MIDTERM 2

Math 114
5/14/2010

Name: $\underbrace{\hspace{6cm}}$

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
Total	100	

1. (*15 points*) Consider a language with a single constant symbol $\mathbf{e}$, a unary function $\mathbf{S}$, and a binary predicate $\mathbf{R}$. Consider the model $\mathfrak{A}$ with $|\mathfrak{A}| = \mathbb{N}$, $\mathbf{e}^{\mathfrak{A}} = 0$, $\mathbf{S}^{\mathfrak{A}}$ the function taking n to $n+1$, and $\mathbf{R}^{\mathfrak{A}} = \{(n,m) \mid n < m\}$. Let $s(v_i) = i$ for all i. (Recall that the notation $s(x \mapsto n)$ has the same meaning as $s(x \mid n)$.)

Find: (**a**) $\overline{s}(\mathbf{S}v_3)$

(**b**) $\overline{s(v_2 \mapsto 4)}(\mathbf{Se})$

(**c**) $\overline{s(v_2 \mapsto 4)}(\mathbf{S}v_2)$

(**d**) Does $\models_{\mathfrak{A}} \forall v_2 \mathbf{ReS}v_2[s]$?

(**e**) Does $\models_{\mathfrak{A}} \forall v_3 \mathbf{ReS}v_2[s]$?

2. (*15 points*) Consider the first-order language with a single 2-place relation symbol $<$. Let $\mathfrak{Z}$ be a structure with $|\mathfrak{Z}| = \mathbb{Z}$ and $<^{\mathfrak{Z}}$ the usual ordering on the integers. Let $\mathfrak{N}$ be a structure with $|\mathfrak{N}| = \mathbb{N}$ and $<^{\mathfrak{N}}$ the usual ordering on the integers.
Show that the structures $\mathfrak{Z}$ and $\mathfrak{N}$ are not isomorphic.

3. (*20 points*) Prove that there is not a derivation of

$$\forall x \neg \forall y \mathbf{R} xy \to \neg \forall y \mathbf{R} yy.$$

4. (*20 points*) Prove, using induction on deductions, that whenever $\Gamma; \beta \vdash \gamma$, also $\Gamma; \alpha \to \beta \vdash \alpha \to \gamma$.

5. (*15 points*) Consider the first-order language with a single 2-place relation symbol $<$, a unary relation symbol $\mathbf{P}$, and a constant symbol $\mathbf{0}$. Consider the model $\mathfrak{N}$ where $|\mathfrak{N}| = \mathbb{N}$, $<^{\mathfrak{N}}$ is the usual ordering on the integers, $\mathbf{P}^{\mathfrak{N}}$ is the set of primes, and $\mathbf{0}^{\mathfrak{N}}$ is 0. Express the following sentences with formulas in this language:

(**a**) Every prime number is greater than 0.

(**b**) There is only one prime number.

6. (*15 points*) Prove that there is a deduction of

$$\exists x(\mathbf{P}x \wedge \mathbf{Q}x) \to \exists x \mathbf{P}x.$$

Be sure to justify each step.