

Problem of the Week #2

Around King Arthur's Table

King Arthur was a happy king in Camelot who had all those knights and a round table. He loved inviting those knights over for parties around his round table.

If there was something pleasant he could only give to one knight, (an extra dessert, the hand of his daughter in marriage, a dragon to chase, etc.) he had them play a game, and it went like this.

First, King Arthur numbered the chairs around the table. At the start every chair was occupied by a knight. (King Arthur himself did not sit at the table.)

Then he stood behind the knight in chair one and said "You're in." Next he moved to the knight in the second chair and said "You're out." And the knight left his seat and went off to stand at the side of the room and watch the rest of the game. Next, he moved to the knight in chair three and said "You're in." Then he said "You're out." To the knight in chair four, and the knight left his seat and went to the side of the room.

He continued around the table in this matter. When he came back to the knight in chair one, he said either "You're in." or "You're out." Depending on what he said to the last knight. (If the last knight was "in", then the first knight was now "out", and vice versa.)

The king kept moving around and around the table saying "You're in." or "You're out" to the knights that remained at the table. (If the chair was now empty, he just skipped it.) He continued until only one knight was left sitting at the table. That knight was the winner.

Here is the question:

Which chair number would you try to sit at King Arthur's table?

Of course, you may want to know how many knights were at the table to begin with. Well that was always changing. So figure out a general rule, in terms of the number of knights. Your rule does not have to be a formula; it may be a clear set of directions that will give the proper answer.

Solution:

- 1) Every time the population of knights doubles from one (A power of two.) The first seat will be selected. For example: 1, 2, 4, 8, 16, 32, 64 and so on the winning seat is seat number 1.
- 2) Count the number of knights over the closest double that is less than the total number of knights. For example: If there are a total number of 30 knights, then the nearest double less than 30 is 16. That would give you a number of 14 knights over 16.
- 3) Multiply that number by 2 and add it to seat number 1. The result will be the seat selected by the king.

Method for solving the problem

I made a chart with the number of knights across one axis, and seats marked by the king on the next axis. This let me see who would be selected if there were only one knight or if there were over 30 knights. A regular pattern emerged from that data, and I was able to put it into words.