712, 2021: Labor, Health & Family

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Penn 2021

1 Life Cycle Stuff

Introduction: What people care

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$$E\left\{\sum_{i=1}^{I} \beta_i \; u(c_i, \ell_i, h_i) \; 1_{\text{alive}}\right\} \text{and health}$$

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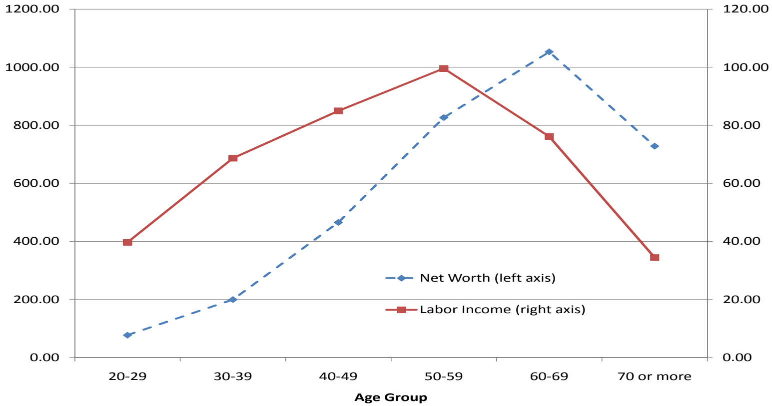
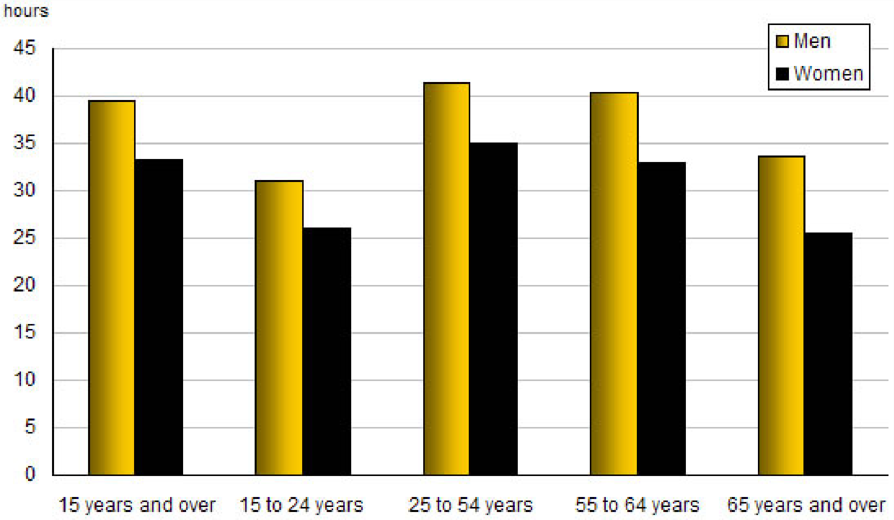
Motivating Facts: Income and Wealth Over Life Cycle

Figure: Labor Income and Net Worth by Age, SCF 2007 ($1,000)



Details of the Data

Hours worked by Age and Sex



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- Non-separability as in Cobb-Douglas

- Family composition details $u(c, n, z)$ or $u_i(c, n)$

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Theories of Men vs Women

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Theories of age profile of wages

1. Biology, i.e. exogenous.

$$w_i = w\ e_i, \quad y_i = w_i\ n_i$$

2. Learning by doing

$$w_i = w\ f(e_{i-1}, n_{i-1}) = w\ f(e_{i-1}, 1 - \ell_{i-1}),$$

3. learning by watching (not) doing

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HWK: Put it in the computer and solve them for $I = 45$.

Problems Stochastic, Recursive

$$v^i(\epsilon, a, e) = \max_{c,\ell,n,a} u(c_i, \ell_i) + \beta_i \; \mathrm{E}\{v^{i+1}(\epsilon', a', e') \mid \epsilon\} \qquad \text{s.t.}$$

$$a' = c + a(1+r) + w \; e \; n$$

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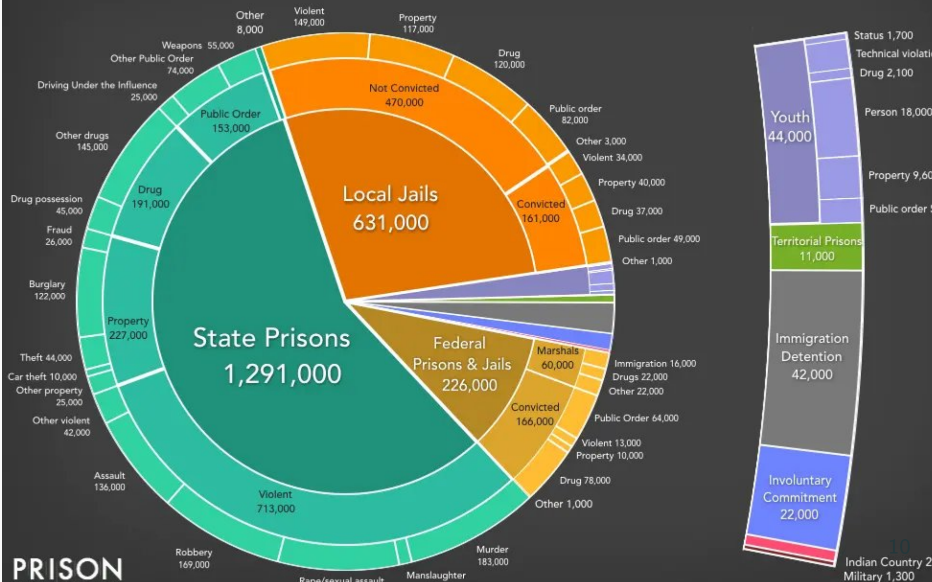
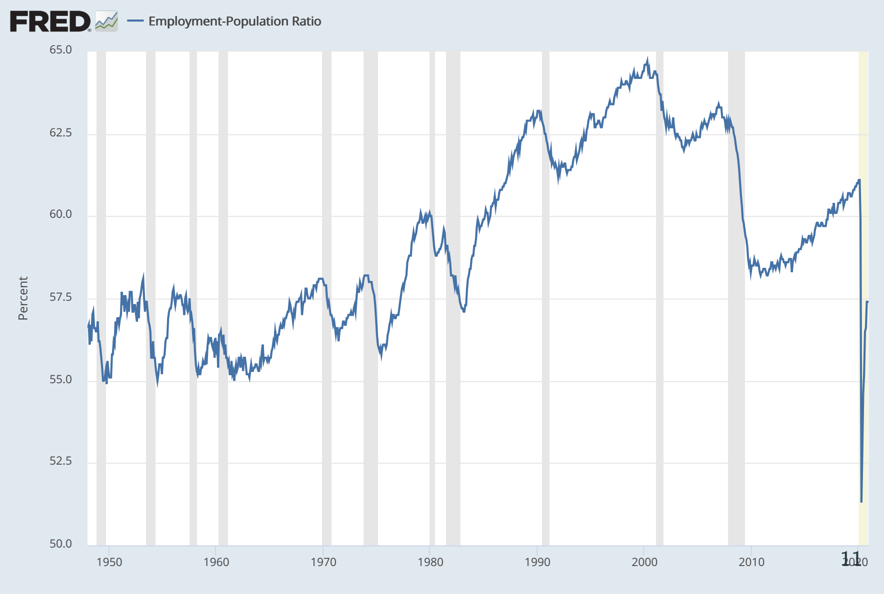
HWK: Solve it for $I = 45$, ϵ a Marvov chain and a random walk.

How many people are locked up in the United States?

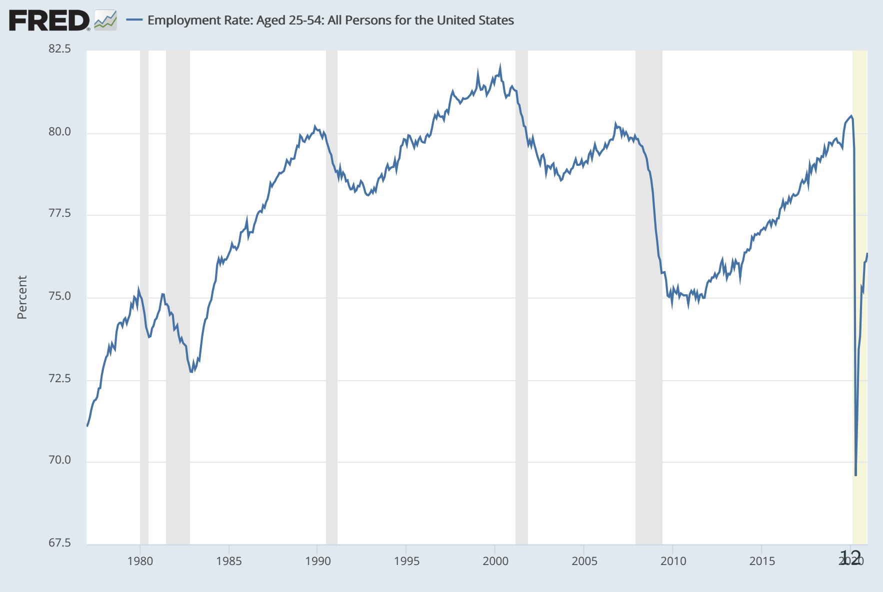
The U.S. locks up more people per capita than any other nation, at the staggering rate of 698 per 100,000 residents. But to end mass incarceration, we must first consider *where* and *why* 2.3 million people are confined nationwide.



Employment to Population Ratio What is going on?



Employment Rate 24-54. What is going on?



Motivating Facts: Portfolio Shares by Age from 2007 (in %)

Age Head	(1) Stk	(2) Res. RE	(3) Non bus.	(4) Non RE	(5) Risky NW	(6) Bond +CD	(7) Car	(8) Oth.	(9) Debt	(10) Safe NW
All	30.3	47.0	12.9	3.8	94.0	17.0	3.5	4.2	-18.6	6.0
20-29	13.2	77.7	43.3	1.3	135.5	13.7	15.3	4.5	-68.9	-35.5
30-39	26.3	96.5	12.7	5.0	140.4	13.8	9.7	4.2	-68.2	-40.4
40-49	30.4	57.6	12.6	3.8	104.4	15.2	4.4	4.5	-28.5	-4.4
50-59	32.7	42.4	13.5	3.7	92.4	17.0	2.8	4.0	-16.1	7.7
60-69	32.2	35.6	13.4	4.1	85.3	17.5	2.4	4.7	-9.9	14.7
70+	27.1	39.8	9.0	3.3	79.2	19.3	1.8	3.7	-3.9	20.8

Risky Net Worth (5) is equal to sum of columns (1)+(2)+(3)+(4). Safe Net Worth (10) is sum of columns (6)+(7)+(8)+(9). Total Net Worth is sum of (5)+(10)

2 What do People like in Each Other (RR Seitz & Tanaka)

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- Why?
- Is there a Systematic Difference in what Men and Women get from marriage?

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  3. yield large changes in marriage patterns.

     (age gap -Δ32%, married Δ20%, never-married -Δ33%, divorce rate Δ642%)

Men versus Women

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- We want to use revealed preference to infer from people's behavior how large are the gains that they perceive they have from marriage and at what ages these gains accrue.

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  3. Divorce is costly.

	Birth Cohort			% Change
	1870	1930	1950	in the Data
Demographics				
Sex ratio	1.040	0.997	0.939	-9.7
Life expectancy from 15 (female)	43.4	55.0	60.4	37.8
Life expectancy from 15 (male)	43.9	51.4	54.4	25.3
Age at first marriage				
Females	21.9	20.3	22.0	0.5
Men	25.9	22.8	24.7	-4.6
Percentage of never-married by age 50				
Females	10.4	5.7	7.6	-32.6
Men	12.9	6.6	9.3	-28.1
Married as a % of those aged 16-49				
Females	59.7	75.4	60.8	1.8
Men	50.8	69.4	56.1	10.4
Divorce rate, per 1,000 people	0.7	2.2	5.2	642.9

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Workings of the Stationary Population

- $i' = \begin{cases} i & \text{with prob } \Gamma^g_{ii} \\ i+1 & \text{with prob} \Gamma^g_{i,i+1} \\ \text{dead} & \text{with prob} \pi^g \end{cases}$

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- The measure of new adolescent single females is

$$n^f = \frac{\left[1 - \Gamma^f_{a,a}(1-\pi^f)\right]\left[1 - \Gamma^f_{y,y}(1-\pi^f)\right]\pi^f}{\left[1 - \Gamma^f_{y,y}(1-\pi^f) + \Gamma^f_{a,y}(1-\pi^f)\right]\pi^f + \Gamma^f_{a,y}(1-\pi^f)\Gamma^f_{y,o}(1-\pi^f)}.$$

The Model: Notation, Meeting and Marriage

Marital status: Single, dating or married $q \in \{0, 1, 2\}$.

Random dating: Prob $\psi^f = min\{1, \frac{x^m}{x^f}\}$. x^g measure of singles. Everybody that can, meets.

The Model: Notation, Meeting and Marriage II

Preferences: If single $u_i^g(0) = 0$. If married, $u_i^g(i^*) = \alpha_{i^*}^g + z$.

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Match quality z: $z = \mu + \epsilon$, $\mu \in \{\theta, 0, -\theta\}$ Markov w transition matrix Λ. Initial draws from Λ_0. ϵ, extreme value shock.

$$\Lambda = \begin{pmatrix} 1-\lambda_1 & \lambda_1 & 0 \\ \lambda_2 & 1-\lambda_2-\lambda_3 & \lambda_3 \\ 0 & \lambda_4 & 1-\lambda_4 \end{pmatrix}$$

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Divorce: Agents pay a cost c^d upon divorce.

States and Aggregates

- An agent is characterized by its sex and maturity i, g as well as by whether it is matched and how z, and if so, what is the maturity of the partner i and the quality of the match as assessed by itself q, and its partner q. For the purpose of record keeping in the measures of agents, x, and the value functions, V , we write the state excluding the realization of the extreme value shocks .

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- We have then that

$$x^{f,i}(s) = x^{f,i}(z, i, \mu, \mu^*) = x^{m,i}(z, i, \mu^*, \mu) \qquad \forall z \in \{1, 2\}, i, i^*, \mu, \mu^*.$$

Value for a single unmatched $\{g, i\}$ Before Extreme Value Shocks

$$\Omega^{g,i}(0,0) \;= \beta\;(1-\pi^g)\;\sum_{i'}\Gamma^g_{i,i'}\Bigg\{\;(1-\psi^g)\;V^{g,i'}(0) \\ +\;\psi^g\sum_{i^*,\mu,\mu^*}\frac{x^{g^*,i^*}(1,.)}{x^{g^*}(1,.)}\;\Lambda_0(\mu)\;\Lambda_0(\mu^*)\;V^{g,i'}(1,i^*,\mu,\mu^*)\Bigg\}.$$

Value conditional on being or becoming married I

$$
\begin{aligned}
\Omega^{g,i}(s,1) = u^{g,i}(s,2) \; + \; \beta\,(1-\pi^{g})\Bigg[& \\
& (1-\pi^{g^*}) \sum_{i',i^{*\prime},\mu',\mu^{*\prime}} \Gamma^{g}_{i,i'}\Gamma^{g^*}_{i^*,i^*}\Lambda^{i'}_{\mu,\mu'}\Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}}\; V^{g,i'}(2,i^{*\prime},\mu',\mu^{*\prime}) \\
& \qquad + \pi^{g^*}\sum_{i'}\Gamma^{g}_{i,i'}\Big((1-\psi^{g})V^{g,i'}(0)+ \\
& \qquad\qquad \psi^{g}\sum_{i^{*\prime},\mu,\mu^*} p^{g}(i^{*\prime})\Lambda_0(\mu)\Lambda_0(\mu^*)V^{g,i'}(1,i^{*\prime},\mu,\mu^*)\Big)\Bigg].
\end{aligned}
$$

VALUE CONDITIONAL ON BEING OR BECOMING MARRIED II

- If the matched agent gets divorced, need to add

$$\Omega^{g,i}(s,0) = -c^d + \ \Omega^{g,i}(0,0).$$

The agent will like to choose

$$\max \ \ \left\{\Omega^{g,i}(s,1) + \epsilon^1, -c^d + \epsilon^0 \ V^{g,i}(0)\right\}.$$

- But both have to agree. With extreme value shocks (Gumbel) the probability that an $\{i, g, s\}$ agent prefers to be married is

$$p^{g,i}(s) = \frac{\exp\left[\Omega^{g,i}(s,1)\right]}{\exp\left[\Omega^{g,i}(s,0) + \Omega^{g,i}(s,1)\right]}$$

- We now obtain the beginning of period value for matched or married (given the decision rule of the partner if any) $s = \{z, i^*, \mu, \mu^*\}$ using again the convenient extreme value formulae

Value conditional on being or becoming married III

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$$V^{g,i}(s) = \ln\left[\exp\Omega^{g,i}(s,1) + \exp\Omega^{g,i}(s,0)\right] p^{g^*,i^*}(s^*(s)) + \Omega^{g,i}(s,0)\left[1 - p^{g^*,i^*}(s^*(s))\right]$$

where obviously $p^{i,g}(0) = 0$.

- A steady state is just a set of measures $x^{g,i}(s)$, values $V^{g,i}(s)$, and choices $p^{g,i}(s)$ such that agents choose optimally, and their choices both generate the value functions and yield the measures as steady state distributions of agents. It is standard, double check.

