Definition 0.1. A *sequent* is a sequence of formulas $\sigma_0, \ldots, \sigma_n \vdash \tau$, which we write as $\sigma_0, \ldots, \sigma_n \vdash \tau$. The list $\sigma_0, \ldots, \sigma_n$ may be empty, which we write as $\vdash \tau$.

The rules for the sequent calculus we are using are:

- Axiom:
  - For any wff, $\alpha \vdash \alpha$ is an axiom.
- Structural Rules:
  - (Weakening) From $\sigma_0, \ldots, \sigma_n \vdash \alpha$, deduce $\sigma_0, \ldots, \sigma_n, \tau \vdash \alpha$
  - (Contraction) From $\sigma_0, \ldots, \sigma_n, \alpha, \alpha \vdash \beta$, deduce $\sigma_0, \ldots, \sigma_n, \alpha \vdash \beta$
  - (Exchange) From $\sigma_0, \ldots, \sigma_i, \sigma_{i+1}, \ldots, \sigma_n \vdash \alpha$, deduce $\sigma_0, \ldots, \sigma_{i+1}, \sigma_i, \ldots, \sigma_n \vdash \alpha$
- Logical Rules:
  - ($\wedge_I$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha$ and $\tau_0, \ldots, \tau_m \vdash \beta$, deduce $\sigma_0, \ldots, \sigma_n, \tau_0, \ldots, \tau_m \vdash \alpha \wedge \beta$
  - ($\wedge_E$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha \wedge \beta$, deduce $\sigma_0, \ldots, \sigma_n \vdash \alpha$
  - ($\wedge_E$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha \wedge \beta$, deduce $\sigma_0, \ldots, \sigma_n \vdash \beta$
  - ($\vee_I$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha$, deduce $\sigma_0, \ldots, \sigma_n \vdash \alpha \vee \beta$
  - ($\vee_I$) From $\sigma_0, \ldots, \sigma_n \vdash \beta$, deduce $\sigma_0, \ldots, \sigma_n \vdash \alpha \vee \beta$
  - ($\vee_E$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha \vee \beta$ and $\tau_0, \ldots, \tau_m, \alpha \vdash \gamma$ and $\upsilon_0, \ldots, \upsilon_k, \beta \vdash \gamma$, deduce $\sigma_0, \ldots, \sigma_n, \tau_0, \ldots, \tau_m, \upsilon_0, \ldots, \upsilon_k \vdash \gamma$
  - ($\rightarrow_I$) From $\sigma_0, \ldots, \sigma_n, \alpha \vdash \beta$, deduce $\sigma_0, \ldots, \sigma_n \vdash \alpha \rightarrow \beta$
  - ($\rightarrow_E$) From $\sigma_0, \ldots, \sigma_n \vdash \alpha \rightarrow \beta$ and $\tau_0, \ldots, \tau_m \vdash \alpha$, deduce $\sigma_0, \ldots, \sigma_n, \tau_0, \ldots, \tau_m \vdash \beta$
  - ($\neg_I$) From $\sigma_0, \ldots, \sigma_n, \alpha \vdash \beta$ and $\tau_0, \ldots, \tau_m, \alpha \vdash \neg\beta$, deduce $\sigma_0, \ldots, \sigma_n, \tau_0, \ldots, \tau_m \vdash \neg\alpha$
  - ($\neg_E$) From $\sigma_0, \ldots, \sigma_n \vdash \neg\neg\alpha$, deduce $\sigma_0, \ldots, \sigma_n \vdash \alpha$

Sample deductions:

1. $\alpha \vdash \alpha$ (Ax)
2. $\vdash \alpha \to \alpha$ ($\to_I$)

1. $\alpha \vdash \alpha$ (Ax)
2. $\alpha, \beta \vdash \alpha$ (W)
3. $\alpha \vdash \beta \to \alpha$ ($\to_I$)
4. $\vdash \alpha \to (\beta \to \alpha)$ ($\to_I$)

1. $(\alpha \wedge \beta) \to \gamma \vdash (\alpha \wedge \beta) \to \gamma$ (Ax)
2. $\alpha \vdash \alpha$ (Ax)
3. $\beta \vdash \beta$ (Ax)
4. $\alpha, \beta \vdash \alpha \wedge \beta$ ($\wedge_I$ from 2,3)
5. $(\alpha \wedge \beta) \to \gamma, \alpha, \beta \vdash \gamma$ ($\to_E$ from 1,4)
6. $(\alpha \wedge \beta) \to \gamma, \alpha \vdash \beta \to \gamma$ ($\to_I$)
7. $(\alpha \wedge \beta) \to \gamma \vdash \alpha \to (\beta \to \gamma)$ ($\to_I$)
8. $\vdash ((\alpha \wedge \beta) \to \gamma) \to (\alpha \to (\beta \to \gamma))$ ($\to_I$)