Math 135, Extra Problem Solutions

for Final Exam

1. $\frac{1}{1+p^2} + \frac{2}{p^3}$
2. (a) $\frac{p+2}{1+(p+2)^2}$

   (b) $L[x^{-3/2}] = L[-2 \cdot (x^{-1/2})'] = -2pL[x^{-1/2}] = -2\sqrt{\pi p}$
3. $\cos 2x + \frac{1}{2}\sin 2x$
4. ..
5. x^2e^x
6. $2e^x - x - 1$
7. $y_0 = 1, y_1 = 1 + x, y_2 = \frac{1}{3} + \frac{2}{3}(1 + x)^{3/2}$
8. $y(x) = \int_0^x t + y(t)dt, y_0 = 0, y_1 = x^2/2, y_2 = x^2/2 + x^3/6$
9. Only $\frac{1}{x+1}$ is Lipschitz on $[0, \infty)$. Only $1/x$ is NOT Lipschitz on $[0, 2]$.
10. In general, $a_n = 2\frac{n\cosh(\pi)\sin(n\pi)+\cos(n\pi)\sinh(\pi)}{(1+n^2)\pi}$ and $b_n = 2\frac{-\cosh(\pi)\sin(n\pi)+\cos(n\pi)\sinh(\pi)}{(1+n^2)\pi}$. So a suitable answer might be $a_0 = 2\sinh(\pi)/\pi$, $b_1 = -\sinh(\pi)/\pi$.
11. In general, $a_n = 0$ and $b_n = \frac{2\sin(n\pi)-2n\pi\cos(n\pi)}{n^2\pi}$
12. Only the first function converges at every point. The first and fourth converge at all but finitely many points. The second and third fail Dirichlet's Theorem because they have infinitely many minima and maxima. All four functions have Fourier series which converge to the function in the mean.
13. $b_1 = 2, b_2 = -1$
14. $a_0 = 2\pi^2/3$, $a_1 = -4$
15. $\int_a^b[\theta_n + \theta_m]^2dx = \int_a^b \theta_n^2dx + 2\int_a^b \theta_n\theta_m dx + \int_a^b \theta_m^2 dx = 1 + 0 + 1 = 2$
16. The first series is orthogonal, but not orthonormal. The second series is not orthogonal. The third series is orthonormal.
17. $a = \int_{-1}^1 x/\sqrt{2}dx$, $b = \int_{-1}^1 x \cdot \sqrt{x}dx$

18. $b_n = \frac{2}{\pi}\int_0^\pi -x \sin nx dx = (-1)^n \frac{2}{n}$, so $w(x,t) = \sum_{n=1}^{\infty} (-1)^n \frac{2e^{-n^2a^2t}}{n} \sin nx$

19. For example $y_1 = \frac{x}{2\sqrt{\pi}}$ and $y_2 = \frac{x^2}{\sqrt{48\pi - 8\pi^3}}$, since $\int_{-\pi}^{\pi} \cos x y_1^2 dx \neq 0$, $\int_{-\pi}^{\pi} \cos x y_2^2 dx \neq 0$, but $\int_{-\pi}^{\pi} \cos x y_1 y_2 dx = 0$.

20. $\int_0^\pi \sqrt{x} y_1(x) y_2(x) dx = 0$

21. The only stationary function is $y(x) = 1$

22. Euler's equation gives $y(y')^2 = c$. This gives $\sqrt{y}\frac{dy}{=}\sqrt{c}dx$, so $\frac{2}{3}y^{3/2} = \sqrt{c}x + d$, and therefore $y = (Cx + D)^{2/3}$. Since $1 = D^{2/3}$, $D = 1$, and since $0 = (C+1)^{2/3}$, C must be -1, so $y = (-x+1)^{2/3}$.

23. Euler's equation gives $xz''(x) + z'(x) = 0$ and $xy''(x) + y'(x) = 0$. So $y = c_1 \ln x + c_2$ and $z = d_1 \ln x + d_2$. The solutions are $y = (\pi - 2)\ln x + 2$, $z = -4 \ln x + 3$.