Applied Economics Letters, 2004, **11**, 693–697

Routledge
Taylor & Francis Group

Endogenous versus exogenous efficiency units of labour for the quantitative study of Social Security: two examples

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This paper explores the role of endogenous versus exogenous efficiency units of labour for the quantitative evaluation of the impact of pay-as-you-go Social Security on labour supply. Pension response to a population growth rate change is also studied. Two dynamic general equilibrium models are used: one with human capital accumulation through learning-by-doing, and a second with exogenous efficiency units of labour. The main differences in the results are the following: (a) the shift in the working time-age profile induced by the elimination of Social Security considerably differs in both models. The increase in average hours worked is 4% higher under human capital accumulation than in the alternative model; and (b) the pension falls by a similar percentage in both models when the population growth rate is set to zero. This occurs because the capital–labour ratio changes less under learning-by-doing than with exogenous efficiency units of labour.

I. INTRODUCTION

A well-established result from the empirical literature is that human capital is endogenous to individual's decisions over the life cycle (e.g. Shaw, 1986; Heckman *et al.*, 1998; Imai, 2001). However, most quantitative works on pay-as-you-go (PAYG) Social Security ignore this fact and assume exogenous efficiency units of labour (e.g. Cooley and Soares, 1999; Kotlikoff *et al.*, 1999). Since human capital accumulation extremely affects wage evolution and time allocation, its omission might lead to inaccurate conclusions when quantitatively studying Social Security issues.

This paper analyses the role of human capital accumulation for assessing the PAYG Social Security impact on labour supply and the pension response to a reduction in population growth. Two dynamic general equilibrium models are used for this purpose: one with human capital investment through learning-by-doing (LBD) and the other with exogenous efficiency units of labour. There are two steps in the process. Both models are calibrated following the same criteria. The calibrated model economies succeed in matching observed hours worked and hourly wage profiles. Two simple exercises are performed in the second step: the Social Security system is removed and the population growth rate is set to zero. The analysis is limited to the comparison of stationary equilibria.

The results from the first exercise reveal that the elimination of Social Security induces a radically different change in the labour supply–age profile in both models, leading to a higher increase in average hours worked under LBD than with fixed efficiency units of labour. This is because LBD reinforces the substitution effect. When the population growth rate is set to zero, the pension falls by a similar percentage in both models, though slightly larger in the model using endogenous efficiency units. The reason is that the capital–labour ratio, and hence the wage per efficiency unit of labour, varies less if human capital is endogenous.

Applied Economics Letters ISSN 1350–4851 print/ISSN 1466–4291 online
http://www.tandf.co.uk/journals
DOI: 10.1080/1350485042000236548

The rest of the paper is organized as follows. Section II presents the model economies. Section III describes the calibration procedure. Section IV presents and comments the results from the simulations. Lastly, Section V concludes the paper.

II. DESCRIPTION OF THE MODEL ECONOMIES

To simplify the notation, time index and the period at which agents were born are omitted in variables that remain unaltered in steady state.

There is a continuum of identical firms, which operate with a constant returns to scale Cobb–Douglas technology

$$Y_t = \mathrm{B}K_t^{\theta}N_t^{1-\theta} \qquad 0<\theta<1 \tag{1}$$

where K_t is the stock of physical capital, N_t is the labour input in period t, and B is a scale parameter. There is no per capita output growth at long run.[1] Output can be used for consumption or for increasing the physical capital stock. The capital stock depreciates each period at the rate δ_K. The conditions of equality between marginal productivities and factor prices from the maximization of profits by firms are

$$R = \mathrm{B}\theta K_t^{\theta-1}N_t^{1-\theta} - \delta_K \qquad W = \mathrm{B}(1-\theta)K_t^{\theta}N_t^{-\theta} \tag{2}$$

where R and W are interest rate and the wage per efficiency unit of labour, respectively.

The economies have overlapping generations of households who live I periods, with ages denoted by $i=\{1,\ldots,I\}$. Agents work during the first I_r-1 periods of their life and then retire. Retirement is compulsory. The population grows each period at a constant rate n.