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* C (6) Preferences (α_j^g)

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* A (3) Demographic parameters, (mortality immigration). Observable.
* B (1) Divorce (cost)
* C (6) Preferences (α_j^g)
* D (4) Aging Process Γ

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* (12) Divorce rates for each of six groups
* (2) Marriage Indicdence
* (2) Average Age at First Marriage

Version of the Model	(1)		(2)			(3)			(4)
Parameters	None		(A)col:A			(A+B)col:A+B		(A+B+C)col:A-	
Cohort	All	1870	1930	1950	1870	1930	1950	1870	1930
(A) Demographics									
Sex ratio	0.992	1.040	0.997	0.939	1.040	0.997	0.939	1.040	0.997
Life expectancy (female)	52.9	43.4	54.9	60.4	43.4	54.9	60.4	43.4	54.9
Life expectancy (male)	42.9	43.9	51.4	54.4	43.9	51.4	54.4	43.9	51.4
(B) Divorce cost	3.33		3.02		5.55	5.37	3.25	5.63	5.41
(C) Preference									
α_a^f Fem for *a* males	-1.72		-4.12			-1.94		-2.12	-2.88
α_y^f Fem for *y* males	-0.09		0.02			-0.23		-0.52	0.13
α_o^f Fem for *o* males	-0.59		-0.14			-0.55		-0.65	-0.41
α_a^m Males *a* females	-5.14		-5.43			-4.27		-4.46	-4.99
α_y^m Males for *y* females	-0.09		-0.28			-0.06		-0.16	0.11
α_o^m Males for *o* females	-0.78		-0.51			-0.64		-0.67	-0.79
(D) Aging process									
Female									
Age of becoming prime	17.3		17.0			17.3			17.1
Age of becoming old	29.3		29.1			27.8			29.7
Male									
Age of becoming prime	18.8		17.7			17.9			17.7
Age of becoming old	31.0		30.0			29.2			30.9
(E) Love shock process									
Gain in *H*-state (θ)	1.28		0.88			1.25			1.31
Prob. *H*-state to *H*-state	1.00		1.00			1.00			1.00
Prob. *M*-state to *H*-state	0.47		0.47			0.42			0.42
Prob. *M*-state to *L*-state	0.15		0.21			0.20			0.20
Prob. *L*-state to *L*-state	0.72		0.71			0.70			0.67
(F) Marriage cost	3.11		2.74			3.11			3.10
(WSSE): E_i	0.152		0.117			0.082			0.029
Norm WSSE	0.181		0.139			0.098			0.035
Measure of Fit: $1-(E_i/E_1)$	0.000		0.231			0.459			0.804

Version of the Model	(1)		(4)			(5)	
Parameters	None	(A+B+C)col:A+B+C			(A+B+D)col:A+B+D		
Cohort	All	1870	1930	1950	1870	1930	1950
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α_o^f Fem for *o* males	-0.59	-0.65	-0.41	-0.71		-0.56	
α_a^m Males *a* females	-5.14	-4.46	-4.99	-4.15		-4.32	
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α_o^m Males for *o* females	-0.78	-0.67	-0.79	-0.61		-0.64	
(D) Aging process							
Female							
Age of becoming prime	17.3		17.1		17.3	17.3	17.3
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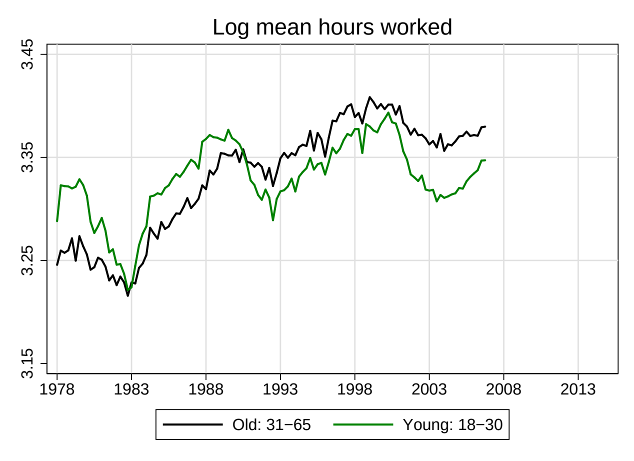
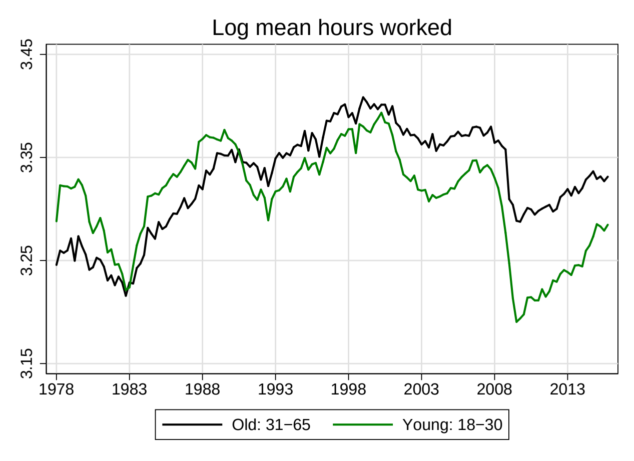
3 Living Arrangements and Labor Market Volatility of Young Workers (Dyrda, Kaplan RR)



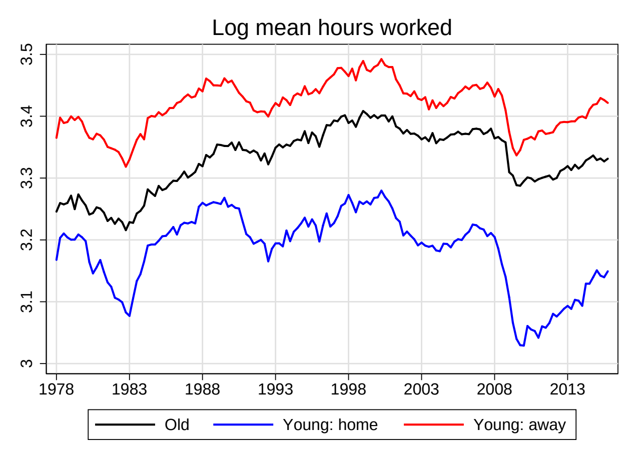
- Young people (18-30) larger cyclical volatility in “normal" cycles

Hours fluctuations for young people



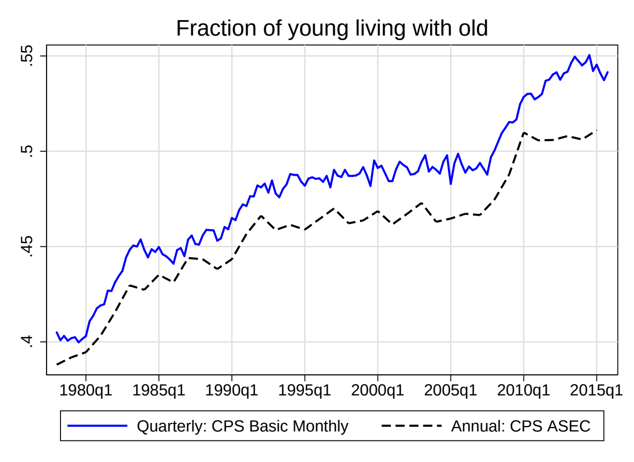
- Young people (18-30) larger cyclical volatility in "normal" cycles
- Harder hit during Great Recession

Living arrangements matter more than age

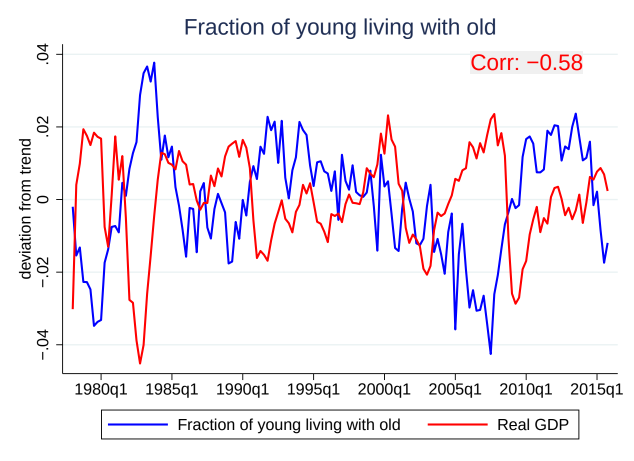


- Roughly half of 18-30 live with a 31-65 (home), half don't (away)
- Young people away: higher average hours, lower volatility
- Additional volatility concentrated among young at home

Living arrangements: endogenous, countercylical



- Secular upward trend since 1980
- Increased by >5pp during Great Recession, barely fallen



- Counter-cyclical pre and post Great Recession

This paper

1. Quantitative theory of fluctuations in living arrangements and hours worked for young relative to old

    - Co-residence trade-off: implicit transfers vs disutility
    - Labor supply more responsive to wages: wedge between Marshallian elasticity of young living away vs together

This paper

1. Quantitative theory of fluctuations in living arrangements and hours worked for young relative to old
2. Estimate model with aggregate data
   - Relative hours, wages by age and coresidence
   - Dynamics of living arrangements
   - De-trended from 1978 to 2006
   - Key identifying assumptions:
     a. Selection: functional forms for dist of unobservables
     b. Labor supply vs demand: conditional on skills, living arrangements do not affect productivity

This paper

1. Quantitative theory of fluctuations in living arrangements and hours worked for young relative to old
2. Estimate model with aggregate data
3. Use estimated model as measurement device
   a. Size of implicit transfers? 16.7% of consumption of the yount
   b. Difference in Marshallian elasticity by living arrangements? 18% higher for young living with old
   c. Importance of coresidence for hours of young?
      - Possibility of in coresidence: 37% of variance
      - Endogeneity in coresidence: 6% of variance
   d. Labor supply vs demand for hours volatility of young?
   e. Implications for Frisch elasticity in RA models? 85% larger

This paper

1. Quantitative theory of fluctuations in living arrangements and hours worked for young relative to old

2. Estimate model with aggregate data

3. Use estimated model as measurement device

4. Interpret Great Recession experience of young relative to old

    - Given dynamics for hours of old, were hours, wages and living arrangements of young in line with expectations based on previous recessions?

Data: 1978-2015

- CPS Basic Monthly Surveys for hours (monthly)

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- CPS ASEC for wages (annual)

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- Detrending:
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  - 2007-2010: Great Recession
  - 2011-2015: Great Recession recovery

Living arrangements and hours of young, 78-06

Definitions:

- Population: 18-65 yr olds not in school
- Young: 18-30
- Old: 31-65
- Young away: no old people in household
- Young together: ≥ 1 old person in household

Living arrangements and hours of young, 78-06

Definitions:

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Quarterly moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean hours	1.00		
St dev log hours	1.58		

Living arrangements and hours of young, 78-06

Definitions:

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Quarterly moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean hours	1.00	1.10	0.88
St dev log hours	1.58	1.32	1.89

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Quarterly moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean hours	1.00	1.10	0.88
St dev log hours	1.58	1.32	1.89

- St dev log fraction young with old $\approx$ 0.8
- Cyclical correlation with hours worked ≈ -0.6

Annual moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean wages	0.65		
St dev log wages	1.07		

- Labor demand mechanism - Jaimovich, Pruitt, Siu (2013):
  - Technology with imperfect substitutability between old and young
  - Quantitative argument requires Frisch for young $= 7$, old $= \infty$

Living arrangements and wages of young, 78-06

Annual moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean wages	0.65	0.75	0.52
St dev log wages	1.07	1.18	1.11

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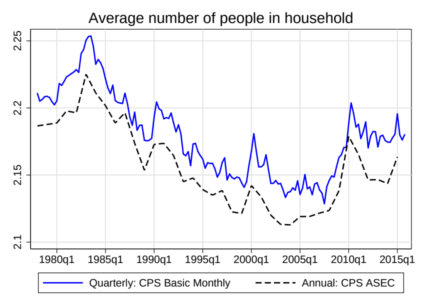
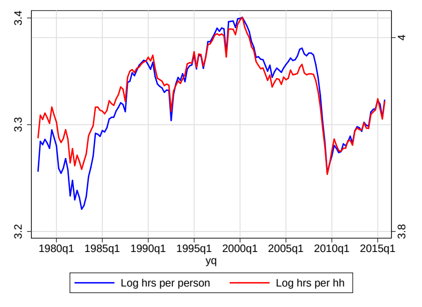
- Labor supply mechanism - this paper:
  - Selection into living arrangements
  - Imperfect substitutability by living arrangements implausible
  - Labor supply elasticities for old disciplined by micro estimates





- Household size moves a lot: trend and cyclical
- Hours per person more volatile than hours per household

- $H =$ total hours
- $N =$ number of individuals
- $F =$ number of households

$$\underbrace{\frac{H}{N}}_{\text{hours per person}} = \underbrace{\frac{H}{F}}_{\text{hours per household}} \div \underbrace{\frac{N}{F}}_{\text{persons per household}}$$

- Cyclical fluctuations

$$V\left(\log\frac{H}{N}\right) = \underbrace{V\left(\log\frac{H}{F}\right)}_{\text{hrs per hh}} + \underbrace{V\left(\log\frac{F}{N}\right)}_{\text{hh size}} \underbrace{-2COV\left(\log\frac{H}{F}, \log\frac{F}{N}\right)}_{\text{covariance term}}$$

Useful decomposition

$$V\left(\log\frac{H}{N}\right) = \underbrace{V\left(\log\frac{H}{F}\right)}_{\text{hrs per hh}} + \underbrace{V\left(\log\frac{F}{N}\right)}_{\text{hh size}} - \underbrace{2COV\left(\log\frac{H}{F}, \log\frac{F}{N}\right)}_{\text{covariance term}}$$

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	Cyclical Variance, 78-06		Great Recession Change, 07-10	
	Quarterly	Annual	Quarterly	Annual
hrs per hh	85%	92%	84%	85%
hh size	5%	3%	16%	15%
covariance	10%	5%		

Useful decomposition

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- Changes in household size offset around 8%-15% of changes in hours per person, at the household level

Useful decomposition 2

- Importance of endogeneity of coresidence: counterfactual series for hours assuming constant $x =$ fraction of young living with old

- All variation in hours is due to variation in hours of two groups:

$$M = \frac{V\left(\log h^{y}\right) - V\left(\log\left[\bar{x}h^{yT} + (1-\bar{x}h^{yA})\right]\right)}{V\left(\log h^{y}\right)} \approx 5\%$$

Model: Demographics

Old agents
- Identical
- Live in unitary households
- Can be invaded by a young agent

Young agents
- Two independent idiosyncratic shocks
- Individual productivity ε
- Distaste for living with old agents η
- Can invade an old households

So three typesof agents
- Old: μ
- Young alone: $(1-\mu)(1-x)$
- Young together (with old): $(1-\mu)x$

Old agents: Standard Rep Agent intertemporal problem

$$\begin{aligned} V^o(a; w^o, r) &= \max_{c^o, h^o, a'} u^o(c^o, h^o) + \beta \mathbb{E}\left[V^o\left(a'; w^{o'}, r'\right)\right] \\ \text{s.t.} \quad & c^o + a' = w^o h^o + (1+r)a \end{aligned}$$

Standard preferences

$$u^o(c, h) = \log c^o - \psi^o \frac{(h^o)^{1+\frac{1}{\nu^o}}}{1+\frac{1}{\nu^o}}$$

Aggregate uncertainty: w^o, r

Non-standard preferences of old

Young agents are hand-to-mouth

$$V^y(\varepsilon,\eta;w^y,c^o) \quad = \quad \max_{A,T}\{V^A(\varepsilon;w^y), V^T(\varepsilon,\eta;w^y,c^o)\}$$

Young alone

$$V^A(\varepsilon;w^y) \quad = \quad \max_{c,h} \frac{c^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}}$$

$$\text{s.t.} \qquad c = w^y \varepsilon h$$

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Young together

$$\begin{aligned} V^T(\varepsilon,\eta;w^y,c^o) &= \max_{c,h} \frac{[c+\zeta(c^o)]^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}} - \eta \\ \text{s.t.} \quad & c = w^y \varepsilon h \end{aligned}$$

Young agents are hand-to-mouth

$$V^y(\varepsilon,\eta;w^y,c^o) = \max_{A,T}\{V^A(\varepsilon;w^y), V^T(\varepsilon,\eta;w^y,c^o)\}$$

Young alone

$$V^A(\varepsilon;w^y) = \max_{c,h} \frac{c^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}}$$
$$\text{s.t.} \quad c = w^y \varepsilon h$$

Young together

$$V^T(\varepsilon,\eta;w^y,c^o) = \max_{c,h} \frac{[c+\zeta(c^o)]^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}} - \eta$$
$$\text{s.t.} \quad c = w^y \varepsilon h$$

Require $\gamma < 1$ for positive co-movement of wages and hours

Implicit transfers from old (economies of scale): $\zeta(c^o)$

Nested CES with capital-experience complementarity (Jaimovich-Pruitt-Siu, AER 2013)

$$F(K, N^y, N^o; Z) = \left[\alpha \left(Z_y N^y\right)^{\sigma} + (1-\alpha)\left(\lambda K^{\rho} + (1-\lambda)\left(Z_o N^o\right)^{\rho}\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}}$$

where N^y and N^o are labor inputs of young and old

- Technology generates higher hours and wage volatility for young

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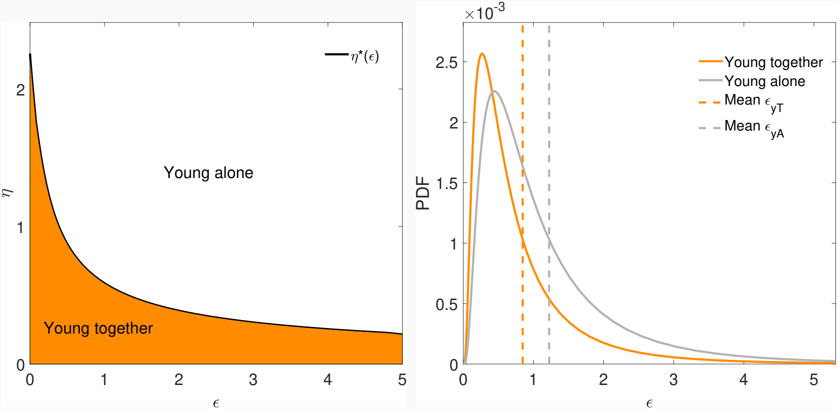
where N^y and N^o are labor inputs of young and old

- Technology generates higher hours and wage volatility for young
- Technology depends on age, but not living arrangements
- Structure on top of standard RBC model: shocks to Z_o and Z_y

Selection into living arrangements for young



Recursive Equilibrium: Aggregate state $s \equiv (K, Z_y, Z_o)$

- An equilibrium is a set functions
  - consumption $\{c^{yA}(\varepsilon, s), c^{yT}(\varepsilon, \eta, s), c^{o}(s\}$
  - hours worked $\{h^{yA}(\varepsilon, s), h^{yT}(\varepsilon, \eta, s), h^{o}(s)\}$
  - threshold for staying at home $\eta^*(s, \varepsilon)$
  - fraction of young that move in with the old $x(s)$

such that:

- old maximize given prices
- young maximize given prices and choice of old
- factor markets clear
- fraction of young living with old satisfies

$$x(s) = \int_0^\infty \int_{-\infty}^{\eta^*(s,\varepsilon)} dF_\eta \, dF_\varepsilon$$

where $\eta^*(s, \varepsilon)$ satisfies the indifference condition for all ε.

