funding level $f_{\min}$. Notice, in fact, that there are two cases to consider according to whether the composite parameter ϕ in equation (3.42) is larger than one or smaller than one. In the former case, which prevails when the political power of the young is sufficiently high (i.e. α is relatively low), the intercept of the function $F(f)$ is positive and the slope is larger than one. Therefore, in this case the policy rule $F(f)$ exceeds one in finite time. In the latter case, instead, $\phi < 1$ and the intercept of the function $F(f)$ is negative while its slope is smaller than one. In this case, the policy rule $F(f)$ will hit its lower bound $f_{\min}$ in finite time. Thus, for any arbitrary non-degenerate political weight $\alpha \in (0, 1)$, the politico-economic equilibrium will converge over time towards one of the two extremes, corresponding to situations in which either the old or the young have all political power.[31] The intuition for this "divergence" of policy outcomes is as follows. Consider the case in which young agents have enough political clout that $\phi > 1$. The policy rule dictates that in this case $F(f) > f$. Since the level of funding increases, next period's old (i.e. the current young) will be able to sell land at a higher price to the new young. As a result their marginal utility of consumption falls, and the candidates optimally select an even higher level of pension funding in order to realign the marginal utilities of young and old. This process continues and perpetuates itself. A similar logic applies to the case $\phi < 1$.

[31] Notice that there is a value of α such that $\phi = 1$, in which case the policy rule is constant over time and such that $F(f) = f$ for all f. That is, any initial funding level is forever preserved. Since this case has measure zero, we ignore it in the main text.

4 The General Case

In this section we consider the case in which the utility function takes the general form in equation (2.1), instead of the logarithmic specification of the previous section. Also in the more general case, absent the downpayment constraint, a policymaker would not be able to affect the utility of either the old or the young generation by manipulating public pension funding. Intuitively, with a perfect capital market, the increase in current taxes associated with a higher level of pension funding is perfectly compensated by the decline in the future price of land induced by higher future taxes.[32] In what follows, we therefore focus on the situation in which the downpayment constraint binds.[33]

4.1 Land Prices and the Indirect Utility Function

In this section we derive the dynamic expression for land prices and the indirect utility function of a young agent and show how they generalize when the utility function takes the form (2.1). Replacing the budget constraints (2.2) and (2.3) into the objective function (2.1), and taking into account the downpayment constraint in equation (2.4), the agent's optimization problem can be written compactly as:

$$L\left(d_t, q_{t+1}\right) = \underset{l \in [0, w/d_t]}{\arg\max}\, U\left(w - d_t l, l, q_{t+1} l \left(1-\kappa\right)\right). \tag{4.1}$$

The function $L\left(d_t, q_{t+1}\right)$ denotes the quantity of land demanded as a function of the down-payment per unit of land d_t and the price of land next period q_{t+1}. The indirect utility

[32] We formally state and prove this result in Appendix B.1.

[33] Providing general sufficient conditions on the model's parameters such that this is the case, both in equilibrium and following a deviation, is difficult because whether the constraint binds or not depends on the equilibrium land prices. In Appendix B.3 we provide sufficient conditions for a binding downpayment constraint that can be verified given a specific functional form and parameter values. A special case in which the downpayment constraint is always binding is one in which consumption when old is not valued at all, or $v'\left(c_o\right) = 0$.

function associated with the optimization problem (4.1) is defined as follows:

$$V(d_t, q_{t+1}) = U(w - d_t L(d_t, q_{t+1}), L(d_t, q_{t+1}), q_{t+1} L(d_t, q_{t+1})(1-\kappa)). \qquad (4.2)$$

Notice that we express $L(d_t, q_{t+1})$ and $V(d_t, q_{t+1})$ as a function of the downpayment per unit of land d_t and its future price q_{t+1} in order to keep comparability with the logarithmic example of Section 3 and to illustrate the separate impact of each variable on land demand and utility. It is useful to keep in mind that with perfect capital markets $L(d_t, q_{t+1})$ and $V(d_t, q_{t+1})$ would only depend on the user cost of land and not independently on its future price q_{t+1}.

A key difference of the general utility case relative to the logarithmic example we considered earlier is that in equilibrium the downpayment depends on the future price of land. In order to characterize this relationship, write down the first order condition associated with problem (4.1) and impose in it the land market equilibrium condition $L(d_t, q_{t+1}) = 1$:

$$-d_t u_1(w - d_t, 1) + u_2(w - d_t, 1) + v'(q_{t+1}(1-\kappa)) q_{t+1}(1-\kappa) = 0. \qquad (4.3)$$

At the optimal demand for land, the marginal utility cost of the downpayment d_t equals the marginal utility of consuming land services plus the marginal utility value of the agent's equity stake in land when old, $q_{t+1}(1-\kappa)$. Assumption 1 guarantees that equation (4.3) is monotonically decreasing in the downpayment d_t. It follows that it can be implicitly expressed as:

$$d_t = g(q_{t+1}), \qquad (4.4)$$

where the function g maps the future price of land q_{t+1} into the unique downpayment d_t such that the land market is in equilibrium.[34] We use the relationship between d_t and q_{t+1}

[34] In general, due to the ambiguous relationship between q_{t+1} and the demand for land, the function $g(.)$ in (4.4) might be either increasing, decreasing, or independent of q_{t+1}. For example, in the logarithmic utility case of Section 3.2, the function $g(q_{t+1})$ is independent of q_{t+1}, and takes the simple form:

$$d_t = d = (\psi + \beta) w.$$

in equation (4.4) to derive the equilibrium equation that land prices have to satisfy. To do so, replace the definition of d_t (equation (2.6)) and the property tax revenue $\tau_t q_t$ (equation (2.8)) into equation (4.4) and solve for to q_t:

$$q_t = -w^g - \frac{f_{t+1} b^g}{R} - b^g (1 - f_t) + \frac{\kappa q_{t+1}}{R} + g(q_{t+1}). \tag{4.5}$$

This is the equilibrium law of motion for the current price of land as a function of its future value and the locality's pension funding policy. Notice that the only difference relative to its counterpart in the logarithmic case of Section 3.2, equation (3.19), is the presence of the term $g(q_{t+1})$ on the right-hand side of equation (4.5).

It is also convenient to re-write the equilibrium lifetime utility of a young agent as a function of q_{t+1} only, replacing $d_t = g(q_{t+1})$ and $l_t = 1$ into equation (4.2):

$$\overline{V}(g(q_{t+1}), q_{t+1}) = U(w - g(q_{t+1}), 1, (1 - \kappa) q_{t+1}). \tag{4.6}$$

Formally, $\overline{V}(g(q_{t+1}), q_{t+1})$ represents the indirect utility function (4.2) evaluated at the downpayment $d_t = g(q_{t+1})$ such that the land market is in equilibrium.

In what follows we use an important property of the function $g(.)$, summarized in Lemma 1 below, to first show that young agents' lifetime utility $\overline{V}(g(q_{t+1}), q_{t+1})$ is strictly increasing in q_{t+1} and later to prove that the equilibrium price of land declines following an increase in pension funding (Proposition 7). For convenience, let the marginal rate of substitution between young and old age consumption be denoted by

$$MRS_t^c \equiv \frac{v'(q_{t+1}(1 - \kappa))}{u_1(w - d_t, 1)}.$$

Then, we can prove that the derivative of the function $g(q_{t+1})$ is bounded from above as follows.

Lemma 1 *The function $g(q_{t+1})$ is such that:*

$$g'(q_{t+1}) < (1-\kappa)\, MRS_t^c. \tag{4.7}$$

Lemma 1 is instrumental in showing that indirect utility is increasing in q_{t+1}, even taking into account the equilibrium relationship between d_t and q_{t+1} in equation (4.4).

Proposition 5 *The indirect utility function in equation (4.6) is strictly increasing in q_{t+1}.*

Thus, from the perspective of young agents, maximizing lifetime utility is the same as maximizing the price of land when they are old. We can now turn to the recursive representation of the equilibrium pricing equation (4.5) in order to characterize its dependence on the locality's pension funding policy.

4.2 Recursive Formulation and Characterization of Politico-Economic Equilibrium

Using the same notation as in Section 3, we can re-write equation (4.5) in recursive form as:

$$Q(f;F) = -w^g - \frac{F(f)b^g}{R} - b^g(1-f) + \frac{\kappa Q(F(f);F)}{R} + g(Q(F(f);F)). \tag{4.8}$$

Similarly, the price of land following a deviation from equilibrium takes the form:

$$\widetilde{Q}\left(f,\widetilde{f}';F\right) = -w^g - \frac{\widetilde{f}'b^g}{R} - b^g(1-f) + \frac{\kappa}{R}Q\left(\widetilde{f}';F\right) + g\left(Q\left(\widetilde{f}';F\right)\right). \tag{4.9}$$

These equations are the analogs of equations (3.20) and (3.12) when the utility function takes the general form (2.1). Notice that the only significant difference is the presence of the term $g(.)$ on the right-hand side of the equations above. In what follows we characterize the equilibrium of the model analytically. We first show in Section 4.2.1 that the only feasible equilibrium funding rule is a constant and characterize the model's equilibrium given an arbitrary (and constant) funding rule $f^* = F(f)$. In Section 4.2.2, we consider the equilibrium after a one-period deviation $\widetilde{f}'$ from F and solve for the equilibrium without commitment.

4.2.1 Equilibrium Given a Constant Funding Rule

In this section we show that, similarly to the logarithmic case, the only feasible equilibrium pension funding rule must be a constant, or $F(f) = f^*$ for all f. Consider first the case in which the policymaker is a young agent. By virtue of Proposition 5 a young policymaker seeks to maximize the future price of land $Q\left(\widetilde{f}'; F\right)$. Since the latter depends only on $\widetilde{f}'$ and not on f, the solution to this problem must be independent of f as well. The old policymaker seeks to maximize current land prices in equation (4.9). Different from the future price of land, the current price in equation (4.9) depends on f. However, it depends on f in a way that does not interact with $\widetilde{f}'$, so the optimal $\widetilde{f}'$ is also independent of f. The following proposition summarizes these results.

Proposition 6 (Constant funding rule) *The only possible politico-economic equilibrium of this economy is one in which the funding rule is a constant $F(f) = f^*$ for all f.*

In light of this proposition, we solve for the equilibrium of the economy given a constant funding policy $f^* = F(f)$ for all f. In this case, the current price of land $Q(f; f^*)$ in equation (4.8) is an affine function of the state f and depends on the constant future land price $Q^* = Q(f^*; f^*)$. The latter is a fixed-point of equation (4.8):

$$Q^* = -w^g - \frac{f^* b^g}{R} - b^g(1 - f^*) + \frac{\kappa Q^*}{R} + g(Q^*). \tag{4.10}$$

To discuss policymakers' incentives to fund pensions, in the next section we discuss the impact of a one-period policy deviation from f^* on equilibrium current and future prices.

