# Math 135 Lecture 2

#### Final Examination Information

### March 11, 2009

The final exam will cover material covered through the entire quarter. Approximately one quarter of the points will be for material covered prior to the midterm (the first two sections below), and the remaining three quarters will be on material since then.

The following is a list of material you should know for the exam:

- Laplace Transformations (Sections 48, 50-52)
  - Definition of the Laplace and inverse Laplace transform
  - Laplace transforms of common functions (see table on page 393, but also  $L[\sqrt{x}]$  and the Dirac  $\delta$  "function")
  - Computing new Laplace transforms from existing ones (using sum, product, derivative, integral, etc.)
  - The convolution, Laplace transforms of convolutions
  - Functions with growth of exponential order, and their relationship to th definability of the Laplace transform
  - General shape of Laplace transforms of functions (in particular, that  $F(p) \to 0$  as  $p \to \infty$
  - The  $\Gamma$  function
  - Using the Laplace transform to solve linear differential equations
  - Using the Laplace transform to solve integral equations
  - Indicial and impulsive responses, and how to use them to solve differential equations
- Picard Iteration (Sections 68, 69)
  - The definition of the Picard iteration for an initial value problem
  - The statements of Picard's Theorem (both Theorem A and B from Section 69)
  - The Lipschitz condition
  - Determining whether the Lipschitz condition holds

– Be able to reproduce and understand (that means being able to make small changes to fit a particular case) the proofs of Theorems A and B

#### • Fourier Series (Sections 33-43)

- Computing the Fourier coefficients and Fourier series given a function
- Mean versus pointwise convergence
- Dirichlet's Theorem
- Even and odd functions
- Fourier sine and Fourier cosine series for functions on  $[0,\pi)$
- Orthogonal functions
- Orthonormal series (and the difference between an orthogonal and an orthonormal series)
- Computing the generalize Fourier series given orthogonal functions
- Mean convergence, mean square error
- Theorems 1 through 5 in Section 38
- Finding eigenvalues and eigenfunctions
- The solutions to the vibrating string and heat equations: both how to do each step of the derivation and how those solutions relate to Fourier series
- Orthogonal sequences relative to a weight functions
- Sturm-Liouville equations: how we solve them, the relationship to orthogonal sequences

## • Calculus of Variations (Sections 65-67)

- Euler's equation
- Special cases of Euler's equation
- Solving problems in the calculus of variations
- Euler's equation with multiple functions
- Euler's equation with a side constraint (isoperimetric problems)
- The basic method of the derivation of Euler's equation

#### • Legendre Polynomials (Sections 44,45)

- The Legendre equation
- The Rodriguez formula
- The proof that the Legendre polynomials are orthogonal
- The Legendre series of a function
- The solution to the heat equation and how it relates to Fourier series