Model parameters: 26 parameters to discipline:

- Productivity heterogeneity: $\varepsilon \sim \log N$

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- Production parameters: α, λ, δ, σ, ρ
- Agg. shocks: $\{\rho_i, \sigma_\xi^i\}_{i=y,o}, Corr(\xi_o, \xi_y)$

1. Exogenously imposed parameters (Data + Micro Estimates): ν^o, μ,γ, ζ^o, ζ^y
2. Directly identified from steady-state conditions: β
3. Rest is targeted using Simulated Method of Moments (SMM):
   - There are 20 parameters to determine and we use 24 targets (next slide)
   - Both first and second moments are targeted. Cannot separate their identification in the model.

SMM: Mapping the model to the data

First Moments:

- Macro: I/Y, K/Y, h^o, $w^y h^y/Y$
- Living arrangements: $h^{yA}, h^{yT}, x, w^{yA}/w^o, w^{yT}/w^o$

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Second Moments:

- Properties of Solow Residual: $AutoCorr(TFP)$, $\sigma(TFP)$

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- Living arrangements moments: $\sigma(x)/\sigma(h^o)$, M, $Contr_{HF}$

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- Living arrangements moments: $\sigma(x)/\sigma(h^o)$, M, $Contr_{HF}$
- Correlations: $Corr(x, h), Corr(w^y, w^o), Corr(c, x), Corr(h^y, h^o)$

Model fit (1)

1.2 [b]

Moment	Weight	Data	Model
First Moments			
Capital/Output	1.0	7.50	7.67
Investment/Output	1.0	0.26	0.27
Mean Hours Old	1.0	0.52	0.52
Mean Hours Young Together	1.0	0.25	0.23
Mean Hours Young Alone	1.0	0.31	0.31
Fraction of Young living with Old	20.0	0.47	0.45
Wage of young alone/Wage Old	1.0	0.75	0.70
Wage of young together/Wage Old	1.0	0.52	0.46
Share of Old Labor Income in GDP	1.0	0.53	0.49

Model fit (2)

Moment	Weight	Data	Model
	Second Moments		
$\sigma(h^y)/\sigma(h^o)$	10.0	2.48	2.24
$\sigma(h^{yT})/\sigma(h^o)$	10.0	3.57	2.73
$\sigma(h^{yA})/\sigma(h^o)$	10.0	1.75	1.70
$\sigma(x)/\sigma(h^o)$	10.0	0.56	0.45
$\sigma(w^y)/\sigma(w^o)$	1.0	1.14	0.82
1.2 [b] $\sigma(w^{yA})/\sigma(w^o)$	1.0	1.40	0.77
$\sigma(w^{yT})/\sigma(w^o)$	1.0	1.23	0.77
Corr(x, h)	1.0	-0.59	-0.64
Corr(w^y, w^o)	1.0	0.62	0.80
Corr(h^y, h^o)	10.0	0.89	0.82
Corr(c, x)	1.0	-0.56	-0.23
Contribution H/F	10.0	0.15	0.17
Moment M	1.0	0.05	0.10
Persistence of the AR(1) SR	1.0	0.95	0.95
Std of the AR(1) SR	1.0	0.007	0.007

Size of implicit transfers

$$\zeta(c^o) = \zeta_0 + \zeta_1 c^o$$

1. Average fraction of consumption of old

$$E\left[\frac{\zeta(c^o)}{c^o}\right] = 4.6\%$$

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$$E\left[\frac{\zeta(c^o)}{\zeta(c^o) + c^{yT}}\right] = 16.7\%$$

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2. Average fraction of consumption of young together

$$E\left[\frac{\zeta(c^o)}{\zeta(c^o) + c^{yT}}\right] = 16.7\%$$

3. Average additional hours need to work by young together

$$E\left[\frac{\hat{h}^{yT} - h^{yT}}{h^{yT}}\right] = 18.5\%$$

Why does coresidence affect hours?

- Frisch elasticity for old $= 0.72$
- Marshallian elasticity for young alone

$$e^{yA} = \frac{(1-\gamma)\nu^y}{1+\gamma\nu^y}$$

- Marshallian elasticity for young together

$$e^{yT}(\varepsilon) = e^{yA} \times \frac{1+\frac{1}{1-\gamma}\frac{\zeta(c^O)}{c^{yT}(\varepsilon)}}{1+\frac{1}{1+\gamma\nu^y}\frac{\zeta(c^O)}{c^{yT}(\varepsilon)}}$$

- If $\gamma < 1$, $\zeta > 0$ then $e^{yT}(\varepsilon) > e^{yA}$
- If $\zeta = 0$ then $e^{yT}(\varepsilon) = e^{yA}$. Also e^{yT} increasing in ζ

Why does coresidence affect hours?

- Frisch elasticity for old $= 0.72$
- Marshallian elasticity for young alone

$$e^{yA} \quad = 0.512$$

- Marshallian elasticity for young together

$$E\left[e^{yT}\right] \quad = 0.603$$

- If $\gamma < 1$, $\zeta > 0$ then $e^{yT}(\varepsilon) > e^{yA}$
- If $\zeta = 0$ then $e^{yT}(\varepsilon) = e^{yA}$. Also e^{yT} increasing in ζ

Importance of coresidence for hours volatility

Experiment 1:

- Possibility of coresidence, no endogeneity of coresidence
- $x = \bar{x}$: fix thresholds $\eta^*(\varepsilon, s) = \eta^*(\varepsilon, \bar{s})$
- St dev of log total hours: 5.5% lower
- St dev of log of young hours: 6.4% lower

Experiment 2:

- No possibility of coresidence
- $x = 0$: all young live alone
- St dev of log total hours: 31.4% lower
- St dev of log of young hours: 37.2% lower

Implications for RA Frisch elasticity

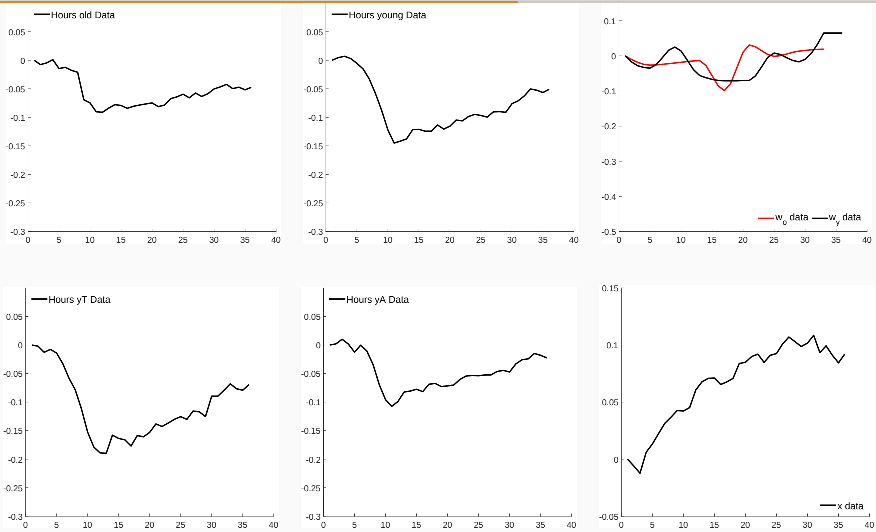
- RA models: Frisch elasticity key for volatility of aggregate hours
  → useful metric for measuring strength of other channels
- What Frisch elasticity would RA model require to generate same volatility of hours as model with young people and coresidence?

Frisch elasticity for old (ν^o)	Implied Frisch in RA RBC model	Proportional Increase
0.72	1.33	85%
0.5	0.87	75%
1.0	2.15	115%
2.0	9.62	381%

4 Great Recession

The Great Recession

The Great Recession in the data



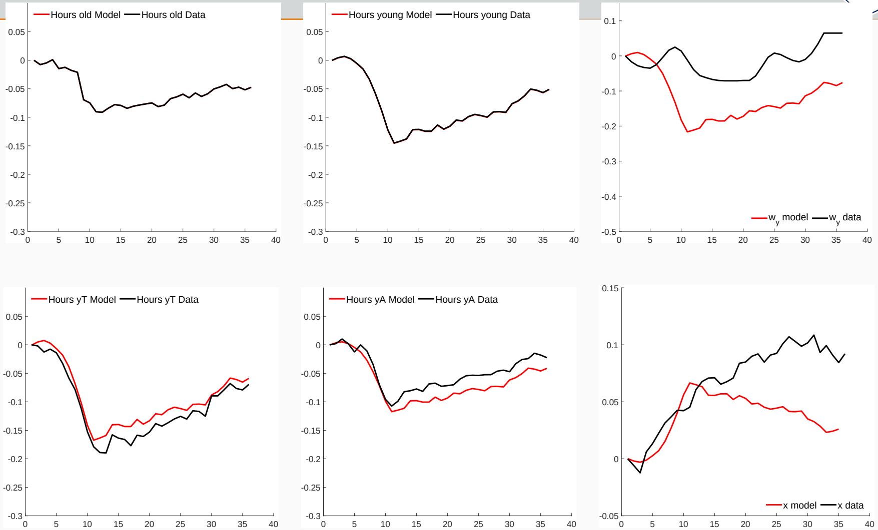
The Great Recession in the model

- Look through the lens of the model at hours of young (alone and together) and living arrangements during the Great Recession.
- Back out values of the shocks, so that the model replicates hours of the young and old between q1:2007 and q4:2015.
- Simulate the model forward with the implied shock values. Agents still have rational expectations about the shock realizations.

What does the model predict about the Great Recession

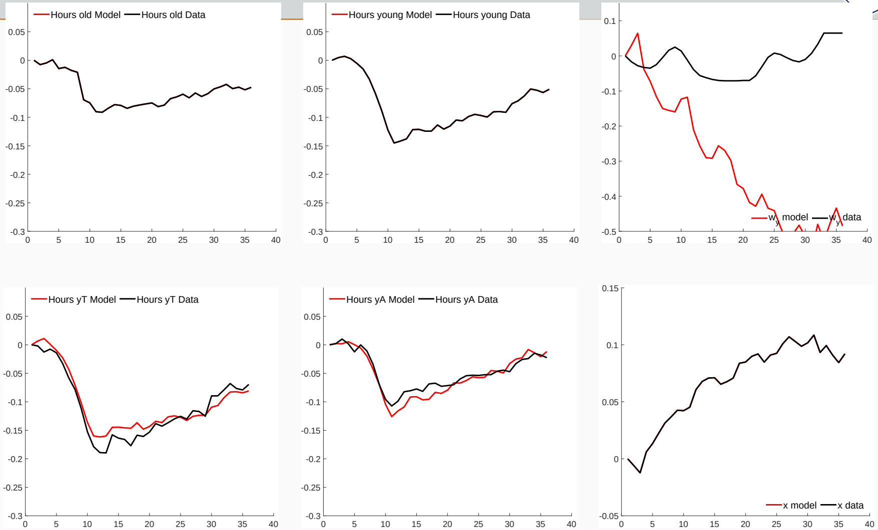
- The baseline model gets coresidence right only up to 10th quarter into the recession; misses the recovery.
- What does it take for the model to account for the data?
- Improved leisure technology: Aguiar, Bils, Charles, Hurst (2018). It becomes less painful to live with parents being equipped with better video games.

Asymmetric TFP shocks to match hours recovery



Model inputs

Asymmetric TFP shocks + improved leisure (ψ_y)



Model inputs

Improved leisure through η

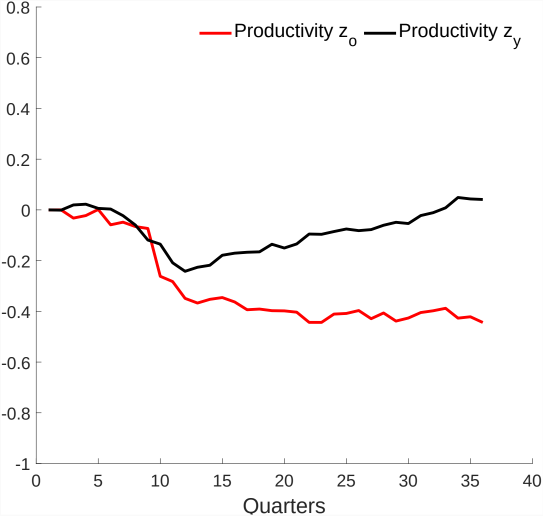
Conclusions

- Young and old have different labor market outcomes. Living arrangements play central role in shaping the behavior of the young.
- We have provided a theory of how it works and mapped it to the data. This theory accounts for the average and cyclical behavior of the young and the old.
- A rational for differences between the micro and the macro (which is 85% larger) Frisch elasticities.
- Our theory + Aguiar et. al. (2018) mechanism accounts for steep rise of coresidence and different outcomes of young and old during the Great Recession.

Model parameters

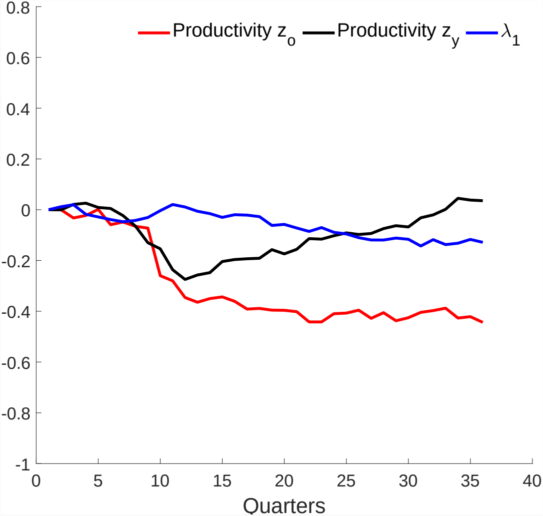
Parameter description	Symbol	Value	Discipline
Params set without solving the model			
Fraction of the old in the Population	μ	0.709	CPS data
Frisch elasticity for the Old	ν^o	0.720	Heathcote et al. (2010)
Equivalence scale within the Old	ζ^o	1.700	OECD data
Equivalence scale for Old with Young	ζ^y	0.500	OECD data
Size of the Old household	γ	1.831	CPS data
Discount Rate	β	0.990	$r = 0.04$
Params requiring solving the model			
Depreciation Rate	δ	0.035	Targeted Moments - Table 57
Production technology elasticity	ρ^f	0.199	Targeted Moments - Table 57
Production technology elasticity	σ^f	0.007	Targeted Moments - Table 57
Disutilty of labor for the Old	ψ^o	4.178	Targeted Moments - Table 57
Disutilty of labor for the Young	ψ^y	4.168	Targeted Moments - Table 57
Curvature in consumption of the Young	ϕ^y	0.409	Targeted Moments - Table 57
Labor elasticity of the Young	ν^y	1.343	Targeted Moments - Table 57
Mean of the prod. distribution of the Young	μ_ε	4.977	Targeted Moments - Table 57
Std of the prod. distribution of the Young	σ_ε	0.870	Targeted Moments - Table 57
Mean of the distate for living with Old	μ_η	0.126	Targeted Moments - Table 57
Std of the distaste for living with Old	σ_η	0.146	Targeted Moments - Table 57
Share of Young in production	μ_F	0.015	Targeted Moments - Table 57
Share of Old in capital-labor CES	λ_F	0.261	Targeted Moments - Table 57
Constant in the transfer function	ζ_0	0.027	Targeted Moments - Table 57
Slope of the transfer function	ζ_1	0.023	Targeted Moments - Table 57
Persistence of the TFP shock for Old	ρ^o	0.970	Targeted Moments - Table 57
Std of the TFP shock for Old	σ^o	0.007	Targeted Moments - Table 57
Persistence of the TFP shock for Young	ρ^y	0.911	Targeted Moments - Table 57

Young hit harder in the GR, but recover faster



Back

Young hit harder in the GR, but recover faster



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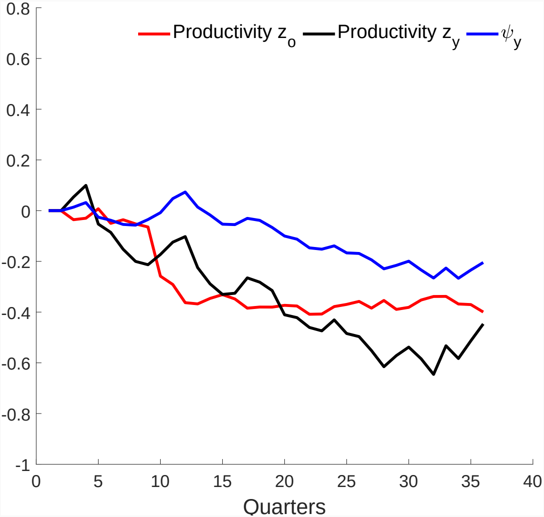

