10 Firms and Equilibrium

What is a firm?

- A technology:

$$y = F(k, l) = A\, k^{\alpha}\, l^{1-\alpha}$$

  for $\alpha \in (0, 1)$.

- Operational definition.

- We are in a static world: we will assume k to be constant.

Properties of the Technology

From lectures in growth we know that:

1. Constant returns to scale.

2. Inputs are essential.

3. Marginal productivities are positive and decreasing.

Problem of the Firms

•Maximizes profits given r and w (we are taking the consumption good as the numeraire!):

$$\max_{k,n} = A\ k^{\alpha} n^{1-\alpha} - rk - wn$$

•We take first order conditions:

$$\begin{aligned} \alpha\ A\ k^{\alpha-1}\ n^{1-\alpha} &= r \\ (1-\alpha)\ A\ k^{\alpha}\ n^{-\alpha} &= w \end{aligned}$$

•Some algebra and we would see that the solution is indeterminate due to constant returns to scale. Only $\frac{k}{n}$ is determined. This means that we do not know the structure of firms but we do know the factor prices.

To make clear that these are the choice of the firms we write $k(f)$ and $n(f)$, so

$$\begin{aligned} \alpha\ A\ k(f)^{\alpha-1}\ n(f)^{1-\alpha} &= r \\ (1-\alpha)\ A\ k(f)^{\alpha}\ n(f)^{-\alpha} &= w \end{aligned}$$

So what happens? EQUILIBRIUM

•Equilibrium is a situation where consumers do what they want, producers do what they want and there actions are compatible. This is achieved via prices.

•How does this apply in our context? Households own some capital k^{hh} that they rent at firms at price r (this plays the role of profits in the previous part so think that $\pi = r\, k^{hh}$ and that for now taxes are zero).

•Equilibrium requires then market clearing

$$\begin{aligned} n^{hh} &= n(f) \\ k^{hh} &= k(f) \end{aligned}$$

•That we can write as

$$\begin{aligned} r &= \alpha \; A \; (k^{hh})^{\alpha-1} \; n^{hh}(w, r\, k^{hh})^{1-\alpha} \\ w &= (1-\alpha) \; A \; (k^{hh})^{\alpha} \; n^{hh}\left(w, r\, k^{hh}\right)^{-\alpha} \end{aligned}$$