6.1

6.1.1

Approximate $\int_{-1}^{1} \arccos x dx$ using 4 equal intervals and left endpoints.

6.1.2

Approximate $\int_0^2 \frac{1}{1+x^2} x dx$ using 6 equal intervals and right endpoints.

6.1.3

Approximate $\int_2^3 \frac{1}{1+e^x} dx$ using 3 equal intervals and midpoints.

6.1.4

Sketch an example of graph and indicate an interval where using Riemann sums with left endpoints will always lead to an *underestimate*.

6.1.5

Suppose you know that f and g are continuous functions such that:

- $\int_0^1 f(x)dx = 1$
- $\int_0^2 f(x)dx = 2$
- $\int_0^1 g(x)dx = 2$
- $\int_0^2 g(x)dx = 1$

What are:

1. $\int_1^2 2f(x) - g(x)dx$?
2. $\int_2^0 f(x) + g(x)dx$?

Is it consistent with the information given that $f(x) + g(x) \leq 2$ for all x?
Is it consistent with the information given that $f(x) - g(x) \leq 2$ for all x?

6.1.6

Recall that for $t > 0$, $\ln t = \int_1^t \frac{1}{x} dx$. Show using the geometric definition of the integral that $\ln(1 + 1/t) \leq 1/t$ for all $t > 0$.

6.2

6.2.1

Find

$$\frac{d}{dx} \int_{e^x}^{e^{2x}} x^2 dx.$$

6.2.2

Find

$$\frac{d}{dx}\int_{x^2}^{x} f(u)du$$

in terms of f and f'.

6.3

6.3.1

Find the area bounded by the curves $y = x^2$ and $y = 1 - x^2$.

6.3.2

If $w(t)$ represents the rate that water is evaporating from a lake at time t, what does $\int_0^{10} w(t)dt$ represent?

6.3.3

What is the average value of e^x on the interval $[0, 1]$?

6.3.4

A child grows by one foot over the course of a year (not a leap year). Show that there is some moment where the child is growing at the rate of exactly $\frac{12}{365}$ inches per day.

Integrals

Finally, some mixed integrals, which could include any method covered. Find the following integrals if the integrand is continuous, otherwise indicate that the integrand is not continuous:

- $\int_0^2 x^4 + 4x + 1 \ dx$
- $\int_{-1}^1 e^x - x \ dx$
- $\int_0^1 x \sin x^2 \ dx$
- $\int_{-2}^{-1} \frac{1}{x} dx$
- $\int_{-1}^1 \frac{1}{1+x^2} dx$
- $\int_{-2}^2 \cos x e^{\sin x} \ dx$
- $\int_0^1 \sqrt{1 - x^2} \ dx$
- $\int_{-4}^0 \tan x \ dx$