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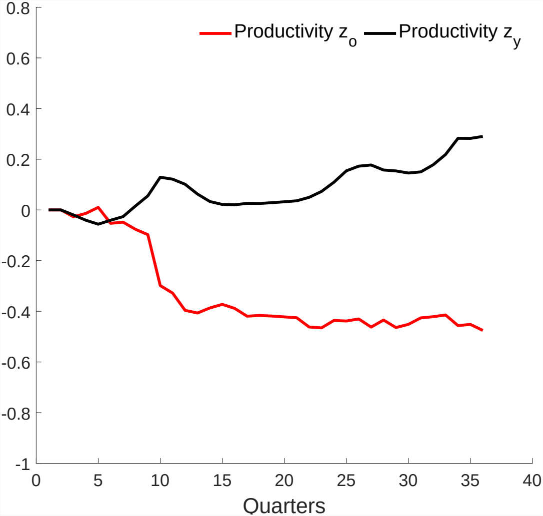

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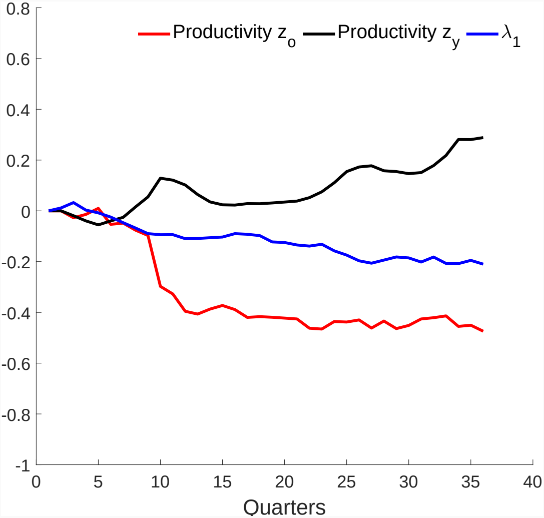
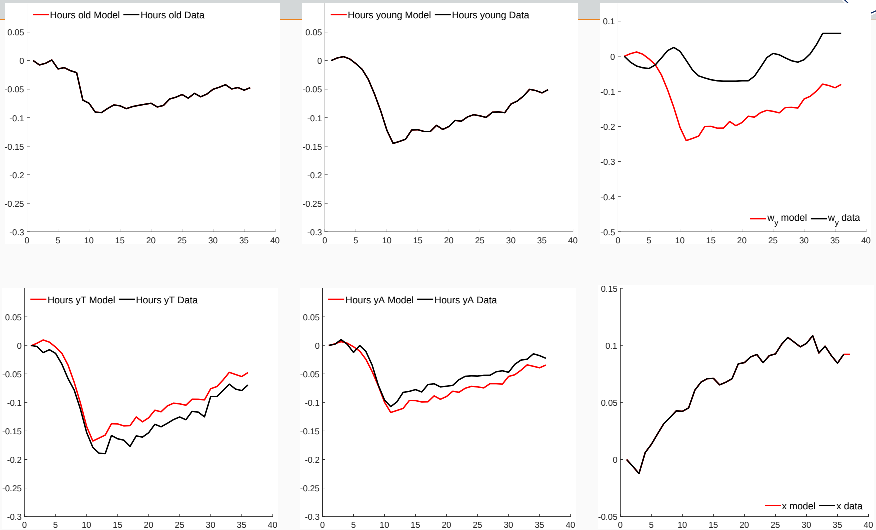


Back

Asymmetric TFP shocks + improved leisure (η)



Model inputs

Back

Contributions

1. We Document hhold sizes and types vary over the business cycle.
    1.1 In expansions hhs get smaller and the head gets younger. (mostly young people emancipate)
2. We Document Living Arrangements of young adults shape hours worked: People that live with parents
    2.1 Have higher average hours worked and lower average wages relative to their peers living alone.
    2.2 Have more volatile hours worked.
3. These two things together imply
    3.1 Volatility of hours person can be decomposed into the volatility of bodies per household and hours per household.
    3.2 15% of volatility of hours per person due to variations in household size in the data.

Contributions

1. We provide a joint quantitative theory of the living arrangements of young adults and their labor market outcomes, which accounts for

   1.1 Relative differences in hours and wages between young living with old and their peers.

   1.2 Business cycle movements in living arrangements of young adults (vars and covars relative to macro aggregates).

   1.3 (First and second moments, then composition) Contribution of movements in hhs size to the variance of hours worked per person. Total hours move more than the hours of individual, in addition to the standard volaitlity moving in expansions makes them work more.

   And it offers:

   4.4 New propagation and amplification (??) channel of the aggregate shocks.

Contributions

1. Implications of our theory:
   1.1 We provide a rational for XX% higher Frisch elasticity of labor supply in RA model due to amplification mechanism (difference between the micro and the macro elasticity).
   1.2 We disentangle and quantify the contributions of labor demand (Jaimovich, Pruitt, Siu (2012)) and the labor supply channel to differences in hours and wages between young and old.
   1.3 We provide a measure of implicit transfers, the average level and dispersion of "joy" of living with parents from old households to young living with them. We quantify the wedge in labor elasticities between young living with old and their peers resulting from different living arrangements.

Contributions

- Hhold sizes and types vary over the business cycle.
    - In expansions hhs get smaller and the head gets younger. (mostly young people emancipate)

- Living Arrangements of young adults shape hours worked: People that live with parents
    - Work less and have lower wages than those alone
    - Have more volatile hours worked.

- These two things together imply
    - Volatility of hours person can be decomposed into the volatility of bodies per household and hours per household.
    - 15% of volatility of hours per person due to variations in household size in the data.

Contributions II

- We provide a joint quantitative theory of the living arrangements of young adults and their labor market outcomes, which accounts for
  - Relative differences in hours and wages between young living with old and their peers.
  - Business cycle movements in living arrangements of young adults (vars and covars relative to macro aggregates).
  - (First and second moments, then composition) Contribution of movements in hhs size to the variance of hours worked per person. Total hours move more than the hours of individual, in addition to the standard volaitlity moving in expansions makes them work more.

  And it offers:
  - New propagation and amplification channel of aggregate shocks.

Contributions III

- Implications of our theory:
  - We provide a rational for XX% higher Frisch elasticity of labor supply in RA model due to amplification mechanism (difference between the micro and the macro elasticity).
  - We disentangle and quantify the contributions of labor demand (Jaimovich, Pruitt, Siu (2012) who argue for imperfect substitution of hours by age) and the labor supply channel to differences in hours and wages between young and old.
  - We provide a measure of implicit transfers, the average level and dispersion of "joy" of living with parents from old households to young living with them. We quantify the wedge in labor elasticities between young living with old and their peers resulting from different living arrangements.

Mapping to data: Functional forms and 24 paramters

- Productivity heterogeneity: $\varepsilon \sim \log N$
- Disutility heterogeneity: $\eta \sim N$
- Implicit transfer function: $\zeta(c^o) = \zeta_0 + \zeta_1 c^o$
- Agg. shocks: $\log Z_i' = \rho_i \log Z_i + \xi_i$, where $\xi_i \sim N(0, \sigma_\xi^i)$, $i = o, y$

- Demographics: μ, ζ
- Preferences old ψ^o, β, ν^o
- Productivity dist μ_ε, σ_ε
- Young preferences γ, ν^y, ψ^y
- Implicit transfers ζ_0, ζ_1
- Disutility dist: μ_η, σ_η
- Production parameters: α, λ, δ, σ, ρ
- Agg. shocks: $\{\rho_i, \sigma_\xi^i\}_{i=y,o}$, $Corr(\xi_o, \xi_y)$

Quantitative Strategy

- Exogenously imposed parameters (Data + Micro Estimates): ν^o, μ, ζ, σ, ρ
- Directly identified from steady-state conditions: β
- Rest is targeted using Simulated Method of Moments (SMM):
  - There are 18 parameters and we use 24 targets (next slide)
  - Both first and second moments are targeted. Can not separate their identification in the model.

Targeted moments

First Moments:

- Macro: I/Y, K/Y, h^o
- Living arrangements: $h^{yA}, h^{yT}, x, w^{yA}/w^o, w^{yT}/w^o$

Second Moments:

- Properties of Solow Residual: $AutoCorr(TFP)$, $\sigma(TFP)$
- Relative hours: $\sigma(h^{yA})/\sigma(h^o)$, $\sigma(h^{yT})/\sigma(h^o)$, $\sigma(h^y)/\sigma(h^o)$
- Relative wages: $\sigma(w^{yA})/\sigma(w^o)$, $\sigma(w^{yT})/\sigma(h^o)$, $\sigma(w^y)/\sigma(h^o)$
- Living arrangements moments: $\sigma(x)/\sigma(h^o)$, M, $Contr_{HF}$
- Correlations: $Corr(x, h), Corr(wy, wo), Corr(c, x)$, $Corr(c, x_{-3}), Corr(c, x_{+3})$

Model fit

	Data	Model
Relative hours		
$E[h^y]/E[h^o]$	1.00	0.98
$E[h^{yA}]/E[h^{yT}]$	1.24	1.35
$\sigma[h^y]/\sigma[h^o]$	1.58	1.57
$\sigma[h^{yA}]/\sigma[h^{yT}]$	0.69	0.71
Relative wages		
$E[w^y]/E[w^o]$	0.65	0.64
$E[w^{yA}]/E[w^{yT}]$	1.44	1.32
$\sigma[w^y]/\sigma[w^o]$	1.07	1.12
$\sigma[w^{yA}]/\sigma[w^{yT}]$	1.06	1.04
Living arrangements		
$\sigma[x]/\sigma[h^o]$	0.75	0.75
$corr(x, h)$	-0.56	-0.56
M (%)	5.0	4.5
Contr F/N (%)	15.3	16.1

*Non-targeted moments

Size of implicit transfers

$$\zeta(c^o) = \zeta_0 + \zeta_1 c^o$$

1. Average fraction of consumption of old

$$E\left[\frac{\zeta(c^o)}{c^o}\right] = 13\%$$

2. Average fraction of consumption of young together

$$E\left[\frac{\zeta(c^o)}{\zeta(c^o) + c^{yT}}\right] = 49\%$$

3. Average additional hours need to work by young together

$$E\left[\hat{h}^{yT} - h^{yT}\right.$$

Why does coresidence affect hours?

- Frisch elasticity for old $= 0.72$
- Marshallian elasticity for young alone

$$e^{yA} \quad = \frac{(1-\gamma)\nu^y}{1+\gamma\nu^y}$$

- Marshallian elasticity for young together

$$e^{yT}(\varepsilon) \quad = e^{yA} \times \frac{1+\frac{1}{1-\gamma}\frac{\zeta(c^o)}{c^{yT}(\varepsilon)}}{1+\frac{1}{1+\gamma\nu^y}\frac{\zeta(c^o)}{c^{yT}(\varepsilon)}}$$

- If $\gamma < 1$, $\zeta > 0$ then $e^{yT}(\varepsilon) > e^{yA}$
- If $\zeta = 0$ then $e^{yT}(\varepsilon) = e^{yA}$. Also e^{yT} increasing in ζ

Why does coresidence affect hours?

- Frisch elasticity for old $= 0.72$
- Marshallian elasticity for young alone

$$e^{yA} \quad = 0.45$$

- Marshallian elasticity for young together

$$E\left[e^{yT}\right] \quad = 0.73$$

- If $\gamma < 1$, $\zeta > 0$ then $e^{yT}(\varepsilon) > e^{yA}$
- If $\zeta = 0$ then $e^{yT}(\varepsilon) = e^{yA}$. Also e^{yT} increasing in ζ

Importance of coresidence for hours volatility

Experiment 1:

- Possibility of coresidence, no endogeneity of coresidence
- $x = \bar{x}$: fix thresholds $\eta^*(\varepsilon, s) = \eta^*(\varepsilon, \bar{s})$
- St dev of log total hours: 5.5% lower
- St dev of log of young hours: 6.4% lower

Experiment 2:

- No possibility of coresidence
- $x = 0$: all young live alone
- St dev of log total hours: 31.4% lower
- St dev of log of young hours: 37.2% lower

Demand vs. Supply channel

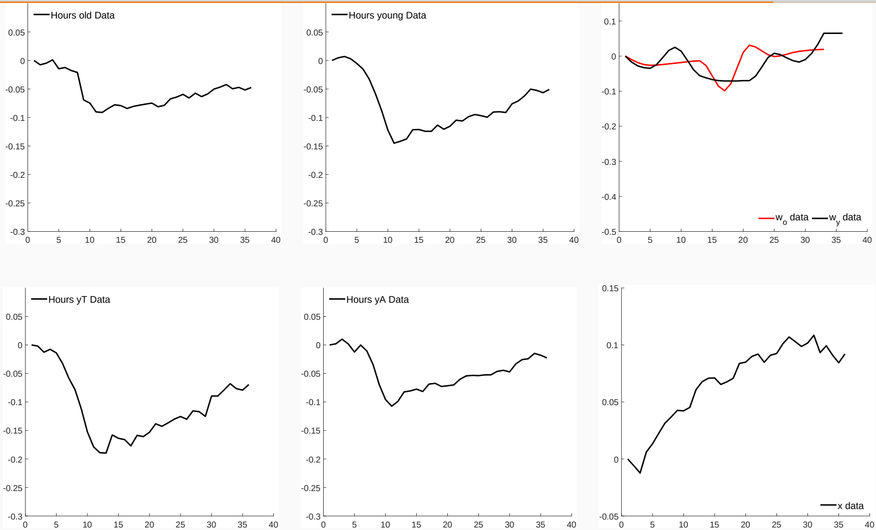
- RA models: Frisch elasticity key for volatility of aggregate hours
  → useful metric for measuring strength of other channels
- What Frisch elasticity would RA model require to generate same volatility of hours as model with young people and coresidence?

Frisch elasticity for old (ν^o)	Implied Frisch in RA RBC model	Proportional Increase
0.72	1.33	85%
0.5	0.87	75%
1.0	2.15	115%
2.0	9.62	381%

5 Great Recession

The Great Recession

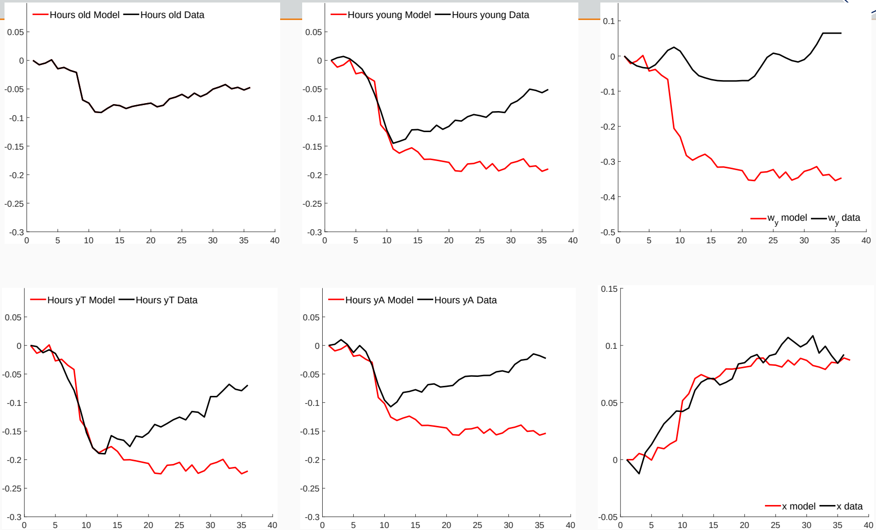
The Great Recession in the data



What does the model predict about the Great Recession

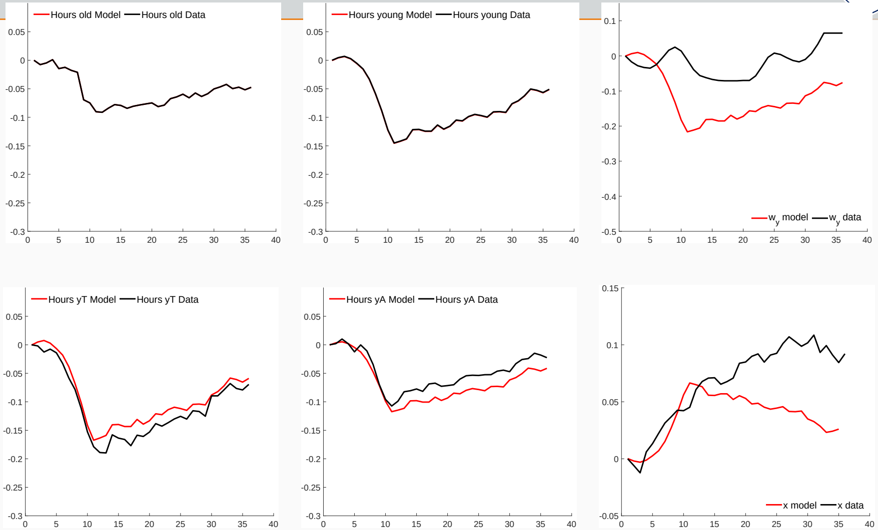
- The baseline model gets coresidence right and hours of the young only up to 10th quarter into the recession; misses the recovery.
- Reason: hours of the young recover faster than of the old
- What does it take for the model to account for these patterns?
  - Asymmetric TFP processes for young and old; fixes the hours but messes up the composition among young
  - Improved leisure technology: Aguiar, Bils, Charles, Hurst (2018). It becomes less painful to live with parents being equipped with better video games.

