MIDTERM 1

Math 3A
10/19/2009

Name: $\underbrace{\hspace{8cm}}$

Signature: $\underbrace{\hspace{8cm}}$

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Whenever you invoke a theorem to justify a result, make sure to clearly identify all premises of the theorem, show that they are true, and specify which theorem you are using.
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
7	10	
Total	100	

1. (*15 points*) Find the following limits if they exist; if not, state that they do not exist, and indicate if they go to ∞ or $-\infty$, indicate this as well. You may use any method you like, but clearly indicate intermediate steps and how you obtain your answer.

(**a**) $\lim_{x\to\infty} \tan x$

(**b**) Find $\lim_{x\to\pi/2^-} e^{\tan x}$

(**c**) Find $\lim_{x\to\infty} \frac{5x^4-7x^2+2}{3x^2+4}$

(**d**) Find $\lim_{x\to 1} \frac{1}{x+1}$

(**e**) Consider the sequence $a_n = \sin n\pi$. What is $\lim_{n\to\infty} a_n$?

2. (*15 points*) Compute

$$\lim_{x\to 0^-} \sin(2x)\cos\frac{2}{x}$$

3. (*15 points*) Show that the polynomial

$$x^3 - 100x^2 + 10$$

has at least two roots.

4. (*15 points*) For what value of a is the function

$$f(x) = \begin{cases} \frac{x^2-x-20}{x-5} & \text{if } x \neq 5 \\ a & \text{if } x = 5 \end{cases}$$

continuous everywhere?

5. (*15 points*) Given an example of three sequences, a_n, b_n, and c_n, such that:

- None of the limits $\lim_{n\to\infty} a_n$, $\lim_{n\to\infty} \frac{a_n}{b_n}$, $\lim_{n\to\infty} \frac{a_n}{c_n}$ exist
- The limit $\lim_{n\to\infty} \frac{a_n}{b_n c_n} = 1$

6. (*15 points*) (**a**) Find the derivative of $\sqrt{2x+1}$ using the chain rule.

(**b**) Find the derivative of $\sqrt{2x+1}$ using the definition of the derivative.

7. (*10 points*) Consider the equality

$$f(x+h)g(x+h)-f(x)g(x) = f(x)[g(x+h)-g(x)]+g(x)[f(x+h)-f(x)]+[f(x+h)-f(x)][g(x+h)-g(x)].$$

Draw a diagram illustrating this equality; each of the quantities in the equation should be represented as the area of some shape in this diagram.