MIDTERM 1

Math 114
4/23/2010

Name:

Read all of the following information before starting the exam:

- Check your exam to make sure all pages are present.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	15	
3	25	
4	25	
5	20	
Total	100	

1. (*15 points*) (**a**) Prove that $(A_1 A_2 \wedge A_3)$ is not a wff. (You may use any theorems proven in class or in the textbook if you wish.)

(**b**) Give an example of an infinite set of wffs Σ such that Σ is not tautologically equivalent to any finite set of wffs.

2. (*15 points*) Consider the following three formulas. Indicate all tautological implications among them:

1. $\neg A \rightarrow B$
2. $\neg((A \rightarrow B) \rightarrow (\neg(B \rightarrow A)))$
3. $\neg((A \vee B) \wedge (\neg A \vee \neg B))$

3. (*25 points*) Recall the ternery connective $\mathbb{I}$, with the property that $\mathbb{I}\alpha\beta\gamma$ is assigned the value T if exactly one of the formulas α, β, γ is assigned the value T.

(**a**) Prove that $\{\mathbb{I}, \top\}$ is complete.

(**b**) Prove that $\{\mathbb{I}\}$ is not complete. (Hint: can you make a formula α with sentence symbols A, B so that $\overline{\nu}(\alpha) = T$ when $\nu(A) = \nu(B) = F$?)

4. (*25 points*) (**a**) Prove that for every finite set $\Sigma = \{\sigma_1, \ldots, \sigma_n\}$ of wffs and every α, $\Sigma \models \alpha$ iff $\models ((\sigma_1 \wedge \cdots \wedge \sigma_n) \to \alpha)$.

(**b**) Show that if Σ is any (not necessarily finite) set of wffs and α is a wff such that $\Sigma \models \alpha$ then there are finitely many $\sigma_1, \ldots, \sigma_n \in \Sigma$ such that $\models ((\sigma_1 \wedge \cdots \wedge \sigma_n) \to \alpha)$.

5. (*20 points*) (**a**) Give a new (that is, one not used lecture or the text book) example of a first order language containing at least one predicate symbol which is not equality, and at least one function symbol. Describe an intended interpretation for this language.

(**b**) Write down a formula in this language, with at least one quantifier, whose intended interpretation is true.

(**c**) Write down a formula in this language, with at least one quantifier, whose intended interpretation is false.