Only aiming hours of the old



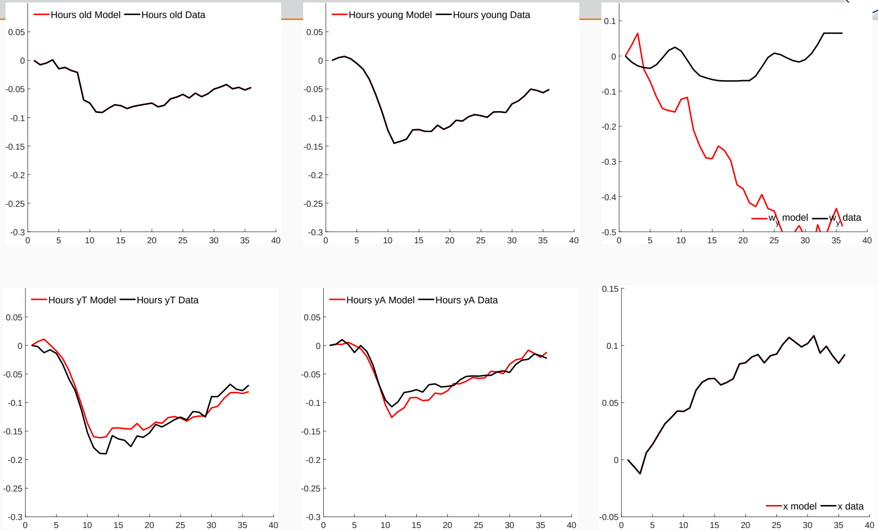
Model inputs

Asymmetric TFP shocks to match hours recovery



Model inputs

Asymmetric TFP shocks + improved leisure (ψ_y)



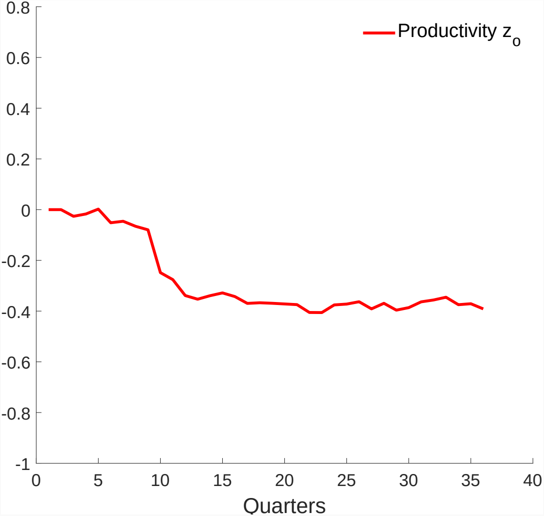
Model inputs

Improved leisure through η

Conclusions

- Young and old have different labor market outcomes. Living arrangements play central role in shaping the behavior of the young.
- We have provided a theory of how it works and mapped it to the data. This theory accounts for the average and cyclical behavior of the young and the old.
- A rational for differences between the micro and the macro (which is 85% larger) Frisch elasticities.
- Our theory + Aguiar et. al. (2018) mechanism accounts for steep rise of coresidence and different outcomes of young and old during the Great Recession.

Young hit harder in the GR, but recover faster



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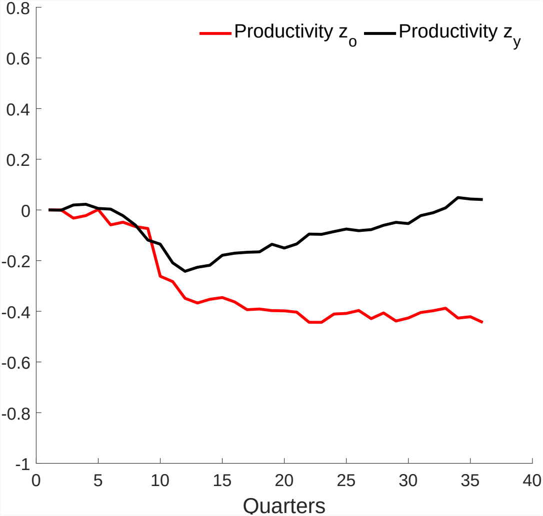
Young hit harder in the GR, but recover faster



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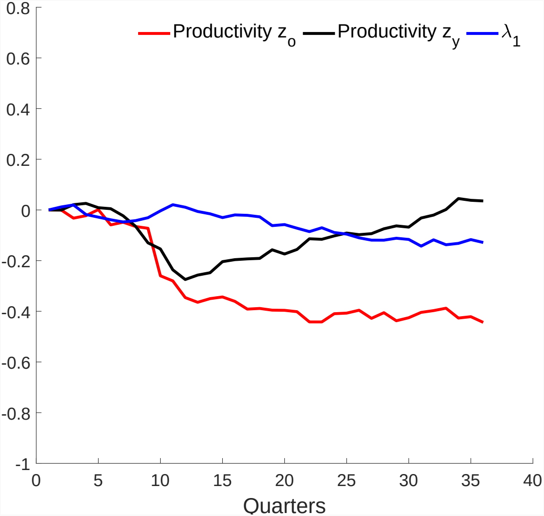
YOUNG HIT HARDER IN THE GR, BUT RECOVER FASTER



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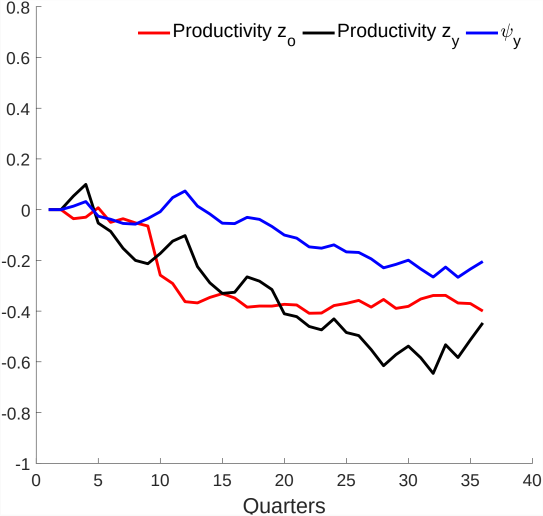
Young hit harder in the GR, but recover faster



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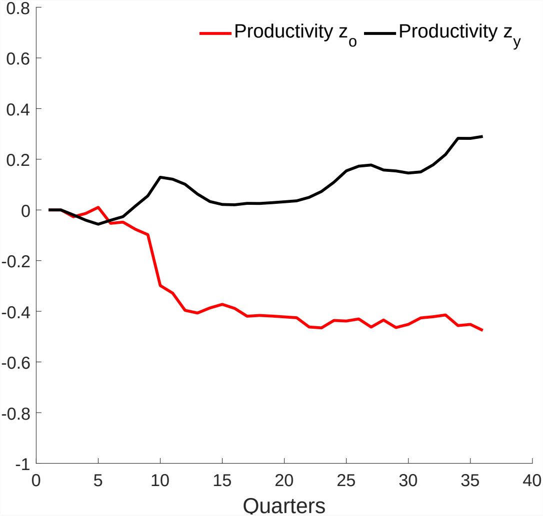

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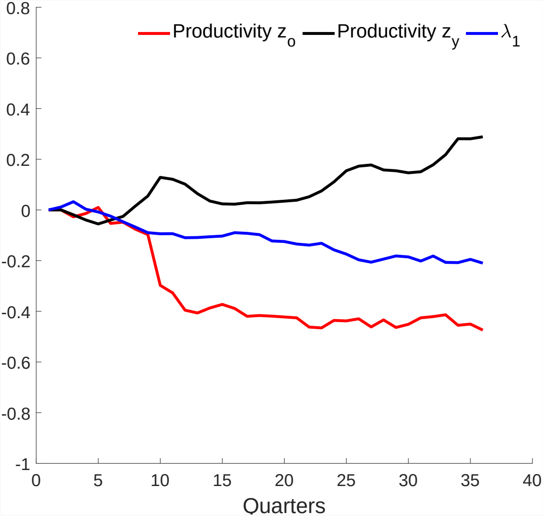

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Data: 1978-2015

- CPS Basic Monthly Surveys for hours (monthly)
- CPS ASEC for wages (annual)
- Individuals: 18-65 year olds, not in school, not in group quarters
- Households: households with at least one such person
- Household size: number of 18-65 year olds not in school
- Quarterly series: de-seasonalize using X12-ARIMA from BLS
- Detrending:
  - 1978-2006: Hodrick-Prescott and various other filters,
  - 2007-2010: Great Recession
  - 2011-2015: Great Recession recovery

Back

- Importance of endogeneity of coresidence: counterfactual series for hours assuming constant $x =$ fraction of young living with old
- All variation in hours is due to variation in hours of two groups:

$$M = \frac{V(\log h^y) - V\left(\log\left[\bar{x} h^{yT} + (1 - \bar{x} h^{yA})\right]\right)}{V(\log h^y)} \approx 5\%$$

Back

Demand vs. Supply channel

	Data	RBC + Imp. Subst.	RBC + Liv. Arr.	Baseline Model
Relative hours				
$E[h^y]/E[h^o]$	1.00	1.01	0.99	0.98
$E[h^{yA}]/E[h^{yT}]$	1.24	-	1.37	1.35
$\sigma[h^y]/\sigma[h^o]$	1.58	1.58	1.60	1.57
$\sigma[h^{yA}]/\sigma[h^{yT}]$	0.69	-	0.72	0.71
Relative wages				
$E[w^y]/E[w^o]$	0.65	0.87	0.63	0.64
$E[w^{yA}]/E[w^{yT}]$	1.44	-	1.33	1.32
$\sigma[w^y]/\sigma[w^o]$	1.07	1.32	1.00	1.12
$\sigma[w^{yA}]/\sigma[w^{yT}]$	1.06	-	1.15	1.04
Living arrangements				
$\sigma[x]/\sigma[h^o]$	0.75	-	0.77	0.75
$corr(x, h)$	-0.56	-	-0.57	-0.56
M (%)	5.0	-	4.6	4.5

*Frisch for the old across experiments is 0.72. Back

Calibration strategy

Two sets of parameters from outside model:

1. Production function elasticities: Jaimovich-Pruitt-Siu (2013)
2. Frisch elasticity of old: baseline $= 0.72$
   Heathcote-Storesletten-Violante (2014)

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Estimate remaining parameters using cyclical fluctuations, 1978-06

1. Standard aggregates (r, I/Y, Capital Share, Solow residual)
2. Mean hours of old, young alone, young together
3. Mean wages of young alone, young together

Calibration strategy

Two sets of parameters from outside model:

1. Production function elasticities: Jaimovich-Pruitt-Siu (2013)
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Estimate remaining parameters using cyclical fluctuations, 1978-06

1. Standard aggregates (r, I/Y, Capital Share, Solow residual)
2. Mean hours of old, young alone, young together
3. Mean wages of young alone, young together
4. St dev hrs of young along, young together relative to st dev hrs old
5. Mean fraction of young living with old
6. St dev fraction of young living with old relative to st dev hrs old
7. Correlation between fraction of young living with old and hours

Back

Old agents

- Standard RA intertemporal problem

$$V^o\left(a; w^o, r\right) = \max_{c^o, h^o, a'} u^o\left(c^o, h^o\right) + \beta\mathbb{E}\left[V^o\left(a'; w^{o'}, r'\right)\right]$$
$$\text{s.t.} \quad c^o + a' = w^o h^o + (1+r)a$$

- Preferences taking into account young invasion

$$u(c^o, h^o, x) = \left[1 - \frac{x(1-\mu)\gamma}{\mu}\right]\left[\frac{1}{1-\sigma^o}\left(\frac{c^o}{\zeta^o}\right)^{1-\sigma^o} - \psi^o\frac{(h^o)^{1+\frac{1}{\nu^o}}}{1+\frac{1}{\nu^o}}\right] + \frac{x(1-\mu)\gamma}{\mu}\left[\frac{1}{1-\sigma^o}\left(\frac{c^o}{\zeta^o+\zeta^y}\right)^{1-\sigma^o} - \psi^o\frac{(h^o)^{1+\frac{1}{\nu^o}}}{1+\frac{1}{\nu^o}}\right]$$

- Aggregate uncertainty: w^o, r

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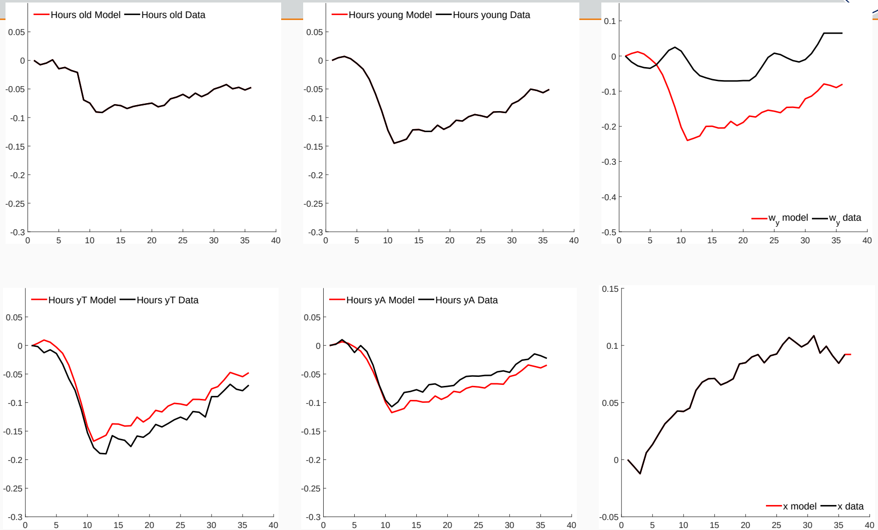
Asymmetric TFP shocks + improved leisure (η)



Model inputs   Back

Wealth, Wages, and Employment

Per Krusell, Jinfeng Luo José-Víctor Ríos-Rull

Introduction

- We want a theory of the joint distribution of employment, wages, and wealth, where
  - Workers are risk averse, so only use self-insurance.
  - Employment and wage risk are endogenous. (More concerned about whether people work than about how long they work.)
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied. In particular, we want to study employment flows jointly with the other standard objects.
- The most sophisticated version compares well with fluctuations data.

Literature

- The steady state of this economy has as its core Aiyagari (1994) meets Merz (1995), Andolfatto (1996) meets Moen (1997).
- Related Lise (2013), Hornstein, Krusell, and Violante (2011), Krusell, Mukoyama, and Şahin (2010), Ravn and Sterk (2016, 2017), Den Haan, Rendahl, and Riegler (2015).
- Specially Eeckhout and Sepahsalari (2015), Chaumont and Shi (2017), Griffy (2017).
- Developing empirically sound versions of these ideas compels us to
    - Add extreme value shocks as a form of accommodating quits and on the job search as choices.
    - Use new potent tools to address the study of fluctuations in complicated economies Boppart, Krusell, and Mitman (2018)

What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

Today: Build the Theory Sequentially and discuss & Fluctuations from two types of shocks

1. No Quits: Exogenous Destruction, no Quits. Built on top of Growth Model. (GE version of Eeckhout and Sepahsalari (2015)): Not a lot of wage dispersion. Not a lot of job creation in expansions.

2. Add Endogenous Quits: Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks).

3. On the Job Search workers may get outside offers and take them. (Similar but not the same as in Chaumont and Shi (2017)).

4. Outside of the Labor Force

5. All of the Above
    - Employers commit both to either a wage or a wage schedule $w(z)$ that depends on the aggregate shock.

Key Findings

- If wages are fully fixed and committed (Drastic Wage rigidity)
  - Both endogenous quits and on-the-job yield counter factual procyclical unemployment and massive on the job search.
  - Allowing the wage of an already formed job match to respond some to aggregate shocks corrects this.
  - Getting the right relative volatility of old and new wages and the amount of job-to-job moves and quits provides a way to measure wage rigidity.
- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

A Brief Look At Data

Relevant Properties in U.S. Data

	Mean Perc	St Dev Relt to Output	Correl w Output	Source
Average Wage	-	0.44-0.84	0.24-0.37	Haefke et al. (2013)
New Wage	-	0.68-1.09	0.79-0.83	Haefke et al. (2013)
Unemployment	4-6	4.84	-0.85	Campolmi&Gnocchi (2016)
Annual Quits (All)	10-40	4.20	0.85	Brown et al. (2017)
Annual Switches	25-35	4.62	0.70	Fujita&Nakajima (2016)
Consumption	75	0.78	0.86	NIPA
Investment	25	4.88	0.90	NIPA

Model 1: No (Endogenous) Quits Model

No (Endog) Quits: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one (z) unit of the good (z is the aggregate state of the economy).
  - Firms pay flow cost $\bar{c}$ to post a vacancy in market $\{w, \theta\}$.
  - Firms cannot change wage (or wage-schedule) afterwards.
  - Think of a firm as a machine programmed to pay w or $w(z)$
  - Plants (and their capital) are destroyed at rate δ^f.
  - Workers quit exogenously at rate δ^h.
- Households differ in wealth and wages (if working) but not in productivity. There are no state contingent claims, nor borrowing.
  - If employed, workers get w and save.
  - If unemployed, workers produce b and search in some $\{w, \theta\}$.
- General equilibrium: Workers own firms.

Order of Events of No Quits Model

1. Households enter the period with or without a job: $\{e, u\}$.

2. Production & Consumption: Employed produce z on the job. Unemployed produce b at home. They choose savings.

3. Firm Destruction and Exogenous Quits :
   Some Firms are destroyed (rate δ^f) They cannot search this period.
   Some workers quit their jobs for exogenous reasons δ^h. Total job destruction is δ.