4.2.2 Effects of a One-Period Deviation and Politico-Economic Equilibrium

Starting from the equilibrium of the model under a constant policy f^*, consider a current-period deviation $\widetilde{f}'$ by the policymaker. Since the equilibrium funding rule is the constant f^*, the current deviation has no impact on future funding. Following a deviation, the equilibrium current price of land $\widetilde{Q}\left(f, \widetilde{f}'; f^*\right)$ is given by equation (4.9). The current price of land depends on the future price of land $Q\left(\widetilde{f}'; f^*\right)$. The latter is given by equation (4.8) with state variable $\widetilde{f}'$ instead of f because next period, the location will have to finance the unfunded portion $1 - \widetilde{f}'$ of pension promises made this period:

$$Q\left(\widetilde{f}'; f^*\right) = -w^g - \frac{f^* b^g}{R} - b^g\left(1 - \widetilde{f}'\right) + \frac{\kappa Q^*}{R} + g\left(Q^*\right). \tag{4.11}$$

From this equation it is straightforward to show that a marginal increase in $\widetilde{f}'$ leads to an increases in the future price of land:

$$\frac{\partial Q\left(\widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = b^g > 0. \tag{4.12}$$

Therefore, a young policymaker would simply set the optimal deviation to $\widetilde{f}' = 1$. In other words, the young policymaker would fully fund pensions in order to maximize the *future* price of land and therefore her lifetime utility.

An old policymaker, instead, would set $\widetilde{f}'$ to maximize the *current* price of land $\widetilde{Q}\left(f, \widetilde{f}'; f^*\right)$. To evaluate the effect of $\widetilde{f}'$ on the latter, take the derivative of equation (4.9) with respect to $\widetilde{f}'$, taking into account equation (4.12):

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = \underbrace{-\frac{b^g}{R}}_{\text{current taxes}} + \underbrace{\frac{\kappa b^g}{R}}_{\text{borrowing}} + \underbrace{g'\left(Q\left(\widetilde{f}'; F\right)\right) b^g}_{\text{effect of resale value of land}}. \tag{4.13}$$

The net effect of a policy deviation $\widetilde{f}'$ on the current price of land depends on the three terms on the right-hand side of equation (4.13). The first two terms are also present in the logarithmic case of Section 3.2, see equation (3.22). To summarize, the first term represents the effect of the higher current taxes associated with an increase in $\widetilde{f}'$ on the price of land. A marginal increase in $\widetilde{f}'$ causes current property taxes to increase by b^g/R. The latter are capitalized in (lower) contemporaneous land prices on a one-for-one basis. The second term (labelled "borrowing") captures the fact that a young agent can borrow $\kappa b^g/R$ units of consumption as a response to a reduction in future taxes by b^g, because the price of land when old increases by b^g as well.

The third term (labelled "effect of resale value of land") on the right-hand side of equation (4.13) reflects the fact that, with a general utility function, an increase in future land prices affects the demand for land. The sign of the third effect can be either positive, if the demand for land is increasing in its future price, or negative, if it is decreasing. Even if the sign of this effect is positive, however, Lemma 1 guarantees that its magnitude is bounded from above by $(1-\kappa)\, MRS_t^c$. As a consequence, the overall effect of an increase in $\widetilde{f}'$ on the current price of land is negative. To make this point formally, notice that Lemma 1, states that:

$$g'\left(Q\left(\widetilde{f}';F\right)\right) < (1-\kappa)\, MRS_t^c. \tag{4.14}$$

Taking this upper bound into account, we conclude that:

$$\frac{\partial \widetilde{Q}\left(f,\widetilde{f}';f^*\right)}{\partial \widetilde{f}'} < -\frac{b^g\,(1-\kappa)}{R}\,(1-R\times MRS_t^c) < 0, \tag{4.15}$$

where the first inequality follows from Lemma 1 and the second one is due to the fact that $R\times MRS_t^c < 1$ because the agent is constrained. It follows from this discussion that the optimal deviation from equilibrium by an old agent is to set pension funding $\widetilde{f}'$ to its minimum allowed value $f_{\min}$ in order to increase the *current* price of land.

Imposing the consistency conditions (3.23) and (3.24), we can easily solve for the equilibrium funding policy under our two alternative assumptions about the identity of the policymaker.

Proposition 7 (Equilibrium policies) *Under a general utility function that satisfies Assumption 1:*

(1) If an old agent sets the funding policy, the only politico-economic equilibrium is one in which pension funding is the minimum allowed, or $f^* = f_{\min}$.

(2) If a young agent sets the funding policy, then the only politico-economic equilibrium is one in which pensions are fully funded, or $f^* = 1$.

The tension between the interests of the old and young generations with regard to funding pensions bears implications for the distributional effects of policies that would force the local government to increase pension funding. One such policy - an increase in $f_{\min}$ - would hurt the first old generation, by reducing the *current* price of land, and improve the welfare of all subsequent generations, by increasing its future price.[35]

5 Empirical Evidence

The version of our model with binding downpayment constraints predicts that, all else equal, localities where political power is concentrated in the hands of young agents have relative lower levels of unfunded liabilities (Proposition 7). In this section, we show that correlations in the data are broadly consistent with this prediction. In our empirical analysis, we use the share of young homeowners in a locality as a proxy for their political power. We draw data from a couple of sources. The first is a report prepared by Munnell and Aubry (2016) that calculates unfunded actuarial accrued liabilities (UAAL) for 173 large U.S. cities.[36] This report also provides data on the ratio of UAAL to own-source revenues. In addition, we use data from the U.S. Census to calculate city-level measures of age distribution, home ownership, home values, income, population density, and population growth.[37]

[35]An example of an increase in $f_{\min}$ is the adoption by some U.S. states of the concept of annual required contribution (ARC) in their statutes and laws. The latter provides an estimate of the flow contributions needed to "adequately" fund a defined benefits pension plan. Brainard and Brown (2015) show that states that have encoded the ARC in their statutes and laws have in recent years made larger contributions to their defined benefits pension plans than states where pension contributions are left at the discretion of plan administrators and policymakers.

[36]The authors of the report collected data from the 2012 Comprehensive Annual Financial Reports of 173 selected U.S. cities. The cities were selected to create a sample that included large cities in each state and provide some variation in institutional arrangements. The authors calculated UAAL using new Governmental Accounting Standards Board guidelines implemented in 2015, which require more transparent reporting of pension liabilities by local municipalities.

[37]Data was drawn from the 2012 American Community Survey and the 1980 Decennial Census. Because of differences in municipal definitions, a few cities were dropped when merging the data, leaving a sample of 168 cities.

We consider four alternative measures of unfunded pension liabilities: UAAL per capita, UAAL divided by aggregate income, UAAL divided by own-source revenues, and UAAL divided by aggregate housing value. As a share of young homeowners, we use the percentage of total households that are headed by owners under 55 years old. Table 1 provides summary statistics of the data on unfunded liabilities and young homeowner households.

Table 1: Summary of Data

	Mean	Std. Dev.	Min	Max
UAAL/population ($)	1,446	1,416	-463	8,775
UAAL/annual income (%)	6.21	6.11	-1.22	29.74
UAAL/annual revenue (%)	71.6	66.3	-17.0	359.0
UAAL/house value (%)	3.81	4.18	-0.71	22.63
Owners under 55 (%)	28.4	6.5	12.0	52.5

Source: Data on UAAL and revenues are from Munnell and Aubry (2016). Demographic data are drawn from the 2012 Annual Community Survey.

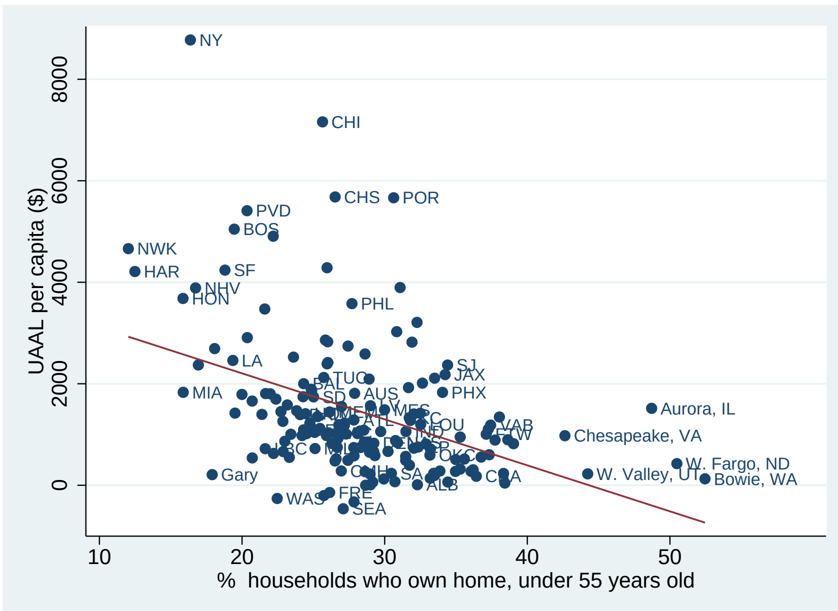
Unfunded liabilities vary greatly across cities. The average unfunded liabilities per capita are $\$1,446$ and range from a high of $\$8,775$ in New York to a surplus of $463 in Seattle. The pattern is similar when liabilities are measured against aggregate income, own-source revenues, and aggregate house values.[38] On average, 28.4 percent of households in cities are headed by homeowners under the age of 55 with considerable cross-sectional dispersion.

Figure 1 plots UAAL per capita against the measure of young homeowners for each city in the sample. There is a strong negative correlation between these two variables with a correlation coefficient of -0.40 and standard error of 0.07.

To further investigate this correlation we run cross-city regressions where the dependent

[38] The rank correlation coefficient across our four measures of unfunded liabilities is always above 0.80.

Figure 1: UAAL and Young Homeownership



Notes: This figure plots the percentage of households who own their home and are headed by someone under 55 years old versus UAAL per capita for 168 large U.S. cities.

variable is one of the four measures of unfunded pension liabilities and the independent variable of interest is the share of homeowners under the age of 55. In order to assess the robustness of our findings, we also allow for a number of control variables in some specifications.

Table 2: Determinants of municipal pension funding

	UAAL/population		UAAL/income	
% owners under 55	-90.6**	-39.6**	-0.43**	-0.28**
	(17.5)	(14.0)	(0.08)	(0.07)
controls[a]	NO	YES	NO	YES
R^2	0.17	0.31	0.21	0.34
	UAAL/revenues		**UAAL/house values**	
% owners under 55	-2.55**	-1.86**	-0.29**	-0.27**
	(0.75)	(0.90)	(0.06)	(0.06)
controls[a]	NO	YES	NO	YES
R^2	0.06	0.16	0.20	0.36
Number of cities	168	165	168	165

Robust standard errors in parenthesis. ** p-value<0.05

[a]Controls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies.