Each individual derives utility from an aggregate consumption good, C_i, and disutility from the time devoted to work, L_i, and maximizes his intertemporal utility discounted at a positive rate β. There is no bequest motive. Preferences are represented by the following discounted utility function

$$\sum_{i=1}^{I}\beta^{i-1}\left(\frac{C_i^{\sigma_C}}{\sigma_C}-\alpha\frac{L_i^{\sigma_L}}{\sigma_L}\right) \tag{3}$$

The budget constraint facing an individual can be written as

$$A_{i+1} = \begin{cases} (1+R)A_i + (1-\tau)WH_iL_i - C_i & \text{if } 1 \leqslant i \leqslant I_r - 1 \\ (1+R)A_i + P - C_i & \text{if } I_r \leqslant i \leqslant I \end{cases} \tag{4}$$

where A_i denotes wealth, H_i represents efficiency units of labour, τ is the Social Security payroll tax and P is the pension.

The only difference between these two models is in the assumption regarding efficiency units of labour. Model 1 incorporates human capital investment through LBD. Human capital technology is taken from Imai (2001),[2]

$$H_{i+1} = a_0 + a_1H_i + (b_0 - b_1(i-1))(c_0 + H_i) \times \left(\left(\frac{L_i}{hn}+\frac{d}{hn}\right)^{e} - c_1\left(\frac{L_i}{hn}+\frac{d}{hn}\right)\right) \tag{5}$$

Efficiency units of labour are exogenously given in the alternative model (Model 2).

Each household maximizes its total discounted utility over the life cycle (Equation 3), subject to the initial conditions as well as constraints Equations 4 and 5 in Model 1, and Equation 4 in Model 2. It is assumed that agents have perfect foresight.

The only role of the government is to administer the PAYG Social Security programme. The system is financed by workers' proportional contributions on earnings and is balanced at each period. The pension is calculated as the sum of all workers' Social Security contributions divided by the total number of retirees.

In equilibrium, capital stock and the labour input in the economy are equal to

$$K_t = \sum_{i=1}^{I}(1+n)^{t-i+1}A_i \qquad N_t = \sum_{i=1}^{I_r-1}(1+n)^{t-i+1}H_iL_i \tag{6}$$

III. CALIBRATION PROCEDURE

Both models are calibrated to replicate some long-run observations of the American economy (calibration targets). Table 1 contains the parameter values and Fig. 1 shows simulated and actual age profiles of hours worked and pre-tax hourly wage.

The calibration process classifies the parameters into two groups. Those in the first group are taken from other works, and the rest of parameters are calibrated to match the targets. In Model 1 six targets are used to calibrate seven parameters ($\beta, \sigma_L, \alpha, \sigma_L, \mathrm{B}, \delta_K$ and b_1), while in Model 2 six parameters (except b_1) are calibrated with five targets.[3]

[1] Human capital technology does not permit the introduction of either exogenous or endogenous growth.
[2] hn is a parameter introduced here to rescale hours worked just to avoid dealing with large numbers in computation. See Imai (2001) for the justification of the functional form.
[3] The non-linear least squares method (Levenberg–Marquardt) used here permits systems with more unknowns than equations to be solved. The value for b_1 in Imai (2001) results too high to replicate the targets, this is why the parameter is calibrated.