4. Search: Firms and the unemployed choose wage w and tightness θ.

5. Job Matching : $M(V, U)$: Some vacancies meet some unemployed job searchers. A match becomes operational the following period.
   Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

No Quits Model: Household Problem

- Individual state: wealth and wage
  - If employed: (a, w)
  - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta) V^e(a',w) + \delta V^u(a')\right]$$

$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c,a',w} u(c) + \beta \left\{\psi^h[\theta(w)]\ V^e(a',w) + [1-\psi^h[\theta(w)]]\ V^u(a')\right\}$$

$$\text{s.t.} \quad c + a' = a(1+r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

Firms Post vacancies: Choose wages & filling probabilities

- Value of wage-w job: uses constant $\overline{k}$ capital that depreciates at rate δ^k ($\Omega = \overline{k}$)

$$\Omega(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega(w) + \delta^h\,\Omega\right]$$

- Affine in w: $\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right]\frac{1+r}{r+\delta^f+\delta^h-\delta^f\delta^h}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)]\;\Omega(w) + [1-\psi^f[\theta(w)]]\,\Omega$

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \psi^f[\theta(w)]\;\frac{\Omega(w)}{1+r} + [1-\psi^f[\theta(w)]]\;\frac{\Omega}{1+r},$$

No (Endog) Quits Model: Stationary Equilibrium

- A stationary equilibrium is functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate r, and a stationary distribution x over (a, w), s.t.

  1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

  2. Zero profit condition holds for active markets

  $$\bar{c} + \overline{k} = \psi^f[\theta(w)] \ \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \ \frac{\overline{k}(1-\delta-\delta_k)}{1+r}, \quad \forall w \ \text{ offered}$$

  3. An interest rate r clears the asset market

  $$\int a \ dx = \int \Omega(w) \ dx.$$

Characterization of a worker's decisions

- Standard Euler equation for savings

$$u_c = \beta\,(1 + r)\; E\,\{u_c'\}$$

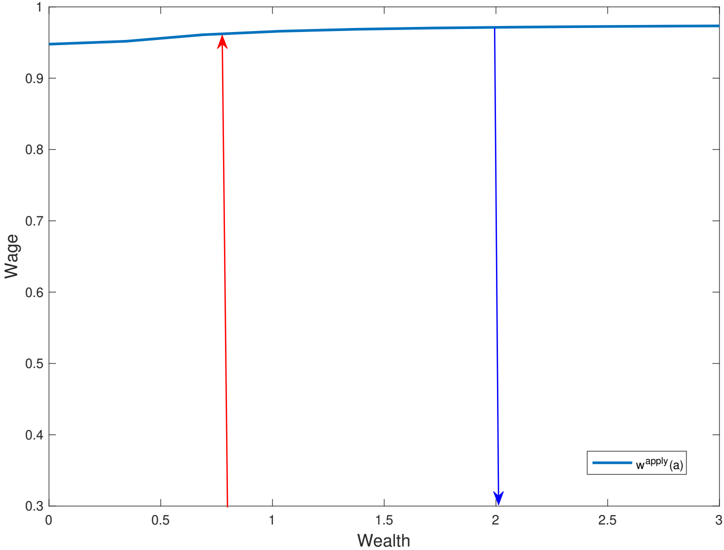
- A F.O.C for wage applicants

$$\psi^h[\theta(w)]\; V_w^e(a', w) = \psi_\theta^h[\theta(w)]\,\theta_w(w)\;\left[V^u(a') - V^e(a', w)\right]$$

- Households with more wealth are able to insure better against unemployment risk.

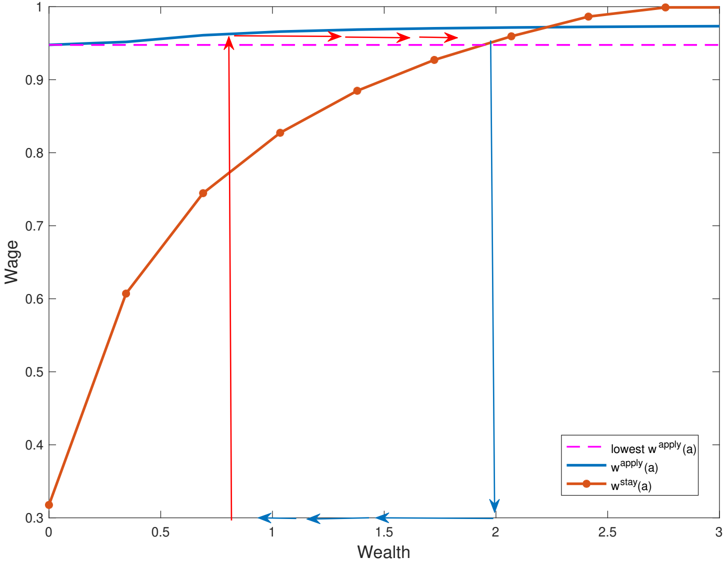
- As a result they apply for higher wage jobs and we have dispersion

Worker's wage application decision



How does the Model Work

Worker's saving decision



Shortcomings of this model

- Silent on Quits and Job-To-Job Movements.
- Low Wage Dispersion
- Small differences in volatility between average and new wages
- Low unemployment volatility

Summary: No (Endog) Quits Model

1. Easy to Compute Steady-State with key Properties
   1. Risk-averse, only partially insured workers, endogenous unemployment
   2. Can be solved with aggregate shocks too
   3. Policy such as UI would both have insurance and incentive effects
   4. Wage dispersion small—wealth doesn't matter too much
   5. · · ·so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings
2. In the following we examine the implications of a quitting choice

Endogenous Quits

Endogenous Quits: Beauty of Extreme Value Shocks

- Temporary Shocks to the utility of working or not working: Some workers quit. (in addition to any intrinsic taste for leisure)
- Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
- Conditional on wealth, high wage workers quit less often.
- But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and those with higher wages have higher wealth which makes them quite more often: Wage inequality collapses.
- We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.

Quitting Model: Time-line

1. Workers enter period with or without a job: $\{e, u\}$.
2. Production occurs and consumption/saving choice ensues:
3. Exogenous job/firm destruction happens.
4. Quitting:
   - e draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision.
     Job losers cannot search this period.
   - u draw shocks $\{\epsilon_1^u, \epsilon_2^u\}$. No decision but same expected means.
5. Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
   Wealth is not observable. (Unlike Chaumont and Shi (2017)).
   Yet it is still Block Recursive
6. Matches occur

Quitting Model: Workers

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\}\, dF^\epsilon$$

  V^e and V^u are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then $\widehat{V}$ has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a,w) - V^u(a)]}}$$

  higher parameter $\alpha \rightarrow$ lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.

Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^e$ for V^e

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta)\widehat{V}^e(a',w) + \delta V^u(a)\right]$$
$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed is like before except that there is an added term $E\{\max[\epsilon_1^u, \epsilon_2^u]\}$

So that there is no additional option value to a job.

Quitting Model: Value of the firm

- $\Omega^j(w)$: Value with with j-tenured worker.
  Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f[\theta(w)]\, \Omega^0(w) + [1 - \psi^f[\theta(w)]]\, \Omega \right\},$$

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$. (One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q^e[g^{e,j}(a,w), w]$$

$g^{e,j}(a,w)$ savings rule of a $j-$ *tenured* worker that was hired with wealth a

- Firm's value

$$\Omega^j(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r} \left\{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)]\, \Omega \right\}$$

Quitting Model: Solving forward for the Value of the firm

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\, Q^1(w) + (1 - \delta^f - \delta_k)k\; Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

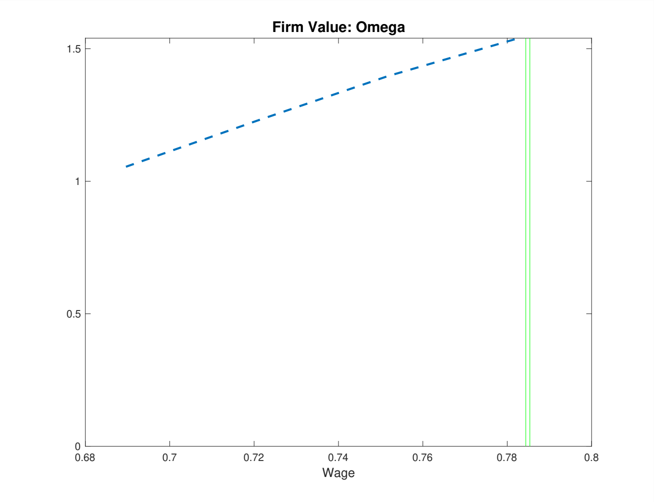
- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- It is Block Recursive because wealth can be inferred from w and j. (No need to index contracts by wealth (as in Chaumont and Shi (2017))).

Do we get More Wage Dispersion?

- This Model has the potential to get more wage dispersion
- Conditional on wealth higher wages lead to less quitting.
- So firms are willing to pay more to keep workers longer
- **BUT** we will see a problem

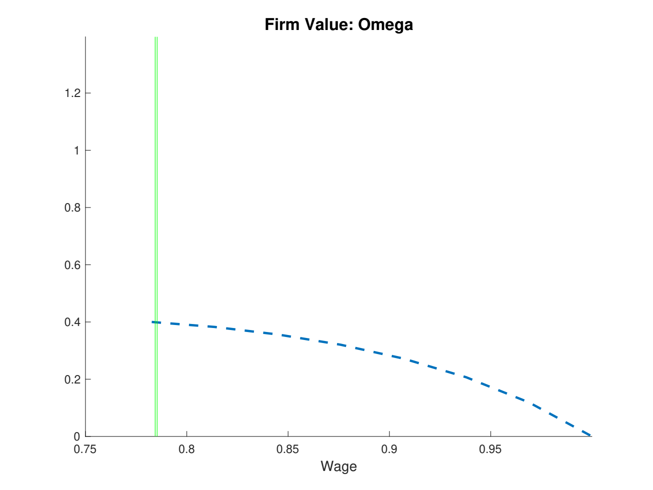
Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



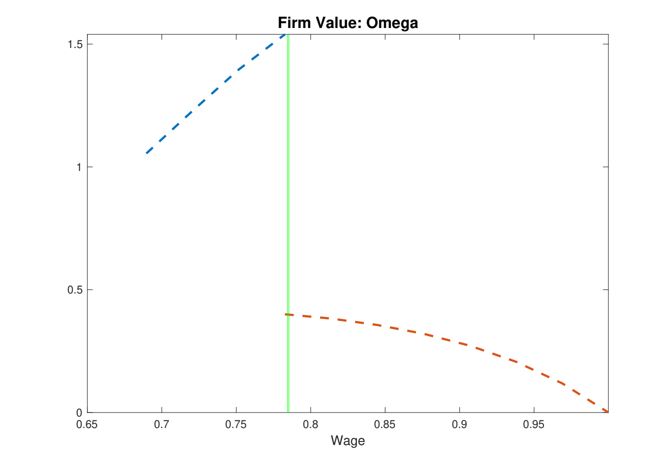
Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



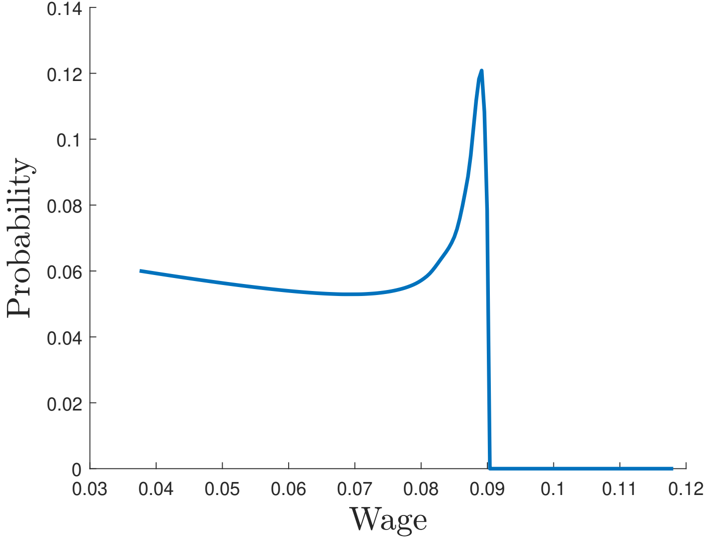
- Related to the Diamond dispersion paradox but for very different reasons.

Effect of Quitting: The Mechanism

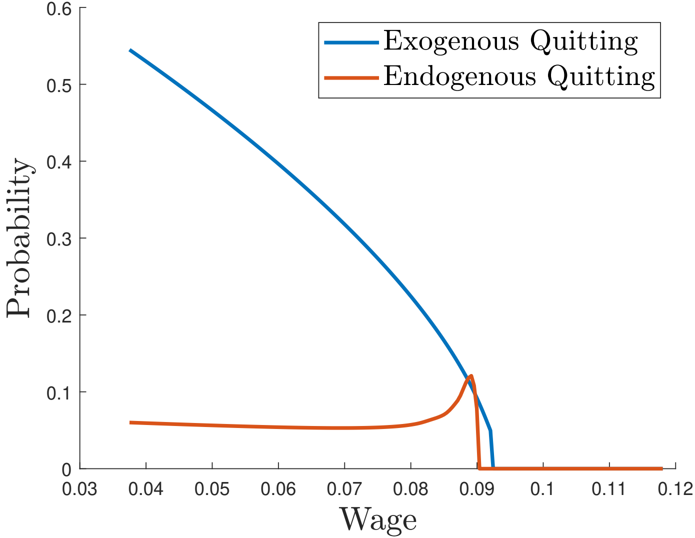
- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates.

Value of the firm: Zero profit Job Finding Probability

- Increasing in Wage (up to Grid calculation): Unique wage.



- Job finding prob with Endo



Shortcommings

- Wage Dispersion Collapses
- Silent on Job-To-Job Movements.
- Unemployment Moves little (but more than the previous one) over the cycle
- No difference in volatility between average and new wages
- Correlation 1 between Wealth when starting to work and wage

A Detour on How to Improve the Correlation Between Wealth and Wages

- Pose *aiming* (extreme value) shocks).
- This reduces the correlation between wages and wealth when first hired.
- It will have many uses, we think.

On the Job Search

On the Job Search Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. Production & Consumption:
3. Exogenous Separation
4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become u', those who search join the u, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain e' with the same wage w and those who neither become e' with w. $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. Search : Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; u and s assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e', otherwise become u'.
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w')V^e(a',w') + (1-\psi^h(w'))V^u(a') + \epsilon^{w'}\right] \, dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a',w) = \int \max\{V^e(a',w)+\epsilon^e, V^u(a')+\epsilon^u, V^s(a',w)+\epsilon^s\} dF^\epsilon$$

- The value of searching is

$$V^s(a',w) = \int \max_{w'} \left[\psi^h(w')V^e(a',w') + [1-\psi^h(w')]V^e(a',w) + \epsilon^{w'}\right] \, dF^\epsilon$$

On the Job Search: Household choices

- The probabilities of quitting and of searching

$$q(a',w) = \frac{1}{1+\exp(\alpha[V^e(a',w)-V^u(a')])+\exp(\alpha[V^s(a',w)-V^u(a')+\mu^s])},$$
$$s(a',w) = \frac{1}{1+\exp(\alpha[V^u(a')-V^s(a',w)])+\exp(\alpha[V^e(a',w)-V^s(a',w)-\mu^s])}.$$

$\mu^s < 0$ is the mode of the shock ϵ^s which reflects the search cost.

- Households solve

$$V^e(a,w) = \max_{a'\geq 0} u[a(1+r)+w-a'] + \beta\left[\delta V^u(a') + (1-\delta)\widehat{V}^e(a',w)\right]$$

$$V^u(a) = \max_{c,a'\geq 0} u[a(1+r)+b-a'] + \beta\widehat{V}^u(a')$$

- The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\ Q^1(w) + (1 - \delta - \delta_k)k\ Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

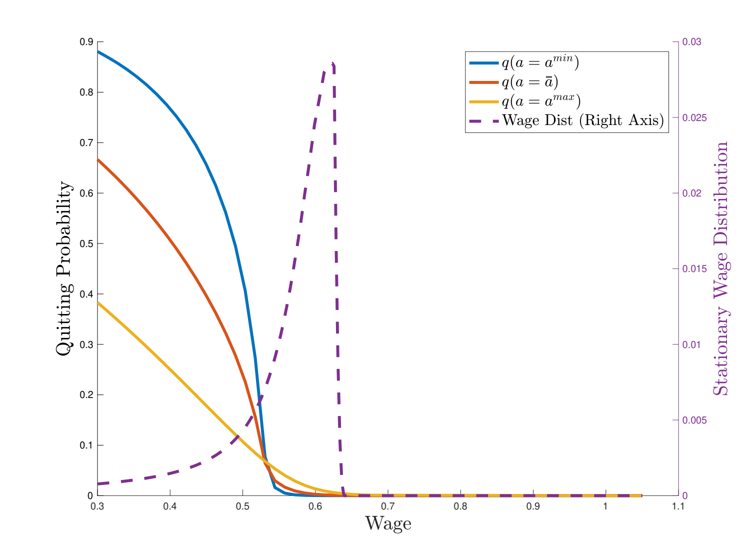
$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\ell^j(w) = 1 - \int h(w; a)\ q[g^{e,j}(a, w), w]\ dx^u(a) -$$

$$\int h(w; a)\ s[w; g^{e,j}(a, w)] \left[\int \hat{h}[\widetilde{w}; g^{e,j}(a, w), w] \xi \phi^h(\widetilde{w})\ d(\widetilde{w}) \right] dx^u(a)$$

OJS Quitting Probabilities, Various wealths & Wage Density



- The rich pursue often other activities (leisure?)

