10.1.6

$f(3,-1,1) = \sqrt{3^3 - 3\cdot(-1)+1} = \sqrt{9+3+1} = \sqrt{13}$

10.1.16

The function $e^{-(x^2+y^2)}$ is defined everywhere, so the domain is $\mathbb{R}^2$. $-(x^2+y^2)$ has range all non-positive numbers, so the range of $e^{-(x^2+y^2)}$ is a number in the interval $(0,1]$.

The level curves are $c = e^{-(x^2+y^2)}$ where $0 < c \le 1$, so $-\ln c = x^2+y^2$, which is the circle with radius $-\ln x$.

10.1.20

Note that $f(-x,y) = \sin(-x)\sin y = -\sin x \sin y$ and $f(x,-y) = \sin x \sin(-y) = -\sin x \sin y$; that is, the function is anti-symmetric along both the x and y axes. Only figure 10.22 has this property.

10.1.22

$f(x,y) = 4 - x^2$ does not depend on y. Only figure 10.21 has this property.

10.2.8

$$\begin{aligned}
\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} &= \frac{\lim_{(x,y)\to(1,1)} xy}{\lim_{(x,y)\to(1,1)}(x^2+y^2)} \\
&= \frac{(\lim_{(x,y)\to(1,1)} x)(\lim_{(x,y)\to(1,1)} y)}{\lim_{(x,y)\to(1,1)} x^2 + \lim_{(x,y)\to(1,1)} y^2} \\
&= \frac{(\lim_{x\to 1} x)(\lim_{y\to 1} y)}{\lim_{x\to 1} x^2 + \lim_{y\to 1} y^2} \\
&= \frac{1\cdot 1}{1^2+1^2} \\
&= \frac{1}{2}
\end{aligned}$$

10.2.20

When $y = mx$, $m \neq 0$,

$$\begin{aligned}
\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{x^3+y^6} &= \lim_{x\to 0} \frac{3x^2(mx)^2}{x^3+(mx)^6} \\
&= \lim_{x\to 0} \frac{3m^2x^4}{x^3+m^6x^6} \\
&= \lim_{x\to 0} \frac{3m^2x}{1+m^6x^2} \\
&= 0
\end{aligned}$$

while when $x = y^2$,

$$\begin{aligned}
\lim_{(x,y)\to(0,0)} \frac{3x^2y^2}{x^3+y^6} &= \lim_{y\to 0} \frac{3(y^2)^2y^2}{(y^2)^3+y^6} \\
&= \lim_{y\to 0} \frac{3y^6}{2y^6} \\
&= 3/2.
\end{aligned}$$

Since these are not equal, the limit does not exist.

10.2.22

$$\begin{aligned}
\lim_{(x,y)\to(0,0)} \sqrt{9+x^2+y^2} &= \sqrt{\lim_{(x,y)\to(0,0)} (9+x^2+y^2)} \\
&= \sqrt{(\lim_{(x,y)\to(0,0)} 9) + (\lim_{(x,y)\to(0,0)} x^2) + (\lim_{(x,y)\to(0,0)} y^2)} \\
&= \sqrt{9 + (\lim_{x\to 0} x^2) + (\lim_{y\to 0} y^2)} \\
&= \sqrt{9+0+0} \\
&= 3
\end{aligned}$$

while $\sqrt{9+x^2+y^2} = \sqrt{9+0+0} = 3$. So the limit and the function both exist, and the values are equal, so the function is continuous.

10.2.26

$\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist, therefore the function is not continuous at $(0,0)$.

10.3.12

$\frac{\partial}{\partial x} e^{-y^2} \cos(x^2 - y^2) = -2xe^{-y^2} \sin(x^2 - y^2)$
$\frac{\partial}{\partial y} e^{-y^2} \cos(x^2 - y^2) = -2ye^{y^2} \cos(x^2 - y^2) + 2ye^{y^2} \sin(x^2 - y^2)$

10.3.14

$\frac{\partial}{\partial x}\ln(3x^2-xy)=\frac{6x-y}{3x^2-xy}$

$\frac{\partial}{\partial y}\ln(3x^2-xy)=\frac{-x}{3x^2-xy}$

10.3.34

$\frac{\partial}{\partial x}\frac{xyz}{x^2+y^2+z^2}=\frac{(x^2+y^2+x^2)yz-2x^2yz}{(x^2+y^2+z^2)^2}$

Since the function is symmetric, the other partial derivatives are the same, change variables accordingly: $\frac{\partial}{\partial y}\frac{xyz}{x^2+y^2+z^2}=\frac{(x^2+y^2+x^2)xz-2xy^2z}{(x^2+y^2+z^2)^2}$

$\frac{\partial}{\partial z}\frac{xyz}{x^2+y^2+z^2}=\frac{(x^2+y^2+x^2)xy-2xyz^2}{(x^2+y^2+z^2)^2}$

10.3.42

$\frac{\partial^2}{\partial y\partial x}\sin(x-y)=\frac{\partial}{\partial y}\frac{\partial}{\partial x}\sin(x-y)=\frac{\partial}{\partial y}\cos(x-y)=\sin(x-y)$

10.3.46

$\frac{\partial^3}{\partial y^2\partial x}e^{x^2-y}=\frac{\partial}{\partial y}\frac{\partial}{\partial y}\frac{\partial}{\partial x}e^{x^2-y}=\frac{\partial}{\partial y}\frac{\partial}{\partial y}2xe^{x^2-y}=\frac{\partial}{\partial y}-2xe^{x^2-y}=2xe^{x^2-y}$