The exam covers the following material:

- Section 7.1
- Section 7.2
- Section 7.3
- Section 7.4.1
- Section 8.1
- Section 8.2

Questions from Section 7.1

Question 7.1.a

$$\int \frac{1}{x}(\sin \ln x)(\cos \ln x)dx.$$

Question 7.1.b

$$\int_1^2 x^5\sqrt{x^2-1}dx$$

Question 7.1.c

$$\int \sin x \sec^2(\cos x)dx$$

Question 7.1.d

$$\int \tan^2 x \sec^2 x dx$$

Question 7.1.e

$$\int \sec x \ dx$$

(Hint: make the substitution $u = \sec x + \tan x$)

Question 7.1.f

$$\int x^2(1-2x)^{2/3} \ dx$$

Question 7.1.g

Suppose $\int \frac{1}{\ln^2 x+1} dx = g(x)$. Express

$$\int \frac{x}{\ln^2 x + 1/4} dx$$

in terms of $g(x)$.

Question 7.1.h

Is the function

$$F(x) = \int_0^x e^{t^2} dt$$

even, odd, or neither?

Questions from Section 7.2

Question 7.2.a.1

Derive a reduction formula for $\int \sin^n x \, dx$ which expresses this integral in terms of $\int \cos^2 x \sin^{n-2} x \, dx$.

Question 7.2.a.2

Using the previous part and the substitution $\cos^2 x = 1 - \sin^2 x$, give a formula for $\int \sin^n x \, dx$ in terms of $\int \sin^{n-2} x \, dx$.

Question 7.2.a.2

Find $\int \sin^6 \, dx$ using the formula from the previous part.

Question 7.2.b

g is an unknown continuous function with the property that $g''' = g$. Find

$$\int g(\ln x) dx$$

(The answer may include using the functions g, g', and g''.)

Question 7.2.c

$$\int \ln x^{3/2} dx$$

Questions from Section 7.3

Question 7.3.a

Find the partial fraction decomposition for

$$\frac{1}{(x-1)^3(x^2+4)^2(4x+1)}$$

(you do not need to solve for the values)

Question 7.3.b

Find and solve the partial fraction decomposition for

$$\frac{-2x^2+4x+15}{(2x+1)(x^2+6)}$$

Question 7.3.c

Find

$$\int \frac{x^2-1}{(x+1)(4x^2+25)}dx$$

Question 7.3.d

Find r, s so that

$$\frac{1}{(x+r)(x+s)} = \frac{1}{x+r} - \frac{1}{x+s}$$

Questions from Section 7.4.1

Find the following integrals or indicate that they diverge:

1. $\int_2^\infty e^{-x}dx$
2. $\int_{-\infty}^{-2} \frac{1}{\sqrt{x}}dx$
3. $\int_{-\infty}^{\infty} \sin x \; dx$
4. $\int_0^\infty e^{x^2}dx$

Question 7.4.a

For which values of $p > 0$ does

$$\int_e^\infty \frac{1}{x\ln^p x}dx$$

converge.

Question 7.4.b

Suppose that f is a function which is continuous everywhere and that for some a,

$$\int_{-\infty}^{a} f(x)dx \text{ and } \int_{a}^{\infty} f(x)dx$$

both exist. Prove that for any b,

$$\int_{-\infty}^{b} f(x)dx \text{ and } \int_{b}^{\infty} f(x)dx$$

both exist and

$$\int_{-\infty}^{a} f(x)dx + \int_{a}^{\infty} f(x)dx = \int_{-\infty}^{b} f(x)dx + \int_{b}^{\infty} f(x)dx.$$

Questions from Section 8.1

Question 8.1.a

Solve the following differential equations:

- $\frac{dy}{dt} = t \sec y$
- $N' = \frac{N^3}{t^2}, N(0) = 1$
- $\frac{dS}{dt} = e^{t+S}, S(0) = 2$

Questions from Section 8.2

Question 8.2.a.1

Identify, and classify as stable or unstable, the equilibria of:

$$\frac{dA}{dt} = (A-4)\ln(|A| + 1/2)$$

Question 8.2.a.2

You know that $A(t)$ satisfies the equation $\frac{dA}{dt} = (A-4)\ln(|A|+1/2)$ and $A(0) = 2$. What is $\lim_{t\to\infty} A(t)$?

Question 8.2.b

Identify, and classify as stable or unstable, the equilibria of:

$$\frac{dy}{dt} = \sin y$$

Mixed Integrals

1. $\int \sqrt{1-3x}\ dx$
2. $\int_0^1 \sqrt{1-3x}\ dx$
3. $\int_{-1}^1 e^{7x+3}\ dx$
4. $\int \frac{e^x+1}{e^x}\ dx$
5. $\int e^{\sqrt[3]{x}}\ dx$
6. $\int \arctan x\ dx$