Outside the Labor Force

Outside the Labor Force Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock. Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
3. Production & Consumption:
4. Exogenous Separation
5. Quitting? Searching? Neither?:
6. Search
7. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
8. Match

Various Economies with added Life Cycle (live 50 years)

- Provides a mechanism for having poor agents

- Right now we have Four Economies

  1. Only Exogenous Quitting
  2. Endogenous Quitting
  3. Exogenous Quitting with On-the-job Search
  4. Endogenous Quitting and On-the-job Search
  5. ... and some agents do not want to work

- Today we will only look at the Economy with Endogenous quitting and On-the-Job-Search (4)

Quantitative Analysis: Steady States

Parameter Values

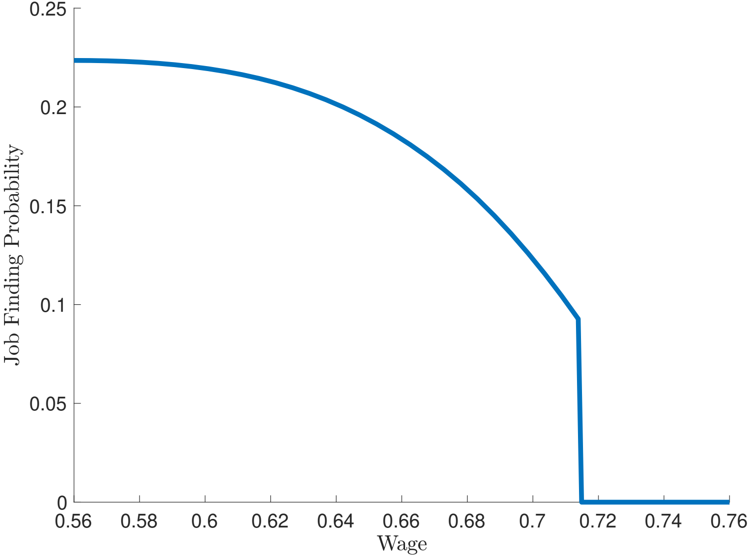
	Definition	Value in Yearly Units
r	interest rate	3%
K	fixed capital required	3
δ^f	firm destruction rate	2.88%
δ^k	capital maintenance rate	6.38%
δ^h	total worker quitting rate	8.56%
c^v	job posting cost	0.03
y	productivity on the job	1
b/w	productivity at home	0.4
σ	risk aversion	2
Matching function	$m = \chi u^\eta v^{1-\eta}$, non-OJS	$\chi = 0.15, \eta = 0.62$
	$m = \chi u^\eta v^{1-\eta}$, OJS	$\chi = 0.3, \eta = 0.5$

- We also explore a lower on the job search economy ()high value of leisure economy $b/w \sim 0.75$

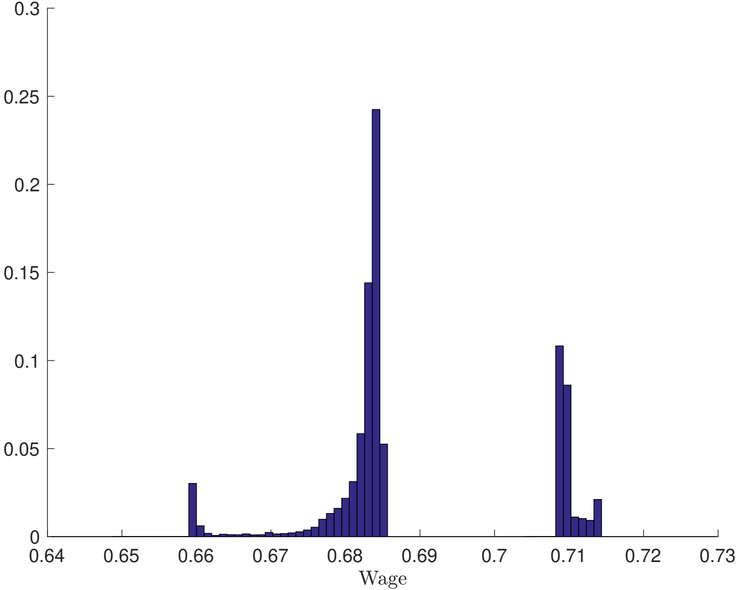
Steady State Allocations in Yearly Units: Endog Quits & OJS

interest rate	0.030
avg consumption	0.651
avg wage	0.689
avg wealth	3.041
stock market value	2.953
avg labor income	0.654
consumption to wealth ratio	0.225
labor income to wealth ratio	0.215
quit ratio	0.090
unemployment rate	0.097
job losers	0.117
wage of newly hired unemp	0.677
std consumption	0.011
std wage	0.002
std wealth	3.606
mean-min consumption	2.051
mean-min wage	1.058
UE transition	0.125
total vacancy	0.578
avg unemp duration	0.773
avg emp duration	7.228
avg job duration	1.898
OJS move rate	0.395

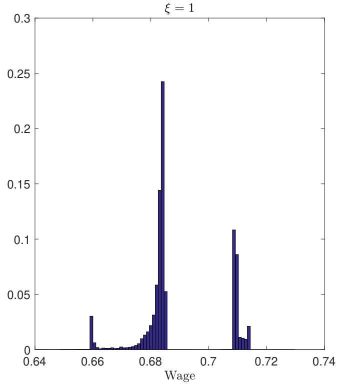
Job Finding Probability Curves



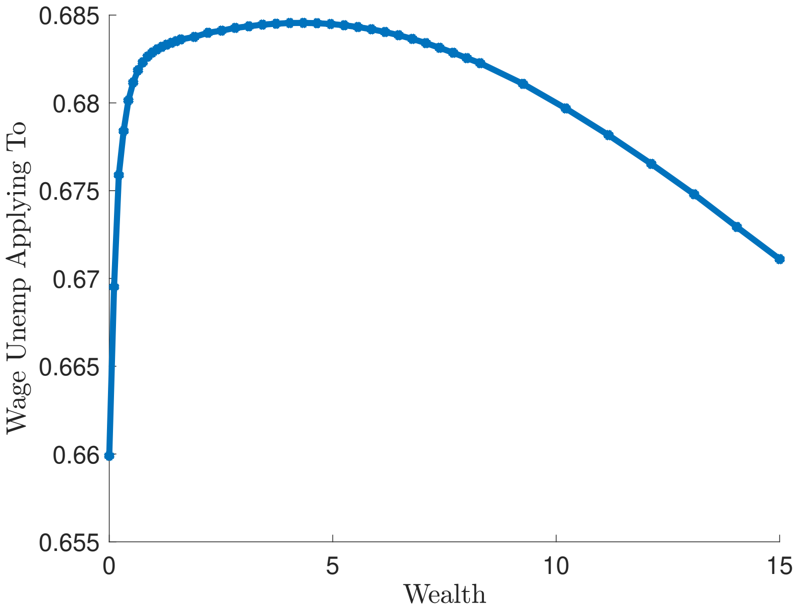
Wage Distributions: Baseline



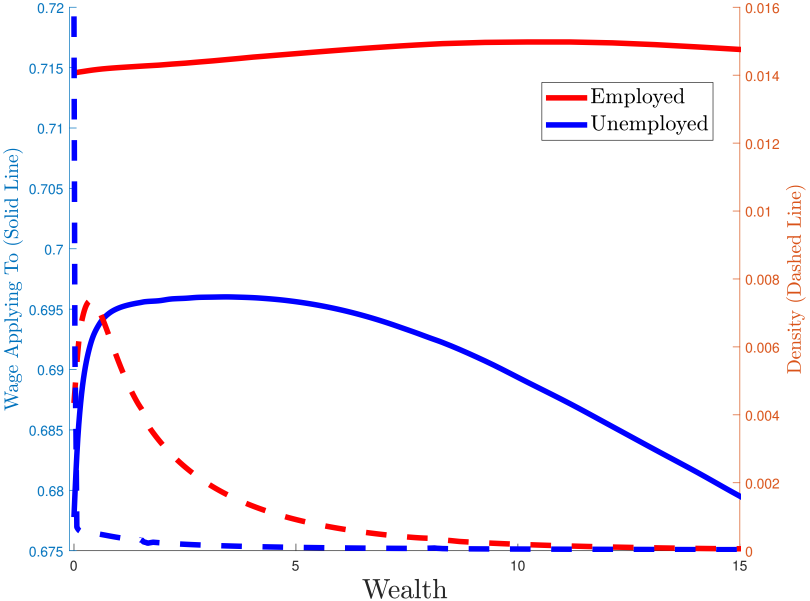
Wage Distributions: Comparing with lower OJS



Wage Applications of the Unemployed by Wealth



Wage Applications of U and $\overline{w}$ and densities of all



Aggregate Fluctuations

Introduce Aggregate Shocks

- We examine the model responses to two type of shocks
  1. Productivity shocks z_t: Output $=$ EmpRate $\times (1 + z_t)$
  2. Firm destruction shocks d_t: Firm Destruction Rate $= \delta^f \times (1 - d_t)$
- We introduce a wage peg assumption:
  - To allow the wage of an already formed job match to respond to z_t shocks directly (by 50%) (but not to d_t shocks)
  - If wages were completely rigid there would be massive quits: counterfactual.

Baseline: IRF to z shock

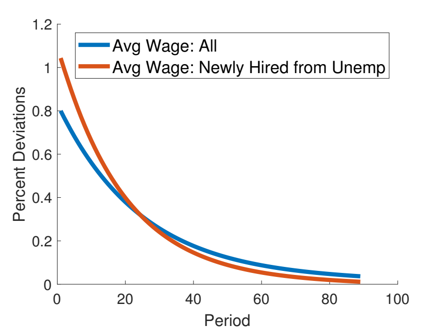


Figure 1: Wages

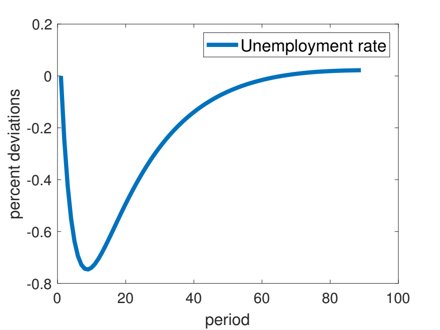


Figure 2: Unemployment Rate

- Responsive new wage (directed search) and average wage (wage peg)
- Non-trivial response of unemployment

Baseline: IRF to z shock

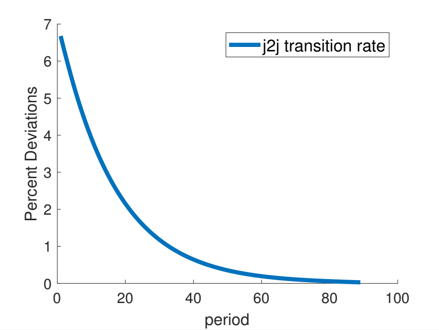


Figure 3: J2J transitions

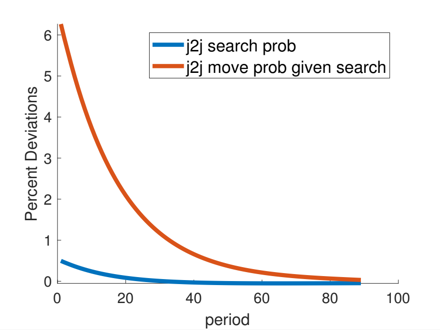


Figure 4: J2J search & JFP

- Too much responsive j2j transitions
- Due to improved job finding probability, not more searchers

Baseline: IRF to d shock

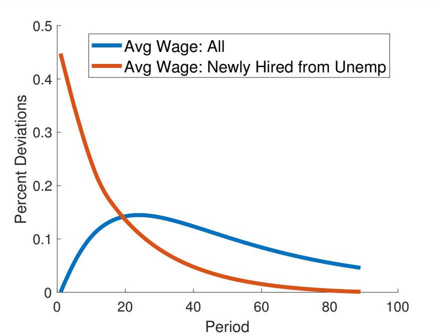


Figure 5: Wages

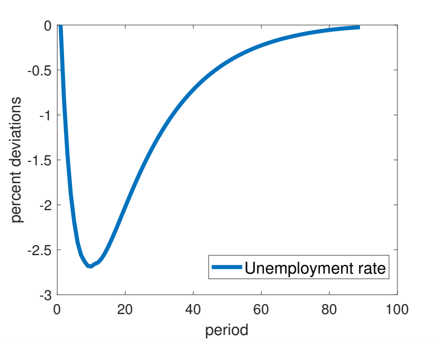


Figure 6: Unemployment Rate

- 1% delta shock = 0.36 base points
- Large response of wage and unemployment to the delta shock
- Note wage is not pegged to the delta shock

Baseline: IRF to d shock

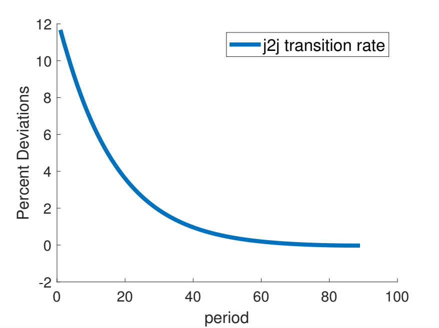


Figure 7: J2J transitions

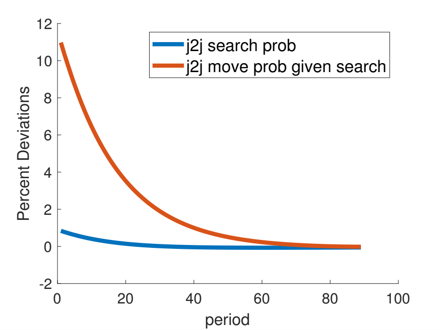


Figure 8: J2J search & JFP

- But too much volatility for job-to-job transitions

Summary, On-the-job Search and Quits

- Pro-cyclical average wages, new wages, and employment, qutting, and job-to-job transitions
- Clear responses of new wages and employment
- Quitting mildly respnds to both shocks
- Job-to-job transitions move too much with both shocks

Assessing Performance in terms of standard hp-filtered 2nd moments

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.
- 2nd order data moments are from Haefke, Sonntag, and Van Rens (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

Productivity Shock: Relative Volatility

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.51	0.44-0.84
New Wage	0.95	0.68-1.09
Unemployment	0.35	4.84
Quits + OJS moves	8.94	4.2
OJS moves	10.66	4.62

Table 1: Standard Deviation Relative to Output: Only Productivity Shock

- Unemployment moves too little and Quits and OJS moves too much

Productivity Shock: Correlation

- Only Productivity Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	1.00	0.24-0.37
New Wage	1.00	0.79-0.83
Unemployment	-0.48	-0.85
Quits + OJS moves	0.99	0.85
OJS moves	0.99	0.70

Table 2: Correlation with Contemprary Output: Only Productivity Shock

- Correlations are on the spot

Delta Shock: Relative Volatility

	Model	Data
Output	1	1
Average Wage	0.09	0.44-0.84
New Wage	2.02	0.68-1.09
Unemployment	4.70	4.84
Quits + OJS moves	41.66	4.2
OJS moves	49.36	4.62

Table 3: Standard Deviation Relative to Output: Only Delta Shock

- Now Unemployment is good but moves are excessive
- Note that relative to output, productivity is very important so employment cannot do that much, but this shock makes employment the only culprit so it has to move a lot

Delta Shock: Correlation

- Only Delta Shock: $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.13	0.24-0.37
New Wage	0.31	0.79-0.83
Unemployment	-0.99	-0.85
Quits + OJS moves	0.40	0.85
OJS moves	0.42	0.70

Table 4: Correlation with Contemprary Output: Only Delta Shock

Both Shocks: Relative Volatility Very correlated

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.49	0.44-0.84
New Wage	1.38	0.68-1.09
Unemployment	3.02	4.84
Quits + OJS moves	25.77	4.2
OJS moves	30.53	4.62

Table 5: Standard Deviation Relative to Output: Both Shocks

Both Shocks: Correlation

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.77	0.24-0.37
New Wage	0.50	0.79-0.83
Unemployment	-0.37	-0.85
Quits + OJS moves	0.28	0.85
OJS moves	0.29	0.70

Table 6: Correlation with Contemprary Output: Both Shocks

Both Shocks: Relative Volatility Uncorrelated

- Interact productivity shock and delta shock
  - Low Correlation of shocks = 0
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.40	0.44-0.84
New Wage	1.35	0.68-1.09
Unemployment	2.59	4.84
Quits + OJS moves	23.98	4.2
OJS moves	28.45	4.62

Table 7: Standard Deviation Relative to Output: Both Shocks

Both Shocks: Correlation Uncorrelated

- Interact productivity shock and delta shock
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.82	0.24-0.37
New Wage	0.62	0.79-0.83
Unemployment	-0.61	-0.85
Quits + OJS moves	0.47	0.85
OJS moves	0.48	0.70

Table 8: Correlation with Contemprary Output: Both Shocks

Clumsy Experiments & Extensions

Several Experiments/Extensions

- Now we move to some experiments/extensions to evaluate the business cycle performance of the model
- We look at the following
  - An extension to allow for different matching function elasticities for UE and EE moves ($\eta^u \neq \eta^e$).
  - On top of that an economy with higher b (from 0.3 to 0.5-0.6) that illuminates the Shimer/Hagedorn-Manowski debate.