The first column of the first panel of Table 2 presents the estimated coefficient on the share of young homeowners when the dependent variable is UAAL per capita. A percentage point increase in the share of young homeowners is predicted to decrease unfunded liabilities by $90.60 per capita. The estimated coefficient is statistically significant at the 5 percent level. Notice that the effect of the share of young homeowners on UAAL is negative and statistically significant independently of how the UAAL variable is standardized, as can be seen in each of the other three panels of Table 2.[39]

The negative correlation between the share of young homeowners and unfunded liabilities could be driven by a number of confounding factors. For example, economically declining cities may be populated by older households and have larger unfunded liabilities. To partially control for alternative mechanisms, in the second column of each panel we include census region dummies, a city's population growth between 2000 and 2012, and a number of other variables (see legend of Table 2). One might also hypothesize that cities populated by younger households have relatively younger public sector employees, reducing overall liabilities and potentially introducing a mechanical relationship between the dependent variable and the share of young homeowners. In order to control for this effect, we also include pension liabilities per capita in the set of control variables. Adding these control variables tends to reduce the absolute magnitude of the coefficient on the share of young homeowners. However, the latter remains negative and statistically significant at conventional levels in all specifications.

We also conducted a variety of additional robustness checks, whose results are reported in Appendix D. First, we used the fraction of homeowners under ages 35, 45 or 65 as explanatory variable and find results similar to those in Table 2. If anything, the correlation between the percentage of young homeowners and the UAAL measure is larger in absolute value if an earlier age cut-off is employed to define a young household. Second, we used fraction of all households (owners and renters) under 35, 45, 55 or 65 as regressor. The findings are similar to our benchmark, but the estimated coefficient is only significant at a conventional level when we use age 55 as the cut-off. Last, we verified that the results are not driven by a number of relatively large cities with particularly low rates of homeownership, such as New York City, San Francisco, and Boston.

[39]It should be pointed out that the measure of young households in a city may, in principle, be endogenous and determined simultaneously with its pension funding policy. As a check, we have run the same regressions with the explanatory variable for young homeowner households measured in 1990 instead of 2012. The magnitude and statistical significance of the relevant coefficients, reported in Table D.1 of the online appendix, is very similar to the estimates of Table 2.

We conclude this section with a caveat. Data limitations restrict the power and scope of the empirical analysis. We do not have access to a panel data set for unfunded liabilities for large cities. But even if one had such a panel, identification is not straightforward. As far as we can tell, there are no obvious policy changes or "natural experiments" that one could exploit here for identification. Moreover, there would still be a need to instrument for the demographic composition of the city, using historical lags or a Bartik-type instrument. While the results of this section provide some support for the predictions of our model, better data and more empirical work is needed to fully answer the questions raised by our theoretical analysis.

6 Conclusions

In this paper, we have explored the determinants of funding of municipal pension plans using a new dynamic politico-economic model. The key insight of the model is that pension funding policies produce distributional effects across generations if agents are subject to binding downpayment constraints when purchasing land. In such a situation, young and old policymakers disagree on the funding policy to pursue, with the former favoring full funding and the latter favoring a pay-as-you-go system. As a result, state-wide policies that mandate binding minimum funding levels hurt the initial old and benefit subsequent cohorts. By contrast, pension funding policies are inconsequential for agents' utilities if the capital market is frictionless. Empirical results based on cross-city comparisons of the magnitude of unfunded liabilities are consistent with the key prediction of the model with binding downpayment constraints.

We conclude with some more general lessons of our analysis and then discuss avenues for future work. One important lesson is that increasing government debt might make young constrained households worse off. This stands in contrast to a standard result in public finance whereby a shift of taxes towards the future alleviates constrained households' borrowing constraints, allowing them to increase current consumption (see, e.g., Yared, 2015).

The reason for this reversal is the adjustment of land prices: following a shift of taxes toward the future, young households increase their demand for land, pushing up land prices. The second lesson is that, in the presence of frictions such as downpayment constraints, land price capitalization might not be sufficient to insulate young generations from the government financing choices made by old generations. More generally, our results suggest that with binding constraints, land price capitalization might not provide sufficient incentives for old generations to invest efficiently in durable public goods (e.g., Conley and Rangel, 2001).

Our analysis can be fruitfully extended in at least three important dimensions. First, we hypothesize that capitalization effects are more likely to operate at the city level, rather than at the state level, because the supply of land is less constrained in the latter case and states rely on income and sales taxes, rather than property taxes, to fund their expenditures. Novy-Marx and Rauh (2009, 2011) have calculated that state governments' unfunded liabilities amount to \$3 trillion, against approximately \$1 trillion of their outstanding debt. Studying the welfare implications of states' unfunded liabilities is therefore an important area for future research. Second, and related to the previous point, we conjecture that land price capitalization effects are more important in cities with a relatively less elastic supply of land. Therefore, it would be interesting - although potentially difficult - to allow the supply of land to respond endogenously to changes in land prices. A third important issue is the extent to which localities might, in the future, be able to change ex-post some of the terms of their pension promises. In our model, localities are assumed to be able to commit to defined pension benefits. Allowing for renegotiation, or even outright default, is another interesting, although not straightforward, extension of our analysis.

References

Albrecht, K. (2012). "Capitalization of Unfunded Public Pension Liabilities," Working Paper, Cornell University.

Azzimonti, M. (2011), "Barriers to Investment in Polarized Societies," *American Economic Review*, Vol. 101, No. 5, pp. 2182-2204.

Azzimonti, M., M. Battaglini, and S. Coate (2016). "The Costs and Benefits of Balanced Budget Rules: Lessons from a Political Economy Model of Fiscal Policy," *Journal of Public Economics*, Vol. 136, pp. 45–61.

Bagchi, S. (2013). "The Effects of Political Competition on the Funding and Generosity of Public-Sector Pension Plans." Working Paper.

Bajari, P., P. Chan, D. Krueger, and D. Miller (2013). "A Dynamic Model of Housing Demand: Estimation and Policy Implications," *International Economic Review*, Vol. 54, No. 2, pp. 409–442.

Barseghyan L., and S. Coate (2015). "Property Taxation, Zoning, and Efficiency in a Dynamic Tiebout Model," *American Economic Journal: Economic Policy*, forthcoming.

Bassetto, M. and T. Sargent (2006). "Politics and Efficiency of Separating Capital and Ordinary Government Budgets," *The Quarterly Journal of Economics*, Vol. 121, No. 4, pp. 1167-1210.

Bassetto, M. (2008): "Political Economy of Taxation in an Overlapping-Generations Economy," *Review of Economic Dynamics*, Vol. 11, No.1, pp. 18–43.

Battaglini, M., and S. Coate (2008). "A Dynamic Theory of Public Spending, Taxation, and Debt," *American Economic Review*, Vol. 98, No. 1, pp. 201–236.

Bohn, H. (2011). "Should Public Retirement Plans Be Fully Funded?" *Journal of Pension Economics and Finance* , Vol. 10, No. 02, pp. 195–219.

Bouton L., Lizzeri A., and N. Persico (2016): "The Political Economy of Debt and Entitlements," working paper.

Brainard, K., and A. Brown (2015). Spotlight on the Annual Required Contribution Experience of State Retirement Plans, FY 01 to FY 13. National Association of State Retirement Administrators.

Campbell, J., and Z. Hercowitz (2005). "The Role of Collateralized Household Debt in Macroeconomic Stabilization." NBER Working Paper 11330.

Conley, J., and A. Rangel (2001). "Intergenerational Fiscal Constitutions: How to Protect Future Generations Using Land Taxes and Federalism," NBER Working Paper 8394.

Conley J., R. Driskill, and P. Wang (2013): "Capitalization, Decentralization, and Intergenerational Spillovers in a Tiebout Economy with a Durable Public Good," Working Paper, Vanderbilt University.

Cukierman A. and Meltzer A. (1989): "A Political Theory of Government Debt and Deficits in a Neo-Ricardian Framework," *American Economic Review*, Vol. 79., No. 4.

Davey M. (2015): "Illinois Supreme Court Rejects Lawmakers' Pension Overhaul", *New York Times*, May 8, 2015.

Davis, M. and M. Palumbo (2007): "The Price of Residential Land in Large US Cities," *Journal of Urban Economics*, vol. 63 (1), p. 352-384.

Epple, D., and K. Schipper (1981). "Municipal Pension Funding: A Theory and Some Evidence." *Public Choice* . Vol. 37, No. 1, pp. 141–178.

Epple D., and C. Spatt (1986). "State Restrictions on Local Debt," *Journal of Public Economics* 29, 199-221.

Favilukis, J., S. Ludvigson, and S. Van Nieuwerburgh (2008). "Macroeconomic implications of housing wealth, housing finance, and limited risk-sharing in general equilibrium." New York University Working Paper.

Glaeser, E,. and G. Ponzetto (2014), "Shrouded Costs of Government: The Political Economy of State and Local Public Pensions." *Journal of Public Economics*, Vol. 116, pp. 89–105.

Glomm, G., and R. Lagunoff (1999), "A Dynamic Tiebout Theory of Voluntary versus Involuntary Provision of Public Goods," , *Review of Economic Studies*, Vol. 66., pp. 659–677.

Glover A., J. Heathcote, D. Krueger, and J.-V. Ríos-Rull (2014),"Intergenerational Redistribution in the Great Recession," Working Paper, University of Minnesota.

Grossman, G., and E. Helpman (1998). "Intergenerational Redistribution with Short-Lived Governments." *Economic Journal*, Vol. 108, No. 450, pp. 1299–1329.

Hassler, J., R. Mora, K. Storesletten, and F. Zilibotti (2003). "The Survival of the Welfare State," *American Economic Review*, Vol. 93, No. 1, 87-112.

Haurin, D., P. Hendershott, and S. Wachter (1996). "Borrowing Constraints and the Tenure Choice of Young Households," NBER Working Paper 5630.

Iacoviello, M., and M. Pavan (2013), "Housing and Debt over the Life Cycle and over the Business Cycle," *Journal of Monetary Economics*, Vol. 60, No. 2, pp. 221–238.

Inman, R. (1982), "Public Employee Pensions and the Local Labor Budget." *Journal of Public Economics*, Vol. 19, No. 1, pp. 49–71.

Jones L. (1989). "Current Wealth and Tenure Choice," *Real Estate Economics*, Vol. 17, No. 1, pp. 7–40.

Kiyotaky, N., and J. Moore (1997). "Credit Cycles," *Journal of Political Economy*, Vol. 105, No. 2, pp. 211–248.

