Math 114, HW 7

Due Friday, June 5th

1. Prove that any two countable dense linear orderings without endpoints are isomorphic. (See the hint in the textbook.)

2. Give prenex normal forms for:

   (a) $\exists x(Px \to \forall y Py)$

   (b) $\forall x \exists y Rxy \to \exists x \forall y Pxy$

3. Assume that σ is true in all infinite models of a theory T. Show that there is a finite number k such that σ is true in all models $\mathfrak{A}$ of T such that $|\mathfrak{A}|$ has at least k elements.

4. Let T_1, T_2 be theories such that $T_1 \subseteq T_2$, T_1 is complete, and T_2 is satisfiable. Show that $T_1 = T_2$. Given examples showing that all three conditions are necessary. (That is, for each of the three conditions, given an example of two non-equal theories satisfying the other two conditions but not that one.)