Hetero-η Economy: Motivation

- For all the above exercises we find that the volatility of j2j transition rate is a magnitude larger than unemployment rate
- In the data unemployment rate is as volatile as (or even more volatile than) the j2j transition rate.
- Difficult to deliver this in the model from aggregate shocks affecting jobs at all wage levels
    - The percentage changes of firm value, vacancy filling probability and job finding probability are similar at all wage levels
    - Thus as a stock, the response of unemployment would thus be a magnitude smaller than the j2j transition rate (a flow)

Hetero-η Economy: Motivation

- Two potential fix
  - Make the firm value at high wages more volatile $\Rightarrow$ hard since high-wage matches feature low profits
  - Make the job finding probability of the employed less responsive to the same percentage change in the firm value $\Rightarrow$ curvature in the matching function controls this

- Motivated by this, we will allow η in the matching function $m = \chi u^{\eta} v^{1-\eta}$ to be low in UE moves but high in EE moves
  - $\psi^h(w) = \chi(\frac{\chi}{\psi^f(w)})^{\frac{1-\eta}{\eta}} \Rightarrow \ln \psi^h(w) = \frac{1}{\eta} \ln \chi - \frac{1-\eta}{\eta} \ln \psi^f(w)$
  - Higher $\eta \Rightarrow$ smaller response of $\psi^h(w)$ to $\psi^f(w)$

- Lower η^u from 0.5 to 0.35 and raise η^e from 0.5 to 0.75

Hetero-η Economy: IRF to z shock

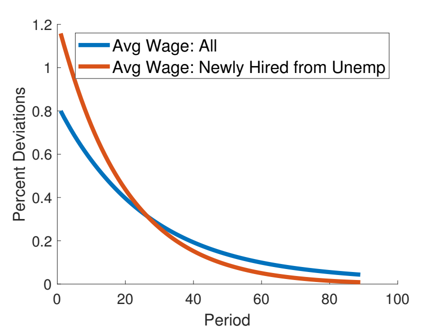


Figure 9: Wages

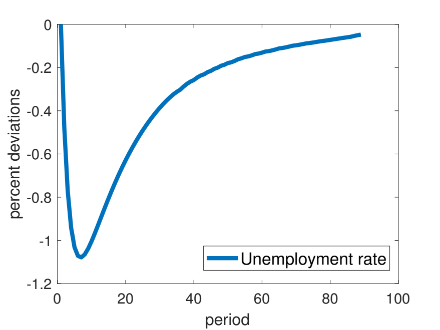


Figure 10: Unemployment Rate

- Similar wage response
- More responsive unemployment (still not enough)

Hetero-η Economy: IRF to z shock

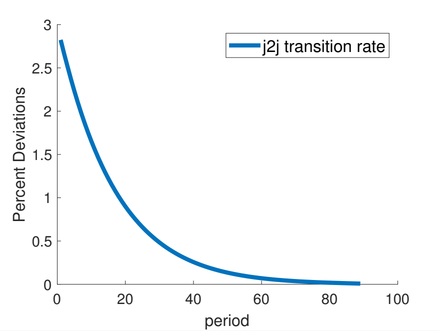


Figure 11: J2J transitions

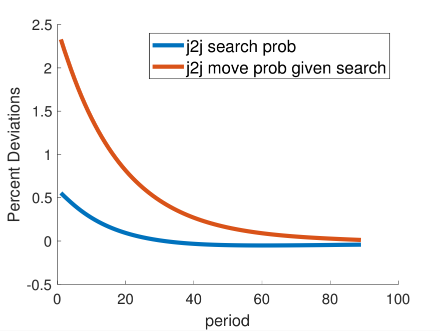


Figure 12: J2J search & JFP

- Response of J2J transition is mitigated
- Due to less responsive job finding probability for the employed workers

Hetero-η Economy: Model Statistics

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1.00
Avg Wage	0.690	0.51	1.00	0.688	0.53	1.00
New Wage	0.689	0.95	1.00	0.654	0.92	1.00
Unemp Rate	10.6%	0.35	-0.48	7.7%	0.78	-0.84
Quits+J2J moves	38.4%	8.94	0.99	34.9%	1.42	1.00
J2J moves	29.2%	10.66	0.99	26.9%	1.98	1.00

Table 9: Productivity Shock ($\rho = 0.95$)

- Allowing for different matching functions for UE and EE moves greatly reduce the gap of volatility between unemployment and j2j transitions
- But they both show insufficient volatility compared to output, in response to the productivity shock

Hetero-η Economy: Model Statistics

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.15	0.13	0.688	0.45	0.47
New Wage	0.689	2.02	0.31	0.654	2.40	0.73
Unemp Rate	10.6%	4.55	-0.99	7.7%	9.37	-0.99
Quits+J2J moves	38.4%	42.41	0.40	34.9%	11.65	0.70
J2J moves	29.2%	49.40	0.42	26.9%	15.55	0.70

Table 10: Delta Shock ($\rho = 0.95$)

- Allowing for different matching functions for UE and EE moves has similar effect on reduce volatility gap between unemployment and j2j transitions
- Unemployment is much more volatile compared to output in response to the delta shock, because the delta shock only affects total output through employment

Hetero-η Economy: Model Statistics

- Two ways to aggregate shocks

	shock corr = 0		shock corr = 0.95	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.48	0.91	0.41	0.94
new wage	1.20	0.80	1.34	0.96
unemployment	3.70	-0.52	3.30	-0.91
quits + j2j movers	4.88	0.60	5.01	0.94
J2J movers	6.50	0.62	6.68	0.96

Table 11: Both Shocks ($\eta^e = 0.75, \eta^u = 0.35, \rho = 0.95$)

- By allowing for two types of shocks, and different matching functions for UE and EE moves, the model delivers a pretty good match to the data

High-b & Hetero-η Economy: Motivation

- The non-market value b is well recognized to be a key driver of the unemployment volatility (Hagedorn and Manovski, 2008).
- We now raise b from 0.3 (Shimer, 2005) to 0.5-0.6 (near the upper limit of our model) in the hetero-η economy.

HIGH-b (0.5) & HETERO-η ECONOMY: IRF TO z SHOCK

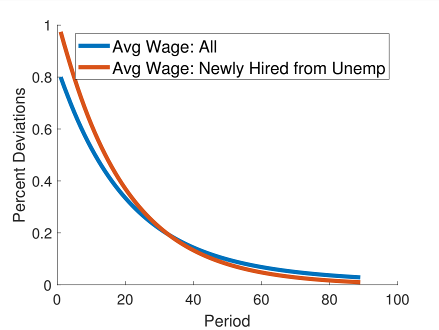


Figure 13: Wage

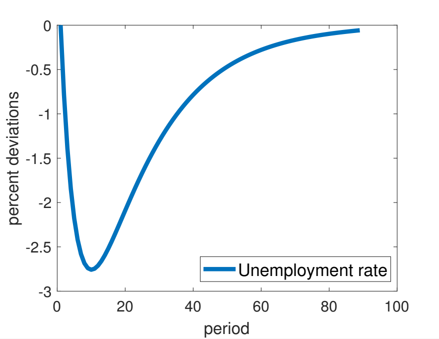


Figure 14: Unemployment Rate

- New wages are a bit less reponsive
- Unemployment drops up to 2.7% for 1% increase in productivity

High-b (0.5) & Hetero-η Economy: IRF to z shock

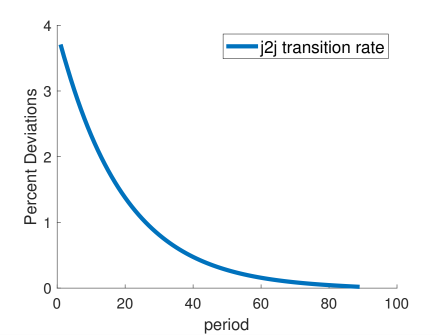


Figure 15: J2J transitions

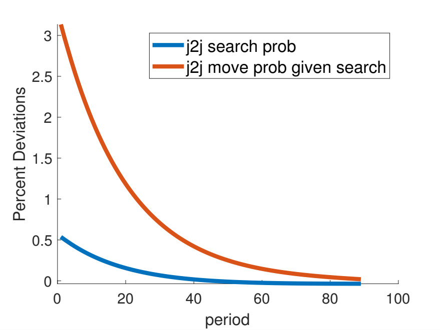


Figure 16: J2J search & JFP

- Response of J2J transition is further mitigated

High-b (0.5) & Hetero-η Economy: Model Statistics

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.66	0.98
New Wage	0.689	0.95	1.00	0.656	0.80	0.97
Unemp Rate	10.6%	0.35	-0.48	9.4%	1.24	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	1.21	-0.82
Quits+J2J moves	38.4%	8.94	0.99	37.7%	2.32	0.98
J2J moves	29.2%	10.66	0.99	28.7%	3.05	0.98

Table 12: High-b & Hetero-η: Productivity Shock ($\rho = 0.95$)

- New: $b = 0.5$, $\eta^u = 0.35, \eta^e = 0.75$. Benchmark: $b = 0.3$, $\eta^u = \eta^e = 0.5$
- All together, these extensions lead to 3.5 times more unemployment volatility, and shrink OJS move volatility to less than 30% the original level.

High-b (0.6) & Hetero-η Economy: IRF to z shock

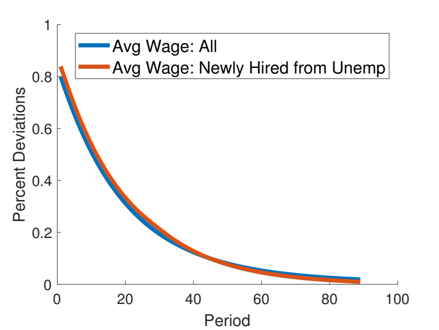


Figure 17: Wage

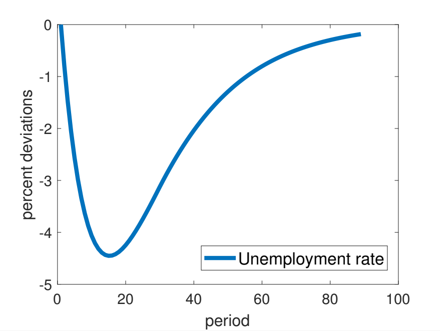


Figure 18: Unemployment Rate

- b to the limit: new wage only slightly more responsive than the average
- Unemployment drops up to 4.5% for 1% increase in productivity

High-b (0.6) & Hetero-η Economy: IRF to z shock

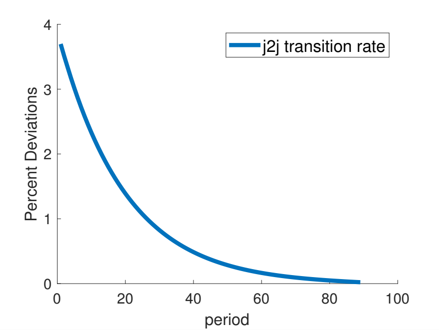


Figure 19: J2J transitions

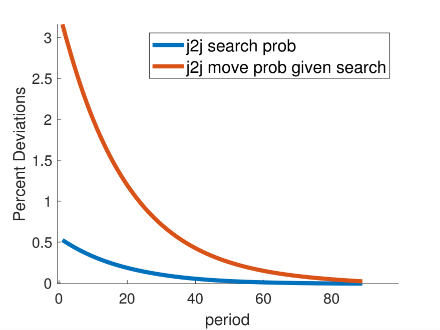


Figure 20: J2J search & JFP

- Response of J2J transition is the same magnitude as unemployment (like in the data)

High-b (0.6) & Hetero-η Economy: Model Statistics

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.44	0.98
New Wage	0.689	0.95	1.00	0.656	0.47	0.97
Unemp Rate	10.6%	0.35	-0.48	20.9%	1.25	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	2.46	-0.84
Quits+J2J Moves	38.4%	8.94	0.99	37.7%	1.54	0.98
J2J Moves	29.2%	10.66	0.99	28.7%	2.06	0.98

Table 13: High-b & Hetero-η Economy: Productivity Shock ($\rho = 0.95$)

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Conclusions I

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to generate quits
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages
- On the Job Search seems to Magnify Fluctuation a lot

Conclusions II

- Exciting set of continuation projects:
  1. Incorporate the movements outside of the labor force.
  2. Endogenous Search intensity on the part of firms
  3. Aiming Shocks to soften correlation between wages and wealth
  4. Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  5. Move towards more sophisticated household structures (more life cycle movements, multiperson households).

Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

Free entry with variable recruiting intensity

- Let $\upsilon(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.

- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c})\, \psi^f[\theta(w)] \; \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c})\, \psi^f[\theta(w)]\right] \; \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

- With FOC given by

$$v_{\overline{c}}(\overline{c}) \; \left\{ \psi^f[\theta(w)] \; \left[\frac{\Omega(w)}{1+r} - \; \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- If $v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_{2\ \overline{c}}$, we have

$$(\upsilon_1 \, \overline{c} \, + \upsilon_2) \left\{ \psi^f[\theta(w)] \, \left[\frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing υ so that for the numbers that have now

$$\frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \, \overline{c} \Big] \, \psi^f[\theta(w)] \, \frac{\Omega(w)}{1+r} + \left[1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \, \overline{c} \right] \, \psi^f[\theta(w)] \, \frac{\overline{k}(1-\delta_k)}{1+r} \Big\} = \overline{c} + \overline{k},$$

- Solving for $\{\upsilon_1, \upsilon_2\}$ that satisfy both equations given our choice of $\overline{c}$ we are done

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A few notes on normalizing Gumbels

Victor et al[1]

February 8, 2021

A structural Interpretation as Taste Shocks

Continuum of Taste Shocks

- Consider a set of possible choices $C = [0, \overline{c}]$
- for each $c \in C$ there is a $\epsilon^c \sim G(0, \alpha)$
- Let

$$X^{\overline{c}} = \max_{c \in [0, \overline{c}]} \{\epsilon^c\}$$

$$V^{\overline{c}} = E\{X^{\overline{c}}\}$$

- Then (see Appendix)

$$X^{\overline{c}} \sim G\left(\ln \overline{c}, \alpha\right)$$

$$V^{\overline{c}} = E\{X^{\overline{c}}\} = \ln \overline{c} + \frac{\gamma}{\alpha}$$

- Both the shock $(\frac{\gamma}{\alpha})$ and having more options $(\ln \overline{c})$ add utility. This should be structural

Adjustment for choice $\widehat{c} < \overline{c}$ (structural)

- Poorer agents have less choice and should have less utility and be more disciplined because of that.
- They face the same shocks ϵ^c but can only choose among ϵ^c, $c \leq \widehat{c}$.
- Then they face

$$X^{\widehat{c}} \sim G(\ln \widehat{c}, \alpha)$$

$$V^{\widehat{c}} = E\{X^{\widehat{c}}\} = \ln \widehat{c} + \frac{\gamma}{\alpha}$$

Discretization over arbitrary grid (not structural)

- Let $j = \{1, \cdots, J\}$ be any discrete state of the agent with associated $[0, c^j]$ being the range of feasible consumptions at that state.
    - Then the appropriate type of the EVShocks should be X^{c^j}.
- Let M^j be the number of grid points that represent the choices associated to state j. Then for each $m^j \in \{1, 2, \cdots, M^j\}$ we should have EVS $\epsilon^{m^j} \sim G\left(0, \alpha^{M^j}\right)$ so that the associated

$$X^{M^j} \sim G\left(\ln c^j, \alpha\right)$$

$$V^{M^j} = E\{X^{c^j}\} = c^j + \frac{\gamma}{\alpha}$$

- Which requires that α^{M^j} V: I think. Please verify the algebra

$$\alpha^{M^j} = \frac{\ln M^j + \gamma}{\ln c^j + \frac{\gamma}{\alpha}}$$

Appendix

Gumbel Distribution

- If ϵ follows i.i.d. $G(\mu, \alpha)$, where the mode μ is non-zero, we have

$$V^1 = E\{\epsilon\} = \mu + \frac{\gamma}{\alpha}$$

$$\text{Mode}\ \ \{\epsilon\} = \mu$$

$$\text{Median}\{\epsilon\} = \mu - \frac{\ln(\ln 2)}{\alpha}$$

$$\text{Var}\{\epsilon\} = \frac{\pi^2}{6\ \alpha^2}$$

$$\text{cdf}\{\epsilon\} = e^{\{-e^{[-\alpha(\epsilon-\mu)]}\}}$$

Expected max: Finitely Many Identically Distributed

- Expected maximum of N Gumbel random variables $G(\mu, \alpha)$. Let

$$X^N = \max\left\{\epsilon^1, \epsilon^2, \cdots, \epsilon^N\right\}$$
$$V^N = \mathbb{E}\left[X^N\right]$$

- We have $$X^N \sim G\left(\frac{1}{\alpha}\ln N\ e^{\alpha\mu}, \alpha\right)$$
$$V^N = \frac{1}{\alpha}\ln N e^{\alpha\mu} + \frac{\gamma}{\alpha} = \frac{1}{\alpha}\left[\ln N + \alpha\mu\right] + \frac{\gamma}{\alpha}$$

- To make V^N independent of the number of choices N, either

$$V^N = \bar{V} \Rightarrow \alpha(N) = \frac{\gamma + \ln N}{\bar{V} - \mu}$$
$$V^N = \bar{V} \Rightarrow \mu(N) = \alpha\bar{V} - \ln N - \gamma$$

Expected max: Location Parameter Heterogeneity

- η^i follows $G(\mu, \alpha)$, let $\epsilon^i = \eta^i + \delta^i$, $\epsilon^i \sim G(\mu + \delta^i, \alpha)$.

$$X^N \sim G\left(\frac{1}{\alpha}\ln\sum_i e^{\alpha\mu^i}, \frac{1}{\alpha}\right) = G\left(\mu + \frac{1}{\alpha}\ln\sum_i e^{\alpha\delta^i}, \frac{1}{\alpha}\right)$$

$$V^N = \mu + \frac{1}{\alpha}\ln\sum_i e^{\alpha\delta^i} + \frac{\gamma}{\alpha}$$

- To make V^N independent of the number of choices, we can require

$$V^N = \bar{V} \Rightarrow \alpha(N) = \frac{\gamma + \ln\sum_i e^{\alpha(N)\mu^i}}{\bar{V} - \mu}$$

$$V^N = \bar{V} \Rightarrow \mu(N) = \bar{V} - \frac{1}{\alpha}\left[\gamma + \ln\sum_i e^{\alpha\mu^i}\right]$$

- No closed-form solution for $\alpha(N)$