Klein, P., P. Krusell, and J.-V. Ríos-Rull (2008). "Time-Consistent Public Policy," *Review of Economic Studies*, Vol. 75, pp. 789–808.

Krusell, P, V. Quadrini and J.-V. Ríos-Rull (1997). "Are Consumption Taxes Really Better Than Income Taxes?" *Journal of Monetary Economics*, Vol. 37, No. 3, pp. 475–503.

Krusell, P., and J.-V. Ríos-Rull (1999). "On the Size of the U.S. Government: Political Economy in the Neoclassical Growth Model." *American Economic Review*, Vol. 89, No. 5, pp. 1156–1181.

Landvoigt T., Piazzesi M., and Schneider M. "The Housing Market(s) of San Diego," *American Economic Review* , April 2015, 105(4), 1371-1407

Linneman, P., and S. Wachter (1989). "The Impacts of Borrowing Constraints on Homeownership," *Real Estate Economics*, Vol. 17, No. 4, pp. 389–402.

Lutz B., Molloy R. and Hui Shan (2011). "The housing crisis and state and local government tax revenue: five channels," *Regional Science and Urban Economics* 41, No. 4, 306-319.

Mumy, G.E. (1978). "The Economics of Local Government Pensions and Pension Funding." *Journal of Political Economy*, Vol. 86, No. 3, pp. 517-527.

Munnell, A. H., and J.-P. Aubry (2016). "GASB 68: How Will State Unfunded Pension Liabilities Affect Big Cities?" *State and Local Pension Plans*, No. 47, Center for Retirement Research at Boston College.

Novy-Marx, R., and J. Rauh (2009). "The Liabilities and Risks of State-Sponsored Pension Plans," *Journal of Economic Perspectives*, Vol. 23, No. 4, pp. 191–210.

Novy-Marx, R., and J. Rauh (2011). "Public Pension Promises: How Big Are They and What Are They Worth?" *Journal of Finance*, Vol. 66, No. 4, pp. 1211–1249.

Oates, W. (1969). "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," *Journal of Political Economy*, Vol. 77, No. 6, pp. 957–71.

Persson, T., and G. Tabellini (2002). *Political Economics: Explaining Economic Policy.* Cambridge, MA: MIT Press.

Pew Charitable Trusts (2013). "A Widening Gap in Cities: Shortfalls in Funding for Pensions and Retiree Health Care."

Poterba J., Venti S. and D. Wise (2011). "The Composition and Drawdown of Wealth in Retirement," *Journal of Economic Perspectives*, Volume 25, Number 4, Pages 95–118.

Schultz, C., and T. Sjostrom (2001), "Local Public Goods, Debt and Migration," *Journal of Public Economics*, Vol. 80, No. 2, pp. 313–337.

Song, Z., K. Storesletten, and F. Zilibotti (2012). "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditures and Debt," *Econometrica*, Vol. 80, No. 6, pp. 2785–2803.

Yared, P. (2010). "Politicians, Taxes, and Debt," *Review of Economic Studies*, Vol. 77, No. 2, pp. 806–840.

Yared, P. (2015). "Optimal Fiscal Policy in an Economy with Private Borrowing Limits," Working Paper, Columbia University.

Zorn P. (1989). "Mobility-Tenure Decisions and Financial Credit: Do Mortgage Qualification Requirements Constrain Homeownership?" *Real Estate Economics*, Vol. 17, No. 4, pp 1–16.

Online Appendix

A Proofs of Lemmas, Propositions, and Numerical Results

A.1 Inequality in Equation 3.25

In order to derive the inequality in equation (3.25), consider the price of land after a deviation, $\widetilde{Q}\left(f, \widetilde{f}'; f^*\right)$, given by equation (3.12). Replace the future price of land $Q(\widetilde{f}'; f^*)$ into this equation:

$$\begin{aligned} \widetilde{Q}\left(f, \widetilde{f}'; f^*\right) &= (\psi+\beta)\, w - w^g - \frac{\widetilde{f}' b^g}{R} - b^g\,(1-f) + \\ &\quad + \frac{\kappa}{R}\left[(\psi+\beta)\, w - w^g - b^g - \frac{f^* b^g}{R} + b^g \widetilde{f}' + \frac{\kappa}{R} Q\left(f^*; f^*\right)\right], \end{aligned} \tag{A.1}$$

where $Q\left(f^*; f^*\right)$ is given by:

$$Q\left(f^*; f^*\right) = \frac{(\psi+\beta)\, w - w^g - b^g}{1-\frac{\kappa}{R}} + b^g f^* \frac{1-\frac{1}{R}}{1-\frac{\kappa}{R}}.$$

Replace $Q\left(f^*; f^*\right)$ into (A.1) and collect terms to obtain:

$$\widetilde{Q}\left(f, \widetilde{f}'; f^*\right) = \left[(\psi+\beta)\, w - w^g - b^g\right] \frac{R}{R-\kappa} - \frac{b^g}{R} \widetilde{f}'\,(1-\kappa) + b^g f - b^g f^* \frac{\kappa}{R}\left(\frac{1-\kappa}{R-\kappa}\right). \tag{A.2}$$

Impose the condition that the downpayment constraint binds for all values of $\widetilde{f}'$ and f (equation 3.16), taking into account the equilibrium downpayment in equation (3.17):

$$\widetilde{Q}\left(f, \widetilde{f}'; f^*\right) > \frac{\beta R}{1-\kappa} w. \tag{A.3}$$

Notice that

$$\widetilde{Q}\left(f, \widetilde{f}'; f^*\right) \geq \widetilde{Q}\left(f_{\min}, 1; f^*\right),$$

so it is necessary and sufficient for (A.3) to hold that:

$$\widetilde{Q}\left(f_{\min}, 1; f^*\right) > \frac{\beta R}{1-\kappa} w.$$

Replace $\widetilde{Q}\left(f_{\min}, 1; f^*\right)$ from equation (A.2) in the inequality above and simplify to obtain equation (3.25). Q.E.D.

A.2 Parameters in the Example of Section 3.3

We have already set $R = 1.22$, $\kappa = 0.80$, and $b^g/R = \$24,000$. According to the U.S. Census, median household income in Chicago in 2010-2014 was $\$47,831$.[40] Nationally, mean income is about 1.3 times median income, so we assume that average household income in Chicago is $w = \$47,831 \times 1.3 = \$62,180$.[41] It follows that:

$$\frac{b^g}{w} = \frac{\$24,000}{\$62,180} \times R = 0.39 \times R.$$

To compute w^g/w, we make the conservative assumption that public sector employees are paid the same as private ones and that in Chicago they account for about 12 percent of aggregate non-farm employment.[42] Hence we set $w^g/w = 0.12$. In the inequality (3.25) we also consider the case $f^* = 1$ which requires a higher cut-off value for ψ. We also notice that, according to the model (see equation (3.13)), a young household spends a fraction

$$\psi + \beta = \frac{l_t\left(\left(1+\tau_t\right) q_t - \kappa q_{t+1}/R\right)}{w}$$

of its income as a downpayment on housing. The average value of a house in Chicago is about $260,000 (Davis and Palumbo, 2007) and property taxes are about 2 percent. If a

[40]This figure is obtained from the U.S. Census' Quick Facts.
[41]The ratio 1.3 is from the Federal Reserve Bank of St Louis' FRED Blog, May 28, 2015.
[42]This figure is taken from the Bureau of Labor Statistics, Chicago Area Summary (2016).

household is able to borrow 80 percent of the value of the house, the downpayment is

$$\begin{aligned} \text{downpayment} &= \$260,000\,(1.02)\,(0.2) \\ &= \$53,040. \end{aligned}$$

As a fraction of household income, the downpayment is:

$$\psi + \beta = \frac{\$53,040}{\$62,180} = 0.85.$$

Replacing these numbers in the inequality (3.25), we conclude that it is satisfied as long as the parameter ψ satisfies the condition $\psi > 0.764$.

A.3 Proof of Proposition 3

Assume that preferences take the form:

$$U\left(c_{yt}, l_{yt}, l_{ot+1}, c_{ot+1}\right) = \left(1 - \psi - \beta\left(1+\theta\right)\right) \ln c_{yt} + \psi \ln l_{yt} + \beta\left(\ln c_{ot+1} + \theta \ln l_{ot+1}\right).$$

A.3.1 Frictionless Asset Market

In this case the budget constraint takes the form:

$$w = c_{yt} + \frac{c_{ot+1}}{R} + \left(\left(1+\tau_t\right) q_t - \frac{q_{t+1}}{R} - r_t\right) l_t + r_t l_{yt} + \frac{r_{t+1} l_{ot+1}}{R}.$$

The optimal choices of the agent are:

$$\begin{aligned} c_{yt} &= \left(1 - \psi - \beta\left(1+\theta\right)\right) w, && \text{(A.4)} \\ l_{yt} &= \frac{\psi w}{r_t}, && \text{(A.5)} \\ c_{ot+1} &= \beta R w, && \text{(A.6)} \\ l_{ot+1} &= \frac{\beta R \theta w}{r_{t+1}}, && \text{(A.7)} \end{aligned}$$

and the optimal choice of l_t must imply that in equilibrium:

$$(1+\tau_t)\, q_t - \frac{q_{t+1}}{R} = r_t. \tag{A.8}$$

Use equations (A.5) and (A.7) into the rental land market clearing condition $l_{yt} + l_{ot} = 1$ to obtain:

$$\frac{\psi w}{r_t} + \frac{\beta R \theta w}{r_t} = 1,$$

which pins down $r_t = r$ for all t. This implies that the indirect utility of a young agent is a constant. Replace r into (A.8) to get the equilibrium user cost of land. This is a version of equation (3.7) and the analysis that follows that equation in the main text applies. In particular, the price of land q_t is independent of a locality's pension funding policy.

