Math 114, HW 6

Due Friday, May 29

1. Consider a language with a single binary predicate symbol P and two constant symbols c, d. Show that there is no formula ϕ such that $\models_{\mathfrak{A}} \phi$ iff there exist $c^{\mathfrak{A}} = y_1, \ldots, y_n = d^{\mathfrak{A}} \in |\mathfrak{A}|$ such that $\langle y_i, y_{i+1} \rangle \in P^{\mathfrak{A}}$ for all $i < n$.

2. Assume a language has only finitely many functions symbols and predicate symbols. Let Σ be a set of sentences such that for any $\sigma \in \Sigma$, if σ has a counterexample (that is, there is a $\mathfrak{A}$ with $\models_{\mathfrak{A}} \neg\sigma$) then σ has such a counterexample with $|\mathfrak{A}|$ finite. Find an effective procedure which, given any $\sigma \in \Sigma$, will decide whether or not σ is valid.

3. Let $\Gamma = \{\neg\forall v_1 P v_1, P v_2, P v_3, \ldots\}$. Is Γ consistent? Is Γ satisfiable?

4. Carefully write up the proof of Step 4 in the proof of the completeness theorem (that when Δ is a complete set of formulas in the expanded language, there is a model $\mathfrak{A}$ satisfying ϕ^* whenever $\phi \in \Delta$). You may refer to the book and your notes, but write the argument up in your own words.