The Rules of Natural Deduction

The only axiom is

$$\frac{}{\Gamma \vdash \phi} \text{ where } \phi \in \Gamma.$$

The structural rule is weakening:

$$W \; \frac{\Gamma \vdash \phi}{\Gamma, \Sigma \vdash \phi}$$

The logical rules are:

$$\bigwedge I \; \frac{\Gamma \vdash \phi \qquad \Sigma \vdash \psi}{\Gamma, \Sigma \vdash \phi \wedge \psi}$$

$$\bigwedge E_L \; \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \phi}$$

$$\bigwedge E_R \; \frac{\Gamma \vdash \phi \wedge \psi}{\Gamma \vdash \psi}$$

$$\bigvee I_L \; \frac{\Gamma \vdash \phi}{\Gamma \vdash \phi \vee \psi}$$

$$\bigvee E \; \frac{\Gamma \vdash \phi \vee \psi \qquad \Sigma, \phi \vdash \theta \qquad \Upsilon, \psi \vdash \theta}{\Gamma, \Sigma, \Upsilon \vdash \theta}$$

$$\bigvee I_R \; \frac{\Gamma \vdash \psi}{\Gamma \vdash \phi \vee \psi}$$

$$\to I \; \frac{\Gamma, \phi \vdash \psi}{\Gamma \vdash \phi \to \psi}$$

$$\to E \; \frac{\Gamma \vdash \phi \to \psi \qquad \Sigma \vdash \phi}{\Gamma, \Sigma \vdash \psi}$$

$$\neg I \; \frac{\Gamma, \phi \vdash \psi \qquad \Sigma, \phi \vdash \neg\psi}{\Gamma, \Sigma \vdash \neg\phi}$$

$$\neg E \; \frac{\Gamma \vdash \neg\neg\phi}{\Gamma \vdash \phi}$$

$$\forall I \; \frac{\Gamma \vdash \phi[y/x]}{\Gamma \vdash \forall x \phi}$$

Where y does not occur free in Γ, ϕ and is substitutable for x in ϕ

$$\forall E \; \frac{\Gamma \vdash \forall x \phi}{\Gamma \vdash \phi[t/x]}$$

Where t is substitutable for x in ϕ

$$\exists I \; \frac{\Gamma \vdash \phi[t/x]}{\Gamma \vdash \exists x \phi}$$

Where t is substitutable for x in ϕ

$$\exists E \; \frac{\Gamma, \phi[y/x] \vdash \psi \qquad \Sigma \vdash \exists x \phi}{\Gamma, \Sigma \vdash \psi}$$

Where y does not occur free in Γ, ψ, ϕ and is substitutable for x in ϕ

Equality adds a new axiom:

$$\text{Refl} \; \frac{}{\Gamma \vdash t = t}$$

and a new family of rules:

$$\text{Equiv} \; \frac{\Gamma \vdash s = t \qquad \Sigma \vdash \phi[s/x]}{\Gamma, \Sigma \vdash \phi[t/x]}$$

where s and t are both substitutable for x in ϕ.