A.3.2 Binding Downpayment Constraint

In this case, the budget constraints are:

$$\begin{aligned} w &= c_{yt} + r_t l_{yt} + (d_t - r_t)\, l_t, \\ c_{ot+1} + r_{t+1} l_{ot+1} &= q_{t+1} l_t, \end{aligned}$$

where we have already incorporated the restriction that $\kappa = 0$. Replace consumption when young and old in the objective function:

$$\begin{aligned} U(c_{yt}, l_{yt}, l_{ot+1}, c_{ot+1}) &= (1-\psi-\beta(1+\theta)) \ln(w - r_t l_{yt} - (d_t - r_t)\, l_t) + \psi \ln l_{yt} \\ &\quad + \beta \left(\ln(q_{t+1} l_t - r_{t+1} l_{ot+1}) + \theta \ln l_{ot+1} \right). \end{aligned}$$

The first-order condition for land consumption when young is:

$$\begin{aligned} l_{yt} &: \frac{(1-\psi-\beta-\beta\theta)\, r_t}{w - r_t l_{yt} - (d_t - r_t)\, l_t} = \frac{\psi}{l_{yt}} \Rightarrow \\ r_t l_{yt} &= \frac{\psi}{1-\beta-\beta\theta} (w - (d_t - r_t)\, l_t). \end{aligned} \tag{A.9}$$

Similarly, land consumption when old satisfies:

$$l_{ot+1} : r_{t+1} l_{ot+1} = \frac{\theta}{1+\theta} q_{t+1} l_t. \tag{A.10}$$

The demand for land as an asset is such that:

$$l_t : \frac{(1-\psi-\beta-\beta\theta)(d_t - r_t)}{w - r_t l_{yt} - (d_t - r_t) l_t} = \frac{\beta q_{t+1}}{q_{t+1} l_t - r_{t+1} l_{ot+1}}.$$

Replacing the first-order conditions (A.9) and (A.10), and simplifying we obtain:

$$l_t = \frac{\beta (1+\theta) w}{d_t - r_t}.$$

Replace into the budget constraint when young and old to obtain:

$$\begin{aligned} c_{yt} &= w - r_t l_{yt} - (d_t - r_t) l_t = w (1 - \psi - \beta (1+\theta)), \\ c_{ot+1} &= \frac{1}{1+\theta} q_{t+1} l_t. \end{aligned}$$

Equilibrium in the market for land ownership ($l_t = 1$) pins down $d_t - r_t$:

$$\frac{\beta (1+\theta) w}{d_t - r_t} = 1 \rightarrow d_t - r_t = \beta (1+\theta) w. \tag{A.11}$$

Equilibrium in the market for land consumption ($l_{yt} + l_{ot} = 1$) together with the above expression implies:

$$\frac{\psi w}{r_t} + \frac{\theta}{1+\theta} \frac{q_t}{r_t} = 1$$

so we can solve for r_t as function of q_t (equation (3.37)):

$$r_t = \psi w + \frac{\theta}{1+\theta} q_t. \tag{A.12}$$

Combine (A.11), (A.12), and the definition of d_t to derive the dynamic equation for land prices:

$$(1+\tau_t)\, q_t = \psi w + \frac{\theta}{1+\theta} q_t + \beta\,(1+\theta)\, w.$$

Replace in it the government's budget constraint (2.8):

$$q_t \left(1 - \frac{\theta}{1+\theta}\right) = \psi w + \beta\,(1+\theta)\, w - w^g - \frac{f_{t+1} b^g}{R} - b^g\,(1-f_t)\,.$$

Simplify to obtain equation (3.36) in the main text:

$$q_t = (1+\theta) \left\{ (\psi + \beta\,(1+\theta))\, w - w^g - \frac{f_{t+1} b^g}{R} - b^g\,(1-f_t) \right\}.$$

Using this equation, it is straightforward to show that a policy deviation that increases pension funding leads to a smaller land price today and a higher land price in the following period. Replacing the optimal choices into the utility function, we derive the utilities of old and young agents in period t (up to some irrelevant constants):

$$\begin{aligned} V_t^{\text{old}} &\approx (1+\theta)\ln q_t - \theta \ln r_t, \\ V_t^{\text{young}} &\approx -\psi \ln r_t + \beta V_{t+1}^{\text{old}}. \end{aligned}$$

We now show that the utility of the old is strictly increasing in q_t. Use equation (A.12) to replace r_t into V_t^{old}:

$$\begin{aligned} V_t^{\text{old}} &= (1+\theta)\ln q_t - \theta \ln r_t \\ &= (1+\theta)\ln q_t - \theta \ln \left(w\psi + \frac{\theta}{1+\theta} q_t \right). \end{aligned}$$

Now, take its derivative with respect to q_t:

$$\frac{\partial V_t^{\text{old}}}{\partial q_t} = \frac{1+\theta}{q_t} - \frac{\theta}{w\psi + \frac{\theta}{1+\theta} q_t} \frac{\theta}{1+\theta}.$$

This is strictly positive if and only if the following condition holds:

$$\frac{(1+\theta)}{q_t} > \frac{\theta}{w\psi + \frac{\theta}{1+\theta}q_t}\frac{\theta}{1+\theta},$$

which can be re-written as:

$$(1+\theta)^2 w\psi + (1+\theta)\,\theta q_t > \theta^2 q_t.$$

The latter always holds. Using the same argument, it is straightforward to show that the young's utility is strictly increasing in the future price of land and strictly decreasing in its current price.

A.4 Proof of Proposition 4

Assume that there is no borrowing, so $\kappa = 0$.

A.4.1 Notation

To save on notation, define:

$$\delta \equiv (\psi + \beta)\frac{w}{b^g} - \frac{w^g}{b^g} - 1,$$

and re-write equations (3.20) and (3.21) as:

$$Q(f;F) = \delta b^g - \frac{F(f)\, b^g}{R} + b^g f, \tag{A.13}$$

$$\widetilde{Q}\left(f, \widetilde{f}'; F\right) = \delta b^g - \frac{\widetilde{f}' b^g}{R} + b^g f. \tag{A.14}$$

A.4.2 Equilibrium Given Policy Rule

Guess that the policy rule takes the affine form:

$$F(f) = \lambda + \phi f,$$

for some parameters (λ, ϕ) , which will have to be determined in equilibrium. Similarly, guess that the equilibrium land pricing function takes the form:

$$Q\left(f;F\right)=\pi b^{g}+\omega b^{g}f, \tag{A.15}$$

for some constants (π, ω) to be determined. To solve for the latter, replace (A.15) into (A.13):

$$Q\left(f;F\right)=\delta b^{g}-\frac{\left(\lambda+\phi f\right)b^{g}}{R}+b^{g}f.$$

Collect the constant terms and those in f:

$$Q\left(f;F\right)=\left(\delta-\frac{\lambda}{R}\right)b^{g}+b^{g}\left(1-\frac{\phi}{R}\right)f.$$

Impose consistency with the guess (A.15). First the slope:

$$\omega=1-\frac{\phi}{R}. \tag{A.16}$$

Then, the intercept:

$$\pi=\delta-\frac{\lambda}{R}. \tag{A.17}$$

A.4.3 Politico-Economic Equilibrium

Now solve for the equilibrium policy rule parameters (λ, ϕ). The policymaker maximizes the weighted average of the old and young utilities:

$$\alpha\ln\widetilde{Q}\left(f,\widetilde{f}';F\right)+\left(1-\alpha\right)\beta\ln Q\left(\widetilde{f}';F\right), \tag{A.18}$$

where:

$$\widetilde{Q}\left(f,\widetilde{f}';F\right)=\delta b^{g}-\frac{\widetilde{f}'b^{g}}{R}+b^{g}f, \tag{A.19}$$

and

$$Q\left(\widetilde{f}';F\right)=\pi b^{g}+\omega b^{g}\widetilde{f}'. \tag{A.20}$$

The interior first-order condition is:

$$-\frac{\alpha}{\widetilde{Q}\left(f,\widetilde{f}';F\right)}\frac{1}{R}+\frac{(1-\alpha)\,\beta\omega}{Q\left(\widetilde{f}';F\right)}=0.$$

Simplify this equation to:

$$\alpha Q\left(\widetilde{f}';F\right)=(1-\alpha)\,\beta R\omega\widetilde{Q}\left(f,\widetilde{f}';F\right).$$

Replace $Q\left(\widetilde{f}';F\right)$ and $\widetilde{Q}\left(f,\widetilde{f}';F\right)$ from equations (A.20) and (A.19):

$$\alpha\left(\pi+\omega\widetilde{f}'\right)=(1-\alpha)\,\beta R\omega\left(\delta+f-\frac{\widetilde{f}'}{R}\right).$$

Solve for $\widetilde{f}'$:

$$\alpha\pi+\alpha\omega\widetilde{f}'=(1-\alpha)\,\beta R\omega\delta+(1-\alpha)\,\beta R\omega f-(1-\alpha)\,\beta R\omega\frac{\widetilde{f}'}{R}$$

or

$$\widetilde{f}'\omega\left(\alpha+(1-\alpha)\,\beta\right)=(1-\alpha)\,\beta R\omega\delta-\alpha\pi+(1-\alpha)\,\beta R\omega f$$

or

$$\widetilde{f}'=\frac{(1-\alpha)\,\beta R\omega\delta-\alpha\pi}{\omega\left(\alpha+(1-\alpha)\,\beta\right)}+\frac{(1-\alpha)\,\beta R}{\alpha+(1-\alpha)\,\beta}f.$$

A.4.4 Impose Consistency

Now, impose consistency to obtain (λ,ϕ):

$$\lambda = \frac{(1-\alpha)\,\beta R\omega\delta-\alpha\pi}{\omega\left(\alpha+(1-\alpha)\,\beta\right)}, \tag{A.21}$$

$$\phi = \frac{(1-\alpha)\,\beta R}{\alpha+(1-\alpha)\,\beta}. \tag{A.22}$$

Solve equation (A.21) after replacing (A.16) and (A.17):

$$\lambda = \frac{(1-\alpha)\,\beta R\left(1-\frac{\phi}{R}\right)\delta - \alpha\left(\delta - \frac{\lambda}{R}\right)}{\left(1-\frac{\phi}{R}\right)\left(\alpha + (1-\alpha)\,\beta\right)}.$$

Simplify:

$$\lambda\left(1-\frac{\phi}{R}\right)\left(\alpha + (1-\alpha)\,\beta\right) = (1-\alpha)\,\beta R\left(1-\frac{\phi}{R}\right)\delta - \alpha\delta + \frac{\alpha}{R}\lambda$$

or

$$\lambda\left[\left(1-\frac{\phi}{R}\right)\left(\alpha + (1-\alpha)\,\beta\right) - \frac{\alpha}{R}\right] = (1-\alpha)\,\beta R\left(1-\frac{\phi}{R}\right)\delta - \alpha\delta,$$

thus:

$$\lambda = \frac{\left((1-\alpha)\,\beta\,(R-\phi) - \alpha\right)\delta}{\left(1-\frac{\phi}{R}\right)\left(\alpha + (1-\alpha)\,\beta\right) - \frac{\alpha}{R}}.$$

Replace ϕ :

$$\lambda = \frac{\left((1-\alpha)\,\beta\left(R - \frac{(1-\alpha)\beta R}{\alpha+(1-\alpha)\beta}\right) - \alpha\right)\delta}{\left(1 - \frac{(1-\alpha)\beta}{\alpha+(1-\alpha)\beta}\right)\left(\alpha + (1-\alpha)\,\beta\right) - \frac{\alpha}{R}}$$

and simplify again:

$$\lambda = \frac{\left(\frac{R\beta(1-\alpha)}{\alpha+(1-\alpha)\beta} - 1\right)\delta}{(1-1/R)}$$

or

$$\lambda = \frac{R\delta}{R-1}\left(\frac{R\beta\,(1-\alpha)}{\alpha + (1-\alpha)\,\beta} - 1\right).$$

More succinctly, using (A.22), it can be written as:

$$\lambda = \frac{R\delta}{R-1}\,(\phi - 1)\,.$$

A.4.5 Equilibrium Policy Rule

The equilibrium policy rule is therefore as follows:

$$f' = \frac{R\delta}{R-1}(\phi - 1) + \phi f,$$

where ϕ is defined in equation (A.22).