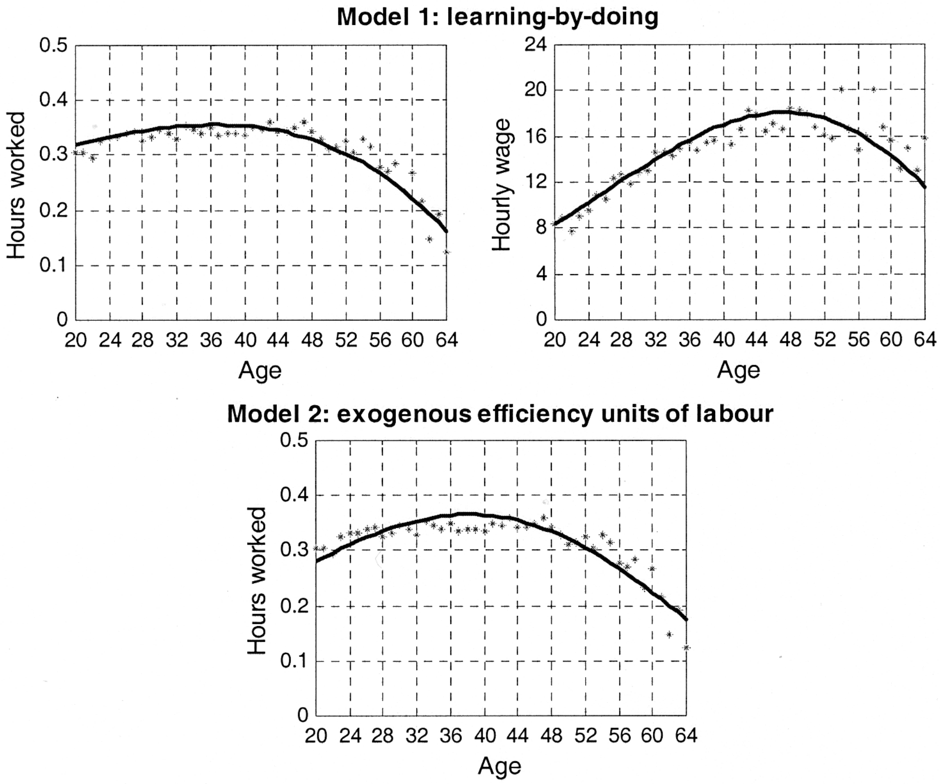

Fig. 1. *Age profiles of hours worked and pre-tax hourly wage delivered by the models confronted to actual data. The solid line represents profiles from the models, while the stars represent actual data from the PSID*

Table 1. *Parameter values (baseline case)*

Population	
	$I = 65$ (age 84); $I_r = 46$ (age 65); $n = 0.012$ (Cooley and Prescott, 1995)
Social Security payroll tax	
	$\tau = 0.112^a$ (U.S. Social Security Administration)
Output technology	
	$\theta = 0.36$ (Ríos-Rull, 1996); $B = 0.9025$; $\delta_K = 0.0737$
Preferences	
	Model 1: $\beta = 0.9624$; $\sigma_C = -0.0988$; $\alpha = 16.7396$; $\sigma_L = 2.2011$
	Model 2: $\beta = 0.9729$; $\sigma_C = -1.1356$; $\alpha = 2.8837$; $\sigma_L = 2.1856$
Human capital technology	
	$a_0 = 0.03322$; $a_1 = 0.3862$; $b_0 = 0.1345$; $c_0 = 0.04244$; $c_1 = 0.0004057$; $d = 404.2/hn$; $e = 0.228$ (Imai, 2001) $hn = 6000$; $b_1 = 0.000393523$
Exogenous efficiency units of labour	
	In Fig. 1

Notes: [a]Old Age and Survivors Insurance (OASI) tax rate corresponding to 1990–1993. *Sources:* Between parentheses.

The targets and sources are as follows. The capital–output ratio (2.94) and the consumption–output ratio (0.748) are taken from Ríos-Rull (1996).[4] The third target is just a normalization of wage per efficiency unit of labour to the unit, which establishes the initial level of human capital to be equal to the hourly wage at age 20. The last three targets refer to mean and standard deviation of annual hours worked divided by 6000 (0.3116 and 0.053, respectively), and the average hourly wage (14.7292) taken from the 1993 Panel Study of Income Dynamics (PSID).[5] Average hourly wage is excluded in the calibration of Model 2. Lastly, human capital-age profile delivered by Model 1 is taken as the exogenous efficiency units of labour–age profile in Model 2.

Auerbach and Kotlikoff's (1987) numerical algorithm was modified to perform the computations of the models. Figure 1 shows that hours worked and hourly wage delivered by the calibrated model economies match quite well with those profiles from the data.

