Suggested practice problems from recent sections:

- 10.4: 3, 4, 7, 8, 13, 14, 19, 20
- 10.5: 17, 18, 27, 28, 35, 36
- 10.6: 1, 2, 3, 4, 40, 41, 42, 43

a

Approximate $\int_1^7 e^x dx$ as a Riemann sum with 3 equal intervals, choosing the left endpoint of each rectangle to be its height.

b

Give the general solution of the differential equation

$$\frac{dy}{dt} = \frac{e^y - 1}{e^y} t^2.$$

c

Recall that $\arcsin x = \int_0^x \frac{1}{\sqrt{1-t^2}} dt$. Show that when $y \geq 1$,

$$\arcsin \sqrt{1 - \frac{1}{y^2}} \leq y \sqrt{1 - \frac{1}{y^2}}.$$

d

You know that $2 \leq f(x) \leq 3$ for all x. Is it possible that $\int_2^5 f(x)dx = 4$?

e

What is $\int_{-1}^{-1} \frac{\cos x}{x} dx$?

f

Find a value $a > 0$ such that $\int_1^a \frac{\sin(x-2)}{(x-2)^2} dx = 0$.

g

Define $F(x) = \int_0^x \frac{\sin t}{t} dt$. What is $\frac{d}{dx} F(\ln x)$?

h

Water is flowing into a container at a rate of $W(t) gal/sec$ (where t is the time). Express the amount of water that enters the container between $t = 0$ and $t = 4$.

i

What is the partial fraction decomposition of

$$\frac{1}{(x^2+4)^3(x^2+1)^2(x-1)^3(x+2)}$$

j

Find and solve the partial fraction decomposition for

$$\frac{1}{(x^2+1)(x^2-1)}$$

k

Integrate:

1. $\int x^2 \ln x^3 dx$
2. $\int \frac{x}{\sqrt{1-x^2}} dx$
3. $\int \arcsin x \ dx$
4. $\int \frac{1}{x^4-1} dx$
5. $\int \frac{1}{4x^2+8x+29} dx$
6. $\int_1^\infty \frac{\ln x}{x} dx$
7. $\int_1^{-\infty} e^x dx$

l

Describe the domain, range, and level curves of $\ln(x^2+y^2-1)$.

m

Find the following partial derivatives:

1. $\frac{\partial}{\partial x}(x^3+xy+\ln x)$
2. $\frac{\partial}{\partial y} e^{xe^{xy}}$
3. $\frac{\partial^2}{\partial x \partial y} e^{xe^{xy}}$
4. $\frac{\partial}{\partial y} \ln xy$
5. $\frac{\partial^3}{\partial y \partial x \partial y} e^{x^2y^2}$
6. $\frac{\partial}{\partial z} \ln(xy+xz+yz)$

n

Indicate whether the following statements are *(A)lways True*, *(S)ometimes True*, or *(N)ever True*.

1. A function that is continuous at (x, y) is also differentiable at (x, y)
2. If f is differentiable at (x, y) then the partial derivative $\frac{\partial f}{\partial x}$ is exists at (x, y)
3. If f is differentiable and $\nabla f \neq 0$, ∇f is the direction in which f decreases most rapidly
4. If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ both exist at (x, y) then $\nabla f(x, y)$ is defined
5. If $f, f_x, f_y, f_{xy}, f_{yx}$ are both defined and continuous at (x, y) then the mixed partials are equal at (x, y)

o

Find and classify all critical points of $x^3y - 4xy^3 + y$

p

Find and classify all critical points of $e^{xy} - e^{2xy}$.

q

Find the candidates for where e^{xy} achieves its minimum on the circle $x^2+y^2 = 1$.

r

Find the candidates for where e^{xy} achieves its minimum on the hyperbola $x = 1/y$.

s

Find the candidates for where $x^2 + y^2$ achieves its minimum on the hyperbola $x = 1/y$.

t

1. Find and classify as stable or unstable the equilibria of
$$\frac{dy}{dt} = (y - 3)(e^y - e).$$
2. y_0 is a solution with $y_0(0) = 0$. What is $\lim_{t\to\infty} y_0$?

3. y_1 is a solution with $y_1(0) = 1$. What is $\lim_{t\to\infty} y_1$?
4. y_2 is a solution with $y_2(0) = 2$. What is $\lim_{t\to\infty} y_2$?
5. y_3 is a solution with $y_3(0) = 3$. What is $\lim_{t\to\infty} y_3$?
6. y_4 is a solution with $y_4(0) = 4$. What is $\lim_{t\to\infty} y_4$?

u

Give an example of an autonomous differential equation which has x^3 as a solution.