A.5 Proof of Lemma 1

Apply the implicit function theorem to equation (4.3) to get:

$$g'(q_{t+1}) = (1-\kappa)\frac{v'(q_{t+1}(1-\kappa)) + v''(q_{t+1}(1-\kappa))\,q_{t+1}(1-\kappa)}{u_1(w-d_t,1) - d_t u_{11}(w-d_t,1) + u_{21}(w-d_t,1)}. \tag{A.23}$$

Notice that:

$$g'(q_{t+1}) < (1-\kappa)\frac{v'(q_{t+1}(1-\kappa))}{u_1(w-d_t,1) - d_t u_{11}(w-d_t,1) + u_{21}(w-d_t,1)} < (1-\kappa)\frac{v'(q_{t+1}(1-\kappa))}{u_1(w-d_t,1)},$$

where the first inequality follows from the facts that $v''(q_{t+1}(1-\kappa)) < 0$ and the denominator of equation (A.23) is positive, while the second inequality follows from the fact that $-u_{11}(w-d_t,1) \geq 0$ and $u_{21}(w-d_t,1) \geq 0$. Q.E.D.

A.6 Proof of Proposition 5

Take the derivative of the indirect utility function in equation (4.6) with respect to q_{t+1} :

$$\frac{\partial U(w - g(q_{t+1}), 1, (1-\kappa)q_{t+1})}{\partial q_{t+1}} = -u_1(w - g(q_{t+1}), 1)\, g'(q_{t+1}) + (1-\kappa)\, v'((1-\kappa)q_{t+1}).$$

Notice that this is strictly positive due to Lemma 1. Q.E.D.

A.7 Proof of Proposition 7

It follows directly from the analysis in the text. Q.E.D.

B Additional Results for the Case of a General Utility Function

B.1 Irrelevance of Pension Funding Policy Under Perfect Capital Markets

Formally, the basic statement is:

Proposition 8 *Without a downpayment constraint (or when the latter never binds), both the price of land and the indirect utility offered by a municipality are independent of the one-period deviation $\widetilde{f}'$ from f^*. As a result, both young and old agents are indifferent about alternative pension funding policies.*

Consider an arbitrary utility function $U(c_{yt}, l_t, c_{ot+1})$ with the standard properties. An agent's lifetime budget constraint is

$$c_{yt} + l_t \left(q_t \left(1 + \tau_t\right) - q_{t+1}/R\right) + c_{ot+1}/R = w.$$

Optimal choices, including land $L(u_t)$, depend on the user cost of land:

$$u_t \equiv \left(q_t \left(1 + \tau_t\right) - q_{t+1}/R\right).$$

Land market equilibrium requires that $L(u_t) = 1$, pinning down $u_t = u^*$ uniquely because $L(u_t)$ is strictly decreasing in u_t. It follows that an agent's lifetime utility, which depends only on u_t, is also a constant independent of pension funding. To verify that the current price of land is also independent of a locality's pension funding policy, write the user cost in recursive form and solve for the current land price:

$$\widetilde{Q}\left(f,\widetilde{f}';f^*\right)=u^*-w^g-\widetilde{f}'b^g/R-b^g\left(1-f\right)+Q\left(\widetilde{f}';f^*\right)/R. \tag{B.1}$$

It is straightforward to verify that the price of land tomorrow is such that:

$$Q\left(\widetilde{f}';f^*\right)=u^*-w^g-f^*b^g/R-b^g\left(1-\widetilde{f}'\right)+Q\left(f^*;f^*\right)/R$$

Replace $Q\left(\widetilde{f}';f^*\right)$ into equation (B.1) and simplify we obtain:

$$\widetilde{Q}\left(f,\widetilde{f}';f^*\right)=u^*-w^g-b^g\left(1-f\right)+\left(u^*-w^g-b^g-f^*b^g/R+Q\left(f^*;f^*\right)\right)/R.$$

This is independent of the policy deviation $\widetilde{f}'$. Hence, the utility of the old generation is independent of the locality's pension funding policy. Q.E.D.

B.2 General Utility Function: Properties of the Demand for Land and the Indirect Utility Function

After replacing the budget constraints (2.2)-(2.3) into the utility function, the optimization problem of an agent is:

$$\max_{l_t,b_{t+1}}U\left(w-\left(1+\tau_t\right)q_tl_t-\frac{b_{t+1}}{R},l_t,q_{t+1}l_t+b_{t+1}\right), \tag{B.2}$$

subject to the downpayment constraint (2.4). If the downpayment constraint binds ($b_{t+1}=-\kappa q_{t+1}l_t$), the optimization problem takes the form in equation (4.1). Formally, the land demand function $L\left(d_t,q_{t+1}\right)$ and the associated indirect utility function $V\left(d_t,q_{t+1}\right)$ have the properties summarized in the following proposition.

Proposition 9 (Properties of the demand for land and of the indirect utility function) *(a) There exists a unique land demand function $L(d_t, q_{t+1})$ that solves problem (4.1).*

(b) If the Inada conditions $u_1(c_y, l) \to +\infty$ as $c_y \to 0$ and $u_2(c_y, l) \to +\infty$ as $l \to 0$ hold, then the land demand function $L(d_t, q_{t+1})$ satisfies the first-order condition for l:

$$-d_t u_1(w - d_t l, l) + u_2(w - d_t l, l) + v'(q_{t+1}(1-\kappa) l) q_{t+1}(1-\kappa) = 0. \tag{B.3}$$

(c) Under the assumptions in part (b), the downpayment constraint binds if and only if the Euler equation for consumption holds as an inequality:

$$u_1(w - d_t L(d_t, q_{t+1}), L(d_t, q_{t+1})) > v'(q_{t+1}(1-\kappa) L(d_t, q_{t+1})) R. \tag{B.4}$$

(d) Under the assumptions in part (b), the land demand function $L(d_t, q_{t+1})$ is strictly decreasing in d_t. The effect of q_{t+1} on the demand for land is ambiguous.[43]

(e) Under the assumptions in part (b), the indirect utility function $V(d_t, q_{t+1})$, defined in (4.2), is strictly decreasing in d_t and strictly increasing in q_{t+1}.

(a) The function $U(w - d_t l, l, q_{t+1}(1-\kappa) l)$ is continuous in l on the interval $[0, d_t/w]$. Therefore it achieves a maximum in this interval.

(b) The Inada conditions on the derivatives at $l = d_t/w$ and $l = 0$ rule out corner solutions in which the optimal demand for land is either d/w or zero. Therefore, the solution must be interior and satisfy the first-order condition (B.3):

$$-d_t u_1(w - d_t l_t, l_t) + u_2(w - d_t l_t, l_t) + v'(q_{t+1} l(1-\kappa)) q_{t+1}(1-\kappa) = 0. \tag{B.5}$$

This equation admits a solution because of the Assumptions in part (b) of Proposition 9. Notice that the objective function $U(w - d_t l, l, q_{t+1}(1-\kappa) l)$ is strictly concave in l_t because

[43] Specifically, it is strictly decreasing in q_{t+1} if and only if the absolute value of the elasticity of $v'(c_o)$ with respect to c_o is strictly larger than one.

its second derivative with respect to l_t is negative:

$$\Delta \equiv d_t^2 u_{11}\left(w - d_t l_t, l_t\right) - 2 d_t u_{12}\left(w - d_t l_t, l_t\right) + u_{22}\left(w - d_t l_t, l_t\right) + v''\left(q_{t+1} l\left(1-\kappa\right)\right)\left[q_{t+1}\left(1-\kappa\right)\right]^2 < 0. \tag{B.6}$$

This is true because, by Assumption 1:

$$\begin{aligned} u_{11}\left(w - d_t l_t, l_t\right) &\leq 0, \\ u_{22}\left(w - d_t l_t, l_t\right) &\leq 0, \\ u_{12}\left(w - d_t l_t, l_t\right) &\geq 0, \\ v''\left(q_{t+1} l\left(1-\kappa\right)\right) &\leq 0, \end{aligned}$$

and at least one of the own-second derivatives is strictly negative. Thus, the solution to the first-order condition is unique.

(c) Consider the agent's optimization problem (B.2), without imposing that the downpayment constraint binds. Let μ denote the Lagrange multiplier associated with the constraint $b_{t+1} + \kappa q_{t+1} l_t \geq 0$. The interior first-order conditions of the problem are:

$$l_t \;:\; -\left(1+\tau_t\right) q_t U_{1t} + U_{2t} + U_{3t} q_{t+1} + \mu \kappa q_{t+1} = 0, \tag{B.7}$$

$$b_{t+1} \;:\; -U_{1t}/R + U_{3t} + \mu = 0. \tag{B.8}$$

The downpayment constraint binds if and only if $\mu > 0$. Thus, according to equation (B.8), the Euler equation holds as an inequality (equation (B.4)) if and only if the downpayment constraint binds.

