11 Welfare Theorems

Pareto Optimality

- An allocation is *Pareto Optimal* if there is no way to rearrange production or reallocate goods so that someone is made better off without making someone else worse off.

- Pareto Optimality $\neq$ perfect state of the world or any concept like that.

The Social Planner

• Let us imagine we have a powerful dictator, the Social Planner, that can decide how much the households consume and work and how much the firms produce.

• The Social Planner does not follow prices. But it understands opportunity cost.

• The Social Planner is benevolent. It searches for the best possible allocation.

- Maximizes utility household given a level of government pur-
chases G^*

$$\max_{c,l} \ u\left(c, l\right)$$

such that

$$c + G = A \ k^{\alpha} \ (h - l)^{1-\alpha} = A \ k^{\alpha} \ n^{1-\alpha}$$

$$G = G^*$$

$$k = k^*$$

- Note: we do not have prices in the budget constraint!!!

- Standard Maximization problem.

- We can rewrite the problem as:

$$\max_{n} \quad u\left(Ak^{*\alpha}l^{1-\alpha} - G^*, h - n\right)$$

- First Order Condition with respect to n:

$$u_c\left(A\ k^{*\alpha}\ n^{1-\alpha} - G^*, h - n\right)(1 - \alpha)\ A\ k^{*\alpha}\ n^{-\alpha} =$$

$$u_{h-n}\left(A\ k^{*\alpha}\ n^{1-\alpha} - G^*, h - n\right)$$

- We rearrange as:

$$\frac{u_{h-n}\left(A\ k^{*\alpha}\ n^{1-\alpha}-G^{*},h-n\right)}{u_{c}\left(A\ k^{*\alpha}\ n^{1-\alpha}-G^{*},h-n\right)}=(1-\alpha)\,A\ k^{*\alpha}\ n^{-\alpha}$$

- The lhs is the Marginal Rate of Substitution, MRS while the rhs is the Marginal Rate of Transformation, MRT.

- Thus, optimality implies:

$$MRS = MRT$$

The Big Question

- What is the relation between the solution to the Planners Problem and the Competitive Equilibrium?

- Or equivalently, is the Competitive Equilibrium Pareto-Optimal?

The answer is YES

- Why do we care about this question?

  1. Positive reasons
  2. Normative reasons.

The Intuition

- First think about the case when $G^* = \tau^l = 0$

- Look again at the Social Planner's optimality condition

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)} = (1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

- Remember that the Household first order condition was:

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)} = w$$

- And that firms profit maximization implied:

$$w = (1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

• First order conditions are equivalent!!!

The Formal Statement

- First Fundamental Welfare Theorem: under certain conditions, the Competitive Equilibrium is Pareto Optimal.

- We have the converse.

- Second Fundamental Welfare Theorem: under certain conditions, a Pareto optimum is a Competitive Equilibrium.

Some consequences

- First Fundamental Welfare Theorem states that, under certain conditions, an allocation achieved by a market economy is Pareto-Optimal.

- Formalization of Adam Smith's "invisible hand" idea.

- Strong theoretical point in favour of decentralized allocation mechanisms: prices give good incentives.

- Second Fundamental Welfare Theorem states what is the best way to change allocations: redistribute income. Do not mess with prices!!!

How robust is the First Welfare theorem?

- Not too much.

- Plenty of reasons that deviate the allocation from a Pareto optimum:

  1. Taxes.
  2. Externalities.
  3. Asymmetric Information.
  4. Market Incompleteness.
  5. Bounded Rationality of Agents.

What if taxes are not zero? (say income taxes)

- Now think about the case when $G^* \neq 0, \tau^l \neq 0$

- Look again at the Social Planner's optimality condition

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)} = (1-\alpha)\,Ak^{*\alpha}l^{-\alpha}$$

- But now the Household first order condition is:

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)} = \left(1 - \tau^l\right)w$$

- And since that firms profit maximization implied:

$$w = (1 - \alpha) A k^{*\alpha} l^{-\alpha}$$

- First order conditions are NOT equivalent!!!

Externalities

- What is an externality? When an agents consumption or production decision changes the production or consumption possibilities of other agents.

- Externalities can be good or bad.

- Example:

  1. Cities
  2. Environment
  3. Congestion, Leisure of Others, Weekends.

Asymmetric Information

- **Information** is dispersed in society.

- We may want to change our behavior based on the information we have.

- Adverse Selection and Moral Hazard. (Left and Right).

- Akerlof-Spence-Stiglitz, Nobel Prize Winners 2001.

Market Incompleteness

- We have assumed that we have complete markets.

- Every good can be traded.

- Is that a good representation of the world?

- Closely related with the problem of asymmetric information.

Distributional Concerns

A Pareto Optimal is an allocation such that there is no other that makes somebody better off and NOBODY worse off.

If one person has everything and everybody else has nothing it is a Pareto Optimum.

However, what about making somebody a little bit worse if many get a lot better?

Pareto optimality has nothing to say, so perhaps it is not so useful a concept.