Math 114, HW 3

Due Friday, April 24

- Consider the following corollary of the compactness theorem:

  If $\Sigma \vDash \tau$ then there is a finite $\Sigma_0 \subseteq \Sigma$ such that $\Sigma_0 \vDash \tau$.

  Give a short proof of the compactness theorem from this.

- Let Σ be an effectively enumerable set of wffs. Assume that for each wff τ, either $\Sigma \vDash \tau$ or $\Sigma \vDash \neg\tau$. Show that the set of tautological consequences of Σ is decidable.

- Show that $\{\wedge, \leftrightarrow, +\}$ is complete but that no proper subset is complete. (Recall that $+$ is the exclusive or connective: $\alpha + \beta$ is true if exactly one of α and β is true.)