(d) The properties of the demand function can be proved using the implicit function theorem as applied to equation (B.5). First:

$$\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial d} = \frac{u_1\left(w - d_t l_t, l_t\right) - d_t l_t u_{11}\left(w - d_t l_t, l_t\right) + u_{12}\left(w - d_t l_t, l_t\right) l_t}{\Delta} < 0$$

because the numerator of this expression is positive and $\Delta < 0$. The other derivative is:

$$\frac{dL\left(d_t, q_{t+1}\right)}{\partial q} = \frac{v'\left(q_{t+1} l\left(1-\kappa\right)\right) + v''\left(q_{t+1} l\left(1-\kappa\right)\right) q_{t+1} l\left(1-\kappa\right)}{-\Delta}\left(1-\kappa\right)$$

which can be expressed as:

$$\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial q} = \frac{v'\left(q_{t+1} l\left(1-\kappa\right)\right)\left[1-\varepsilon\left(q_{t+1} l\left(1-\kappa\right)\right)\right]}{-\Delta}\left(1-\kappa\right), \tag{B.9}$$

where

$$\varepsilon\left(q_{t+1} l\left(1-\kappa\right)\right) \equiv -\frac{v''\left(q_{t+1} l\left(1-\kappa\right)\right) q_{t+1} l\left(1-\kappa\right)}{v'\left(q_{t+1} l\left(1-\kappa\right)\right)} > 0$$

is the (positive) elasticity of $v'(c_{ot+1})$ with respect to c_{ot+1}. It follows that the derivative (B.9) is negative if and only if $\varepsilon\left(q_{t+1} l\left(1-\kappa\right)\right) > 1$. In terms of κ, we can write:

$$\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial \kappa} = \frac{\partial L\left(d_t, q_{t+1}\right)}{\partial d_t}\frac{\partial d_t}{\partial \kappa} + \frac{v'\left(q_{t+1} l\left(1-\kappa\right)\right) + v''\left(q_{t+1} l\left(1-\kappa\right)\right) q_{t+1} l\left(1-\kappa\right)}{\Delta} q_{t+1}.$$

Replacing $\partial L\left(d_t, q_{t+1}\right)/\partial d_t$ from above and noticing that $\partial d_t/\partial \kappa = -q_{t+1}/R$, leads to

$$\begin{aligned} &\frac{\partial L\left(d_t, q_{t+1}\right)}{\partial \kappa} \\ = \; &\frac{q_{t+1}}{R\left(-\Delta\right)}\left\{\begin{array}{c} u_1\left(w-d_t l_t, l_t\right) - d_t l_t u_{11}\left(w-d_t l_t, l_t\right) + u_{12}\left(w-d_t l_t, l_t\right) l_t - Rv'\left(q_{t+1} l\left(1-\kappa\right)\right) \\ -Rv''\left(q_{t+1} l\left(1-\kappa\right)\right) q_{t+1} l\left(1-\kappa\right) \end{array}\right\}, \end{aligned}$$

where $(-\Delta) > 0$. The term in brackets is also positive because the agent is constrained and so $u_1\left(w-d_t l_t, l_t\right) > Rv'\left(q_{t+1} l\left(1-\kappa\right)\right).$

(e) By the envelope theorem:

$$\begin{aligned} \frac{\partial V\left(d_t, q_{t+1}\right)}{\partial d_t} &= -u_1\left(w-d_t L\left(d_t, q_{t+1}\right), L\left(d_t, q_{t+1}\right)\right) L\left(d_t, q_{t+1}\right) < 0, \\ \frac{\partial V\left(d_t, q_{t+1}\right)}{\partial q_{t+1}} &= v'\left(q_{t+1} L\left(d_t, q_{t+1}\right)\left(1-\kappa\right)\right) L\left(d_t, q_{t+1}\right)\left(1-\kappa\right) > 0. \end{aligned}$$

Q.E.D.

B.3 Binding Downpayment Constraint

The analysis of Section 4 proceeds under the assumption that the downpayment constraint in equation (2.5) is always binding both in equilibrium and following a deviation. As we had mentioned at the beginning of the section, outside of special cases, such as the logarithmic example of Section 3, there are no simple conditions on the model's parameters that guarantee that this is indeed the case. It is, however, feasible to provide *sufficient* conditions that can be verified given specific utility functions and parameter values, such that the downpayment constraint is guaranteed to always be binding. These are contained in the following proposition.

Proposition 10 (Sufficient conditions for binding downpayment constraint) *Consider a politico-economic equilibrium characterized by the constant funding rule* $f^* = F(f)$. *Then, sufficient conditions for the downpayment constraint to be always binding, both in equilibrium and following a deviation from it, are:*
(a) If the function $g(q_{t+1})$ *is weakly increasing in* q_{t+1}*:*

$$u_1\left(w - g\left(Q\left(f_{\min}; f^*\right)\right), 1\right) > Rv'\left(Q\left(f_{\min}; f^*\right)(1-\kappa)\right).$$

(b) If the function $g(q_{t+1})$ *is weakly decreasing in* q_{t+1}*:*

$$u_1\left(w - g\left(Q\left(1; f^*\right)\right), 1\right) > Rv'\left(Q\left(f_{\min}; f^*\right)(1-\kappa)\right).$$

As proved in Proposition 9, part (c), the downpayment constraint is binding when

$$u_1\left(w - d_t, 1\right) > v'\left(q_{t+1}(1-\kappa)\right)R,$$

where we are considering the local economy in a situation in which the land market is in equilibrium ($l_t = 1$). Replace d_t from equation (4.4) and re-write this equation using the recursive notation:

$$u_1\left(w-g\left(Q\left(\widetilde{f}';f^*\right)\right),1\right)>Rv'\left(Q\left(\widetilde{f}';f^*\right)(1-\kappa)\right).$$

For the downpayment constraint to always be binding we need the inequality above to hold for all $\widetilde{f}'\in[f_{\min},1]$, which includes the politico-economic equilibrium case $\widetilde{f}'=f^*$. Notice that, since $Q\left(\widetilde{f}';f^*\right)$ is strictly increasing in $\widetilde{f}'$ and $v''\leq 0$, we can write:

$$v'\left(Q\left(f_{\min};f^*\right)(1-\kappa)\right)\geq v'\left(Q\left(\widetilde{f}';f^*\right)(1-\kappa)\right).$$

(a) Since $u_{11}\leq 0$, if the function $g\left(.\right)$ is increasing in its argument, we know that:

$$u_1\left(w-g\left(Q\left(\widetilde{f}';F\right)\right),1\right)\geq u_1\left(w-g\left(Q\left(f_{\min};f^*\right)\right),1\right)$$

for all $\widetilde{f}'\in[f_{\min},1]$. Therefore, if the function $g\left(.\right)$ is increasing, we can write:

$$\begin{aligned}u_1\left(w-g\left(Q\left(\widetilde{f}';F\right)\right),1\right) &\geq u_1\left(w-g\left(Q\left(f_{\min};f^*\right)\right),1\right)>Rv'\left(Q\left(f_{\min};f^*\right)(1-\kappa)\right)\\ &\geq Rv'\left(Q\left(\widetilde{f}';f^*\right)(1-\kappa)\right).\end{aligned}$$

Thus, as long as the middle inequality holds, the agent is constrained.

(b) Conversely, if the function $g\left(.\right)$ is decreasing in its argument, we know that:

$$u_1\left(w-g\left(Q\left(\widetilde{f}';F\right)\right),1\right)\geq u_1\left(w-g\left(Q\left(1;f^*\right)\right),1\right)$$

and the same argument applies with:

$$\begin{aligned}u_1\left(w-g\left(Q\left(\widetilde{f}';F\right)\right),1\right) &\geq u_1\left(w-g\left(Q\left(1;f^*\right)\right),1\right)>Rv'\left(Q\left(f_{\min};f^*\right)(1-\kappa)\right)\\ &\geq Rv'\left(Q\left(\widetilde{f}';f^*\right)(1-\kappa)\right).\end{aligned}$$

Q.E.D.

For example, if the utility function takes the quasi-linear form:

$$U = c_{yt} + \phi(l_t) + \beta c_{ot+1}, \tag{B.10}$$

with $\phi'(l) > 0$ and $\phi''(l) < 0$, the marginal utility of consumption when young is a constant equal to 1, while the marginal utility of consumption when old is simply β. Thus, the downpayment constraint is always binding if $\beta R < 1$.[44]

C Additional Extensions and Results

In this appendix we consider two extensions of the model. In the first one we consider alternative forms for the downpayment constraint. In the second we consider a version of the model with geographic mobility.

C.1 Alternative Forms of the Downpayment Constraint

In this section we consider and analyze two alternative formalizations of the downpayment constraint considered in the main text. The first one makes the constraint a function of the current price of land, instead of its future price. In the second one, agents face a higher interest rate when borrowing than when lending. In the former case, we are able to solve for equilibrium under a binding constraint only if utility is logarithmic. In the latter case we are able to solve for equilibrium for a general utility function. In this case, differently from the benchmark model, young agents' utility is always independent of the locality's funding policy. Old agents always prefer to maximize current land prices by setting pension funding at its lower possible level.

[44]For this quasi-linear case the function $g(q_{t+1}) = \phi'(1) + \beta(1-\kappa)q_{t+1}$. Notice that, in this example, the condition $\beta R < 1$ is also necessary for the downpayment constraint to bind.

C.1.1 Downpayment Constraint with Current Land Price (Log Utility Case)

The downpayment constraint is now:

$$b_{t+1} \geq -\kappa q_t l_t.$$

For the case of log utility, the analysis in the main text goes through after redefining

$$d_t = (1 + \tau_t - \kappa/R)\, q_t. \tag{C.1}$$

Land market clearing ($l_t = 1$) then pins down $d_t = d^*$, which can be used to solve for q_t:

$$q_t = \frac{d^*}{1-\kappa/R} - \frac{\tau_t q_t}{1-\kappa/R}.$$

Replacing the government's budget constraint (2.8) for $\tau_t q_t$ we obtain:

$$q_t = \frac{d^*}{1-\kappa/R} - \frac{w^g}{1-\kappa/R} - \frac{1}{1-\kappa/R}\frac{f_{t+1}b^g}{R} - \frac{1}{1-\kappa/R} b^g \left(1 - f_t\right).$$

The analysis of the politico-economic equilibrium is then straightforward and confirms our results. Specifically, the current price of land decreases in pension funding (f_{t+1}), while the future price increases.

The lifetime utility of a young agent can be written as:

$$V_t^{\text{young}} = (1-\psi-\beta)\ln\left(1-\psi-\beta\right) w + \beta \ln\left(q_{t+1} - \kappa q_t\right).$$

It follows that young agents prefer the maximum funding policy in order to maximize q_{t+1} and minimize q_t.

