MIDTERM 2

Math 114
5/15/2009

Name:

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	15	
3	20	
4	15	
5	15	
6	20	
Total	100	

1. (*15 points*) In each of the following formulas, determine whether $fxyz$ is substitutable for x. If not, explain why not.

(**a**) $\forall x \exists y (Rxy \to \forall z Qxyz)$

(**b**) $\exists w (Rxy \to \forall z Qwyz)$

(**c**) $\forall y (Rxy \to \exists z Qwyz)$

(**d**) $\exists w (Rxy \to \forall z Qwwx)$

2. (*15 points*) Consider a language with a single constant symbol, $\mathbf{0}$, and a single binary function, $\mathbf{F}$. Let $\mathfrak{A} = (\mathbb{N}, 0, +)$ and let $s(v_i) = i$. (Recall that the notation $s(x \mapsto n)$ has the same meaning as $s(x \mid n)$.) Find:

(a) $\overline{s}(\mathbf{F}v_7\mathbf{F}\mathbf{0}v_3)$

(b) $\overline{s(v_2 \mapsto 4)}(\mathbf{F}v_7\mathbf{F}\mathbf{0}v_3)$

(c) $\overline{s(v_3 \mapsto 4)}(\mathbf{F}v_7\mathbf{F}\mathbf{0}v_3)$

(d) Does $\models_{\mathfrak{A}} \forall x \mathbf{F}\mathbf{F}xv_1v_3 = \mathbf{F}xv_4[s]$?

(e) Does $\models_{\mathfrak{A}} \forall x \mathbf{F}\mathbf{F}xv_1v_3 = \mathbf{F}xv_4[s(v_3 \mapsto 4)]$?

3. (*20 points*) State whether the following are axioms. If they are, state which group they belong to; if not, change a *single symbol* to make it an axiom.

(**a**) $\forall v_4(\forall v_2\neg\forall v_3\neg\mathbf{P}v_2v_3 = \mathbf{P}v_3\mathbf{0} \to \neg\forall v_3\neg\mathbf{P}\mathbf{P}v_0v_3v_3 = \mathbf{P}v_3\mathbf{0})$

(**b**) $\forall v_5(\mathbf{P}v_5\mathbf{0} = \mathbf{0} \to \forall v_4 v_4 = v_5) \to \forall v_5\mathbf{P}v_5\mathbf{0} = \mathbf{0} \to \forall v_5\forall v_4 v_4 = v_5$

(**c**) $\mathbf{P}v_5 = \mathbf{0} \to \forall v_5\mathbf{P}v_5 = \mathbf{0}$

(**d**) $\forall v_6\forall v_2\forall v_3(\mathbf{P}v_2\mathbf{0} = \mathbf{P}v_3v_6 \wedge \forall v_1\mathbf{P}v_2\mathbf{0} = v_1 \to \mathbf{P}v_2\mathbf{0} = \mathbf{P}v_3v_6)$

4. (*15 points*) Prove that there is a derivation of the formula $\forall x(\alpha \vee \beta) \to \exists x\alpha \vee \exists x\beta$.

5. (*15 points*) Prove that there is not a derivation of the formula $\forall x(\alpha \vee \beta) \to \forall x\alpha \vee \forall x\beta$.

6. (*20 points*) Consider a language containing a single binary predicate $\mathbf{Q}$, and consider some model $\mathfrak{A}$ where the universe is the natural numbers, $\mathbb{N}$. Suppose that

$$\{n, m \mid \models_{\mathfrak{A}} \mathbf{Q}xy[s(x \mapsto n, y \mapsto m)]$$

is decidable. Show that

$$\{n \mid \models_{\mathfrak{A}} \exists y \mathbf{Q}xy[s(x \mapsto n)]$$

is effectively enumerable.