[4] These are the observations constructed by Cooley and Prescott (1995), but do not reflect government data.
[5] The sample is composed of 20–64 year olds, both sex family heads. Only individuals who belong to the employment status categories 'currently working' and 'retired' are included in the sample. The hourly wage is computed as annual labour income divided by annual hours worked. Data are collapsed in means by age.

IV. RESULTS

The calibrated models are used to perform two simple exercises: to remove the Social Security from the economy and to set the population growth rate to zero. Table 2 and Fig. 2 contain the findings.[6]

The Social Security payroll tax distorts time allocation. Removing the distortionary taxation has an income and substitution effect on the labour supply. Table 2 shows that the substitution effect dominates and average hours worked increases in both models, though that increase is 4% higher in Model 1 than in Model 2. The termination of the PAYG system also induces a shift in the age profile of labour supply. Figure 2 shows that in both models, young agents work less and old agents work more than in the initial steady state, reflecting that individuals have to save for their retirement. However, the reduction in hours worked by young agents is considerably lower in Model 1 than in Model 2. The results suggest that human capital accumulation plays an important role in explaining the different response of the labour supply in Model 1. Human capital investment takes place mainly at the beginning of the working life because the cost in terms of forgone leisure is lower, and the period to enjoy investment returns is longer. This is precisely why the working time barely falls at early ages and, consequently, average hours worked increases by a higher percentage in Model 1 than in the alternative model.

When the population growth rate is set to zero, the pension decrease is similar in both frameworks, though about 1% greater in the first one. The result is due to the lower increase in the capital–labour ratio, and hence in the wage per efficiency unit of labour, under LBD than with exogenous efficiency units (Table 2). This is because in the first model both working time and the human capital vary, while in the second model the efficiency units are fixed. The reduction in the population growth rate makes labour relatively scarcer than physical capital, raising its market value and generating incentives to work more. As in the previous exercise, substitution and income effects drive the labour supply response. However, in Model 1 the average working time increase is higher and average hourly wage raise is lower than in Model 2. This explains why the pension falls by a similar percentage in both models. Even though additional hours worked lead to more human capital, the increase in this variable is not large enough to compensate for the lower variation of wage per efficiency unit of labour.

V. CONCLUSION

General equilibrium models of Social Security typically ignore human capital investment and assume exogenous efficiency units of labour, which is in contradiction with the empirical evidence. If human capital is endogenous to individual's decisions, its omission might lead to inaccurate results when quantitatively analysing Social Security issues.

This paper has presented two examples, which consider whether human capital accumulation through LBD or

Table 2. *Endogenous versus exogenous efficiency units of labour*[a]

	Model 1			Model 2		
Variable	Baseline	$\tau = 0\%$	$n = 0\%$	Baseline	$\tau = 0\%$	$n = 0\%$
K/N	4.5937	5.0314 (9.5282)	4.8930 (6.5154)	4.5937	5.3793 (17.1016)	5.1159 (11.3677)
P	1.7257	0 (−100)	1.2363 (−28.3595)	1.7316	0 (−100)	1.2553 (−27.5063)
R	4.87%	4.18% (−14.1683)	4.39% (−9.8562)	4.87%	3.70% (−24.0246)	4.06% (−16.6324)
W	1.0000	1.0333 (3.3300)	1.0230 (2.3000)	1.0000	1.0585 (5.8500)	1.0395 (3.9500)
$\overline{L}$[b]	0.3116	0.3274 (5.0706)	0.3210 (3.0166)	0.3116	0.3150 (1.0911)	0.3189 (2.3427)
$\overline{H}$[b]	14.7292	14.9849 (1.7360)	14.8618 (0.9002)	14.7292	14.7292 (0)	14.7292 (0)
$\overline{WH}$[b]	14.7292	15.4839 (5.1238)	15.2033 (3.2187)	14.7292	15.5908 (5.8500)	15.3110 (3.9500)
$\overline{WHL}$[b]	4.6101	5.0971 (10.5637)	4.9058 (6.4141)	4.6406	5.0294 (8.3782)	4.9815 (7.3460)

Notes: [a]Variations in percentage respect to the baseline case between parentheses; [b]The dash over the variables denotes means.

[6] Data, codes and results are available upon request via e-mail.