C.1.2 Higher Borrowing Rate (General Utility Case)

Suppose that preferences take the form:

$$U\left(c_{yt}, l_t, c_{ot+1}\right) = u\left(c_{yt}, l_t\right) + v\left(c_{ot+1}\right).$$

Agents face an interest rate R/κ when borrowing and R when lending (with $\kappa < 1$). Consider, for the sake of the argument, that a young agent would like to borrow, in which case the budget constraint is:

$$w = c_{yt} + d_t l_t + \frac{c_{ot+1}}{\kappa^{-1}R},$$

where the d_t is defined as in the main text:

$$d_t = \left(1+\tau_t\right) q_t - \frac{q_{t+1}}{\kappa^{-1}R}.$$

Replace the budget constraints into the utility function:

$$U\left(c_{yt}, l_t, c_{ot+1}\right) = u\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + v\left(c_{ot+1}\right).$$

The first-order condition for land and consumption when old are:

$$\begin{aligned} l_t &: -d_t u_1\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + u_2\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right) + v'\left(c_{ot+1}\right) = 0, \\ c_{ot+1} &: -u_1\left(w - d_t l_t - \frac{c_{ot+1}}{\kappa^{-1}R}, l_t\right)\frac{1}{\kappa^{-1}R} + v'\left(c_{ot+1}\right) = 0. \end{aligned}$$

Imposing land market clearing $l_t = 1$, we obtain:

$$\begin{aligned} l_t &: -d_t u_1\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right) + u_2\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right) + v'\left(c_{ot+1}\right) = 0, \\ c_{ot+1} &: -u_1\left(w - d_t - \frac{c_{ot+1}}{\kappa^{-1}R}, 1\right)\frac{1}{\kappa^{-1}R} + v'\left(c_{ot+1}\right) = 0. \end{aligned}$$

One can solve for c_{ot+1} from the second equation as a function of d_t :

$$c_{ot+1} = C(d_t)$$

and replace the resulting function in the first equation:

$$-d_t u_1\left(w - d_t - \frac{C(d_t)}{\kappa^{-1}R}, 1\right) + u_2\left(w - d_t - \frac{C(d_t)}{\kappa^{-1}R}, 1\right) + v'(C(d_t)) = 0.$$

This equation pins down the equilibrium downpayment:

$$d_t = d^*.$$

Thus, in equilibrium:

$$(1+\tau_t)\, q_t - \frac{q_{t+1}}{\kappa^{-1}R} = d^*,$$

and the analysis follows in the same way as in the main text. Specifically, the current land price increases as pension funding declines and an old policymaker would want to set pension funding to the minimum allowed level. Differently from the main text, however, the lifetime utility of a young agent is:

$$V^{\text{young}}(d^*) = u\left(w - d^* - \frac{C(d^*)}{\kappa^{-1}R}, 1\right) + v(C(d^*)),$$

so it is not affected by the locality's funding policy.

C.2 Geographic Mobility

Intuitively, geographic mobility should act as a force that dampens the effect of reducing pension funding on young agents' utility and, consequently, on the price of land. This intuition is correct within the context of our model. In particular, with perfect mobility, lower pension funding leads to a smaller increase in land prices in a locality than with a geographically fixed population. In addition, young agents are insulated from the effect of reduced pension funding by a locality on utility. However, young agents are *not* insulated if *all* localities follow the same policy, as they do in the symmetric general equilibrium of the model. In such case, a policy intervention by a higher authority that dictates minimum funding levels produces the same effect as in Section 4.2.2, i.e. it increases the welfare of the young at the expense of the utility of the old generation.

We consider the case in which young agents' labor mobility is perfect, in the sense that a locality would not be able to attract *any* young agents if it offered less than some lifetime utility V^*. We start by considering one locality in isolation taking V^* as given, but later endogenize V^* by imposing an economy-wide market clearing condition for the young population. The timing of events is as follows. A location funding policy is chosen first, then young agents choose where to reside, and how much land and consumption to demand. Finally, the market for land clears. Thus, the policymaker fully anticipates the effect of her choices on the measure of young agents that choose to reside in the locality. Given that young agents are fully mobile and always attain lifetime utility V^*, they are indifferent about the pension funding policy of any locality they are considering as potential place of residence. Thus, we only focus on the case in which the policymaker is an old agent.

Relative to the case of exogenous population, with endogenous young population the land market equilibrium condition becomes:

$$n_t L\left(d_t, q_{t+1}\right) = 1, \tag{C.2}$$

where n_t denotes the endogenous measure of young agents who are attracted to the location. In equilibrium, the young have to be indifferent between living in the locality or living elsewhere and obtaining lifetime utility V^* :

$$V\left(d_t, q_{t+1}\right) = V^*, \tag{C.3}$$

where the indirect utility function is defined in equation (4.2). Since the indirect utility function is strictly decreasing in d_t (Proposition 9, part (e)), equation (C.3) can be written as:

$$d_t = h\left(q_{t+1}\right). \tag{C.4}$$

The function $h(.)$ plays the same role as the function $g(.)$ introduced in Section 4.1 for the case of fixed population. Apply the implicit function theorem to equation (C.3) to compute the derivative of $h(.)$:

$$h'\left(q_{t+1}\right) = (1-\kappa)\frac{v'\left((1-\kappa)\,q_{t+1}L\left(d_t, q_{t+1}\right)\right)}{u_1\left(w - d_t L\left(d_t, q_{t+1}\right), L\left(d_t, q_{t+1}\right)\right)}. \tag{C.5}$$

Follow the same steps as in Section 4.2, to obtain the derivative of land prices with respect to pension funding $\widetilde{f}'$ for the case of endogenous young population:[45]

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = \underbrace{-\frac{b^g}{R}}_{\text{current taxes}} + \underbrace{\frac{\kappa b^g}{R}}_{\text{borrowing}} + \underbrace{h'\left(Q\left(\widetilde{f}'; F\right)\right) b^g}_{\text{effect of resale value of land}}, \tag{C.6}$$

with $h'(.)$ replacing $g'(.)$. Since the derivative in equation (C.5) is strictly larger than $g'\left(q_{t+1}\right)$ (Proposition 1), reducing pension funding leads to a smaller increase in current land prices when young agents are mobile than when they are not so. The intuition is that, following a reduction in the future price of land brought about by a decline in $\widetilde{f}'$, the location becomes less attractive to prospective young residents. The loss of young population contributes to reduce current land prices. This additional mechanism reduces, but does not fully offset, the extent of the increase in current land prices following a reduction in pension funding. In order to compute the overall effect on land prices, replace equation (C.5) into (C.6), and rearrange:

$$\frac{\partial \widetilde{Q}\left(f, \widetilde{f}'; f^*\right)}{\partial \widetilde{f}'} = -\frac{b^g(1-\kappa)}{R}\left(1 - R \times MRS_t^c\right) < 0, \tag{C.7}$$

where the negative sign is due to the fact that $R \times MRS_t^c < 1$, since young agents are constrained. The only difference with respect to the case of a fixed population (Section 4.2.2) is that, with a mobile population, the derivative on the left-hand side of equation

[45] See the analog equation (4.13) for the case of fixed population.

(C.7) is equal to - rather than strictly less than - the term on the right-hand side.

Thus, as in the case of fixed population, the current land price declines in response to an increase in pension funding $\widetilde{f}'$. The optimal policy deviation for an old policymaker is therefore to set $\widetilde{f}' = f_{\min}$. This is the same result obtained when population is fixed (Proposition 7).

In general equilibrium, the measure of young agents born in each period needs to settle somewhere in the economy. Therefore, if all locations are homogeneous, each of them absorbs a measure one of young agents:

$$n_t = 1.$$

This condition pins down the equilibrium utility level V^*. That is, geographic mobility does not insulate young agents from the effects of reduced pension funding if *all localities* pursue the same funding policy. Specifically, since in general equilibrium $n_t = 1$, land market clearing requires that:

$$L\left(d_t, q_{t+1}\right) = 1.$$

Thus, the analysis of Section 4.2 applies and the lifetime utility achieved by a young agent must equal to

$$V^* = \overline{V}\left(g\left(q_{t+1}\right), q_{t+1}\right),$$

with $\overline{V}\left(g\left(q_{t+1}\right), q_{t+1}\right)$ defined in equation (4.6). That is, the lifetime utility achieved by a young agent in this economy is increasing in the future price of land q_{t+1} (Proposition 5). In turn, the future price of land Q^*, defined in equation (4.10), is increasing in f^*. Recall that when the policymaker is old, $f^* = f_{\min}$. It follows that the equilibrium utility level V^* achieved by a young agent is lowest when old agents are in charge of setting pension funding policy. This is the same result as in the benchmark model with fixed population.

D Robustness Analysis

In Table D.1 we report regression results corresponding to using as independent variable of interest the homeownership rate of households under 55 in 1990 instead of 2012.

Table D.1: Robustness: Lagged Measures of Age and Ownership

	UAAL/population		UAAL/income	
% owners under 55 (1990)	-79.0**	-38.0**	-0.37**	-0.23**
	(13.1)	(12.7)	(0.06)	(0.06)
controls[a]	NO	YES	NO	YES
R^2	0.18	0.31	0.21	0.34
	UAAL/revenues		UAAL/house values	
% owners under 55 (1990)	-2.30**	-1.66**	-0.25**	-0.22**
	(0.66)	(0.78)	(0.04)	(0.05)
controls[a]	NO	YES	NO	YES
R^2	0.06	0.16	0.20	0.36
Number of cities	160	160	160	160

Robust standard errors in parenthesis. ** p-value<0.05
[a]Controls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies.

In Table D.2 we report regression results corresponding to alternative definitions of the explanatory variable of interest. In particular, we vary the age cutoff between young and old, and we also consider the role of home ownership. Each estimate represents a different regression with UAAL/population as the dependent variable but with different explanatory variables. All controls are included in each regression. The first column shows the sensitivity of the baseline results to age cutoffs. The second column performs the same exercise, but considers the age distribution for all households instead of only homeowners. The third column reports the results of regressing the UAAL/population measure on the percent of households that are renters.

Table D.2: Robustness: Effect of age cutoffs and ownership on baseline estimates.

	Home owners	All households	Renters
% Under 35	-78.03*	-14.42	-6.72
	(45.85)	(18.35)	(18.85)
% Under 45	-58.81**	-26.40	-3.08
	(23.23)	(20.52)	(18.22)
% Under 55	-39.63**	-44.10*	2.35
	(14.00)	(25.55)	(16.46)
% Under 65	-30.32**	-52.64	10.47
	(11.93)	(33.90)	(13.80)

This table shows the robustness of baseline parameter estimates to age cutoffs and home ownership. Each point estimate represents a separate regression using UAAL/population as the dependent variable and different dummy variables as proxies for the young population. Each regression includes controls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies. Robust standard errors in parenthesis. ** p-value < 0.05 , * p-value<0.1

In Table D.3 we report regression results corresponding to dropping from the sample the 10 cities with the lowest homeownership rates. These are: Boston, Hartford, Jersey City, Miami, New Haven, New York City, Newark, Providence, Los Angeles, and San Francisco.

Table D.3: Robustness: Remove Low-Ownership Cities

	UAAL/population		UAAL/income	
% owners under 55	-51.2**	-32.4**	-0.26**	-0.22**
	(15.1)	(13.2)	(0.06)	(0.06)
controls[a]	NO	YES	NO	YES
R^2	0.06	0.12	0.09	0.18
	UAAL/revenues		**UAAL/house values**	
% owners under 55	-0.78	-0.31	-0.18**	-0.21**
	(0.62)	(0.71)	(0.04)	(0.05)
controls[a]	NO	YES	NO	YES
R^2	0.00	0.08	0.09	0.29
Number of cities	158	155	158	155

The ten cities with the lowest overall ownership rates were removed. Robust standard errors in parenthesis.
** p-value<0.05
[a]Controls include population density, liabilities per capita, the ratio of median income and median house values, log city population, percentage population change between 2000 and 2012, and regional dummies.