MIDTERM 1

Math 114
4/24/2009

Name: $\underbrace{\hspace{10cm}}$

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	15	
3	15	
4	20	
5	15	
6	20	
Total	100	

1. (*15 points*) Give translations of each of the following into sentential logic (you may assign any sentence symbols you wish to atomic sentences, but be consistent across different parts). If the sentence is ambiguous, give all reasonable interpretations (but you need not give multiple tautologically equivalent interpretations):

(**a**) If the cat is out of the bag then the king is angry and we will all hang.

(**b**) Unless the king is angry, either there will be a feast tonight or the army will ride (but not both).

(**c**) The cat is out of the bag and there will be no feast tonight and the army will ride and we will all hang.

2. (*15 points*) Given English equivalents to each of the following sentences of sentential logic. Be careful to make the English version unambiguous, even if the phrasing is a bit awkward.

(**a**) $A \wedge (B \vee \neg C) \rightarrow D$

(**b**) $\neg D \rightarrow \neg A \wedge C$

3. (*15 points*) Determine whether the following formulas are tautologically equivalent using a truth table:

- $(A \wedge B) \to C$
- $A \to (B \to C)$

4. (*20 points*) **(a)** During the proof of the compactness theorem, from an arbitrary finitely satisfiable set Σ of wffs, we construct a finitely satisfiable set $\Delta \supseteq \Sigma$ such that for every wff α, either $\alpha \in \Delta$ or $\neg\alpha \in \Delta$. Show that Δ need not be unique by describing an infinite, finitely satisfiable set Σ of wffs such that there is more than one possible extension Δ.

(b) Show that the condition in the Compactness Theorem that Σ be finitely satisfiable cannot be weakened to the property "every subset of Σ of size 3 is satisfiable". That is, give an example of a set Σ such that every subset of Σ of size 3 is satisfiable, but Σ is not satisfiable.

The remaining questions on this exam will refer to a first order logic in which there is a single 0-ary function symbol C (that is, a constant), a single binary function function symbol P, and the equality relation.

5. (*15 points*) Which of the following are well-formed formulas in this first order logic (allowing for the standard abbreviations)? If not, explain why not.

(**a**) $PCPPCv_4$

(**b**) $Pv_1C = PPCCC$

(**c**) $\forall v_2 PCPv_5 = PPv_2v_1v_3$

(**d**) $\forall v_1 \wedge v_2(PCv_1 = C)$

(**e**) $C = C \vee (\forall v_2 PCC = v_2)$

6. (*20 points*) Call a term of this language *closed* if it contains no variables. That is, C is closed, and if α and β are closed then $P\alpha\beta$ is closed.
We wish to define a recursive function, $mirror$, with the following properties:

- $mirror(C) = C$
- $mirror(P\alpha\beta) = P(mirror(\alpha))(mirror(\beta))$

(**a**) Give functions h, h_P such that the corresponding $\overline{h}$ given by the recursion theorem is $mirror$.

(**b**) Prove that $mirror(mirror(\alpha)) = \alpha$ for every closed term α.