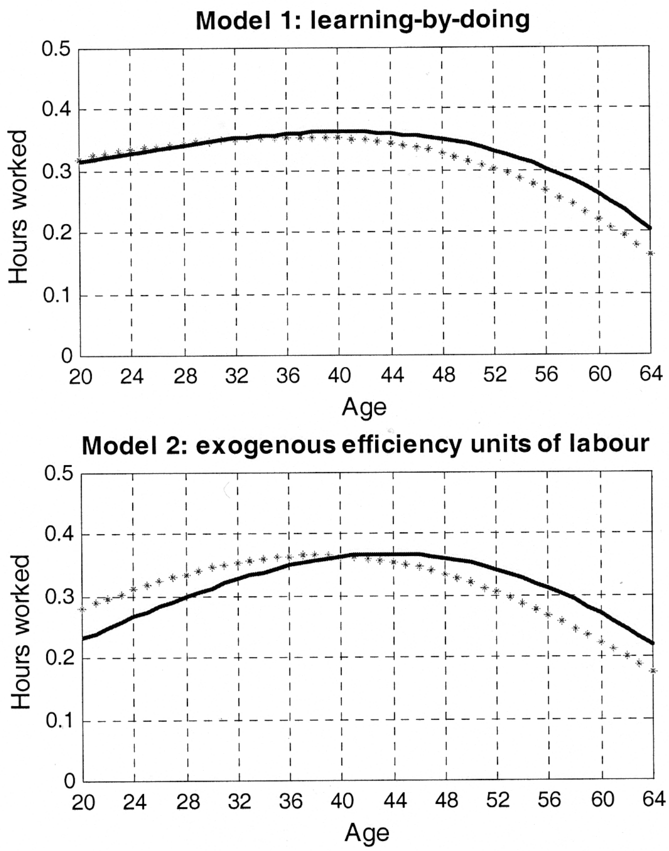

Fig. 2. *Effects on hours worked due to Social Security elimination. The solid line represents the no Social Security case, while the stars represent the baseline case*

exogenous efficiency units of labour considerably modifies the quantitative results. More specifically, it has shown that the elimination of PAYG Social Security induces a higher increase in average hours worked under LBD than with fixed efficiency units of labour. Additionally, when the population growth rate is set to zero, the pension falls by a similar percentage in both cases. However the reduced variation in the capital–labour ratio is the main reason why this occurs.

Though more research is needed on the role of human capital investment in quantitative models of Social Security, the findings obtained here suggest that the typical assumption of exogenous efficiency units of labour should be reconsidered.

ACKNOWLEDGEMENTS

I am deeply indebted to Víctor Ríos for his help and suggestions. I am also grateful to Fernando Perera for his comments.

REFERENCES

Auerbach, A. J. and Kotlikoff, L. J. (1987) *Dynamic Fiscal Policy*, Cambridge University Press, Cambridge.

Cooley, T. F. and Prescott, E. (1995) Economic growth and business cycle, in *Frontiers of Business Cycle Research* (Ed.) T. F. Cooley, Princeton University Press, Princeton, NJ, pp. 1–38.

Cooley, T. F. and Soares, J. (1999) Privatizing social security, *Review of Economic Dynamics*, **2**, 731–55.

Heckman, J., Lochner, L. and Taber, C. (1998) Explaining rising wage inequality: explanations with a dynamic general equilibrium model of labor earnings with heterogeneous agents, *Review of Economic Dynamics*, **1**, 1–58.

Imai, S. and Keane, M. P. (2004) Intertemporal labor supply and human capital accumulation, *International Economic Review*, **45**(2), 601–41.

Kotlikoff, L. J., Smetters, K. and Walliser, J. (1999) Privatizing social security in the United States: comparing the options, *Review of Economic Dynamics*, **2**, 532–74.

Ríos-Rull, J. V. (1996) Life-cycle economies and aggregate fluctuations, *Review of Economic Studies*, **63**, 465–89.

Shaw, K. (1986) Life-cycle labor supply with human capital accumulation, *International Economic Review*, **30**, 431–56.