**Homework 7**

**5.1, 41.** Since $f(x) = -x^2 + 2$ is continuous on $[-1, 2]$, and differentiable on $(-1, 2)$, therefore by MVT, there exists $c$ in $(-1, 2)$, such that $f'(c) = \frac{f(2)-f(1)}{2-(-1)} = -1$.

**42.** Since $f(x) = x^3$ is continuous on $[-1, 0]$, and differentiable on $(-1, 0)$, therefore by MVT, there exists $c$ in $(-1, 0)$ thus in $(-1, 1)$, such that $f'(c) = \frac{f(1)-f(0)}{1-0} = 1$.

**43.** We should draw a function that is continuous on $[0, 1]$ and differentiable on $(0, 1)$, the graph is omitted. For the second statement the reason is that: by MVT, since $f(x)$ is a function that is continuous on $[0, 1]$ and differentiable on $(0, 1)$, therefore there existse a point $c$ in $(0, 1)$, such that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

**44.** We should draw a function that is continuous on$[0, 1]$ and differentiable on $(0, 1)$, and should have two "peaks". The graph is omitted. For the second statement the reason is that: by MVT, since $f(x)$ is a function that is continuous on $[0, 1]$ and differentiable on $(0, 1)$, therefore there existse a point $c$ in $(0, 1)$, such that $f'(c) = \frac{f(1)-f(0)}{1-0}$.

**46.** Since f is continuous on $[a, b]$, and differentiable on $(a, b)$, and $f(b) - f(a) > 0$, then by MVT, there exists $c$ in $(a, b)$, such that $f'(c) = \frac{f(b)-f(a)}{b-a} > 0$.

**47.** since $f(x)$ is not constant, then we can find a point $d$ in $(a, b)$ such that $f(d) \neq 0$, so $f(d) > 0$ or $f(d) < 0$. If $f(d) > 0$, apply MVT on $[a, d]$, we get there exists a point $c_1$ in $(a, d)$, such that $f(c_1) = \frac{f(d)-f(a)}{d-a} > 0$, since $f(d) - f(a) = f(d) > 0$; and then apply MVT on $[d, b]$, we get that there exists a point $c_2$ in $(d, b)$, such that $f(c_2) = \frac{f(b)-f(d)}{b-d} < 0$, since$f(b) - f(d) = -f(d) < 0$. Similarly if $f(d) < 0$, we can also find such two points satisfy the required conditions.

**5.2, 6.** $y = (x - 2)^3 + 3$, $x \in R$.
$y' = 3(X - 2)^2 \geq 0$ for all $x \in R$, therefore $f(x)$ is increasing on $R$.
$y'' = 6(x - 2)$, then when $x > 2$, $y'' > 0$, thus $y$ concave up; then when $x < 2$, $y'' < 0$, thus $y$ concave down.

**7.** $y = \sqrt{x + 1}$, $x \geq -1$.

$y' = \frac{1}{2\sqrt{x+1}} \geq 0$, for all $x \geq -1$, thus $y$ is increasing on $x \geq -1$.

$y'' = \frac{-1}{4(x+1)^{\frac{3}{2}}} < 0$, for $x > -1$, therefore $y$ is concave down for $x > -1$.

**8.** $y = (3x - 1)^{\frac{1}{3}}$, $x \in R$

$y' = \frac{1}{3}(3x - 1)^{-\frac{2}{3}} \geq 0$, thus the function is increasing on $R$.

$y'' = -\frac{2}{9}(3x - 1)^{-\frac{5}{3}}$:when $x < \frac{1}{3}$, $y'' > 0$, thus concave up;
when $x > \frac{1}{3}$, $y'' < 0$, thus concave down.

**9.** $y = \frac{1}{x}$, $x \neq 0$

$y' = -\frac{1}{x^2} < 0$ for all $x \neq 0$, thus the function is decreasing.

$y'' = \frac{2}{x^3}$: when $x > 0$, $y'' > 0$, thus concave up;
when $x < 0$, $y'' < 0$, thus concave down.

**31** $f(P) = e^{-aP}$, then $f'(P) = -ae^{-aP} < 0$, therefore $f(P)$ decreases.
**32** $f(P) = \left(1 + \frac{aP}{k}\right)^{-k}$, then $f'(P) = -a\left(1 + \frac{aP}{k}\right)^{-k-1} < 0$, since $P$ and $k$ are both positive constants,

therefore $f(P)$ decreases.

**5.3 2.** $y = \sqrt{x-1}, \quad 1 \le x \le 2.$
$y' = \frac{1}{\sqrt{x-1}} > 0$, for $1 < x \le 2$, therefore the function is increasing. And for $1 \le x \le 2$, the local maximum is $(2, f(2)) = (2, 1)$, and the local minimum is $(1, f(1)) = (1, 0)$, therefore the absolute maximum is $(2, 1)$, and absolute minimum is $(1, 0)$.

**3.** $y = ln(2x-1)$, $1 \le x \le 2$.
$y' = \frac{2}{2x-1} > 0$ for all $1 \le x \le 2$, thus the local maximum is $(2, f(2)) = (2, ln3)$, and the local minimum is $(1, f(1)) = (1, 0)$, therefore the absolute maximum is $(2, ln3)$, and absolute minimum is $(1, 0)$.

**4.** $y = ln\frac{x}{x+1}, \quad x > 0.$
$y' = \frac{1}{x(x+1)} > 0$, for all $x > 0$. therefore, there is no maximum or minimum.

**(**5. )$y = xe^{-x}, \quad 0 \le x \le 1$
$y' = e^{-x}(1-x) \ge 0$, for all $0 \le x \le 1$, therefore the function is increasing. thus the local maximum is $(1, f(1)) = (1, \frac{1}{e})$, and the local minimum is $(0, f(0)) = (0, 0)$, therefore the absolute maximum is $(1, \frac{1}{e})$, and absolute minimum is $(0, 0)$

**19.** $f(x) = x^3 - 2, x \in R.$
$f''(x) = 6x$, let $f''(x) = 0$. we get $x = 0$, and when $x > 0$, $f''(x) > 0$, and when $x < 0$, $f''(x) < 0$, therefore, at $x = 0$, the concavity changes, therefore, $x = 0$ is the inflection point of $f(x)$.

**20.** $f(x) = (x-3)^5, \quad x \in R.$
$f''(x) = 20(x-3)^3$, let $f''(x) = 0$, we get $x = 3$, and when $x > 3$, $f''(x > 0)$, and when $x < 3$, $f''(x) < 0$, therefore, at $x = 3$, the concavity changes, therefore $x = 3$is the inflection point of $f(x)$.

Draw a function f so that $f'(x)$ is negative when $x$ is negative, $f'(x)$ is positive when $x$ is positive, but $f(0)$ is not a minimum.
The function $f(x) = - \mid \frac{1}{x} \mid$ for $x \neq 0$ works.