## 12 Intertemporal Choice

- So far we have only studied static choices.
- Life is full of intertemporal choices: should I study for my test today or tomorrow? should I save or should I consume now?
- We will present a simple model: the Life-Cycle/Permanent Income Model of Consumption.
- Developed by Modigliani (nobel winner 1985) and Friedman (Nobel winner 1976).


## The Model

- Household, lives 2 periods.
- Utility function

$$
u\left(c_{1}, c_{2}\right)=U\left(c_{1}\right)+\beta U\left(c_{2}\right)
$$

where $c_{1}$ is consumption in first period of his life, $c_{2}$ is consumption in second period of his life and $\beta$ is between zero and one and measures household's degree of impatience.

- Income $y_{1}>0$ in the first period of life and $y_{2} \geq 0$ in the second period of his life
- Initial wealth $A \geq 0$, due to bequests that he received from parents.


## Budget Constraint

- Household can save some of his income in the first period or some of his initial wealth, or he can borrow against his future income $y_{2}$. Interest rate on both savings and on loans is equal to $r$. Let $s$ denote saving
- Budget constraint in first period of life

$$
c_{1}+s=y_{1}+A
$$

- Budget constraint in second period of his life

$$
c_{2}=y_{2}+(1+r) s
$$

- Summing both budget constraints

$$
c_{1}+\frac{c_{2}}{1+r}=y_{1}+\frac{y_{2}}{1+r}+A=I
$$

- We have normalized the price of the consumption good in the first period to 1 . Price of the consumption good in period 2 is $\frac{1}{1+r}$, which is also the relative price of consumption in period 2, relative to consumption in period 1. Gross interest rate $1+r$ is the relative price of consumption goods today to consumption goods tomorrow.


## Key Results

- What variables does current consumption depend on? $y_{1}, y_{2}, A, r$.
- What happens to consumption if $y_{1}, y_{2}$ or $A$ increases?
- Both $c_{1}$ and $c_{2}$ increase.


## Comparative Statics: Changes in the Interest Rate

- Income effect: if a saver, then higher interest rate increases income for given amount of saving. Increases consumption in first and second period. If borrower, then income effect negative for $c_{1}$ and $c_{2}$.
- Substitution effect: gross interest rate $1+r$ is relative price of consumption in period 1 to consumption in period 2. $c_{1}$ becomes more expensive relative to $c_{2}$. This increases $c_{2}$ and reduces $c_{1}$.
- Hence: for a saver an increase in $r$ increases $c_{2}$ and may increase or decrease $c_{1}$. For a borrower an increase in $r$ reduces $r_{1}$ and may increase or decrease $c_{2}$.


## Borrowing Constraints

- So far: household could borrow freely at interest rate $r$
- Now: assume borrowing constraints $s \geq 0$
- If household is a saver, nothing changes
- If household would be a borrower without the constraint, then $c_{1}=y_{1}+A$, $c_{2}=y_{2}$. He would like to have bigger $c_{1}$, but he can't bring any of his second period income forward by taking out a loan. In this situation first period consumption does not depend on second period income or the interest rate.


## MACROECONOMICS

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## Chapter 8

A Two-Period Model: The ConsumptionSavings Decision and Ricardian Equivalence

## Figure 8.1 Consumer's Lifetime Budget Constraint



## Figure 8.2 A Consumer's Indifference Curves



## Figure 8.3 A Consumer Who Is a Lender



## Figure 8.4 A Consumer Who Is a Borrower



## Figure 8.5 The Effects of an Increase in Current Income for a Lender



## Figure 8.6 Percentage Deviations from Trend in GDP and Consumption, 19472003



## Figure 8.7 An Increase in Future Income



## Figure 8.8 Temporary Versus Permanent Increases in Income



## Figure 8.9 Stock Prices and Consumption of Nondurables and Services, 1985-2003



Figure 8.10 Scatter Plot of Percentage Deviations from Trend in Consumption of Nondurables and Services Versus Percentage Deviations from Trend in a Stock Price Index


## Figure 8.11 An Increase in the Real Interest Rate



## Figure 8.12 An Increase in the Real Interest Rate for a Lender



## Figure 8.13 An Increase in the Real Interest Rate for a Borrower



## Figure 8.14 Example with Perfect Complements Preferences



## Figure 8.15 A Consumer's Demand for Current Consumption Goods, c ${ }^{\text {d }}$, as a Function of Current Income



## Figure 8.16 A Shift in a Consumer's Demand for Current Consumption



## Figure 8.17 Ricardian Equivalence with a Cut in Current Taxes for a Borrower



## Figure 8.18 Pay-As-You-Go Social Security for Consumers Who Are Old in Period T



## Figure 8.19 Pay-As-You-Go Social Security for Consumers Born in Period T and Later



## Figure 8.20 Fully Funded Social Security When Mandated Retirement Saving Is Binding



## Figure 8.21 A Consumer Facing Different Lending and Borrowing Rates



## Figure 8.22 Effects of a Tax Cut for a Consumer with Different Borrowing and Lending Rates



## Figure 8.23 Real Consumption of Durables, 1991-1993



## Figure 8.24 Real Consumption of Nondurables, 1991-1993



## Figure 8.25 Real Consumption of Services, 1991-1993



## An Extension of the Basic Model: The Life Cycle Hypothesis

- We can extend to $T$ periods: Franco Modigliani' life-cycle hypothesis of consumption
- Individuals want smooth consumption profile over their life. Labor income varies substantially over lifetime, starting out low, increasing until the 50 'th year of a person's life and then declining until 65, with no labor income after 65.
- Life-cycle hypothesis: by saving (and borrowing) individuals turn a very nonsmooth labor income profile into a very smooth consumption profile.
- Main predictions: current consumption depends on total lifetime income and given initial wealth. as in the simple model. Saving should follow a very pronounced life-cycle pattern with borrowing in the early periods of an economic life, significant saving in the high earning years from 35-50 and dissaving in retirement years.
- One empirical puzzle: older household do not dissave to the extent predicted by the theory. Several explanations

1. Individuals are altruistic and want to leave bequests to their children.
2. Uncertainty with respect to length of life and health status.

## Another Extension: The Permanent Income Hypothesis

- Future labor income is uncertain.
- Income of an individual household, $y$ consists of a permanent part, $y_{p}$ and a transitory part $y_{t}$

$$
y=y_{p}+y_{t}
$$

- Permanent part $y_{p}$ : expected average future income (usual salary)
- Transitory part $y_{t}$ : random fluctuations around this average income (win in the lottery)
- Individuals react differently to an increase in permanent and an increase in transitory income
- Increase in the permanent component of income brings about an (almost) equal response in consumption
- Individuals would smooth out transitory income shocks over time
- It follows that individual consumption is almost entirely determined by permanent income

$$
c=\alpha y_{p}
$$

where $\alpha$ is a parameter close to 1

- Data seem quite favorable to these theories


## An Application of the Theory: Social Security

- Personal saving rate -the fraction of disposable income that private households save- has declined from about $7-10 \%$ in the 60's and 70 's to $2.1 \%$ in 1997
- Is expansion of the social security system responsible for this?
- Use simple life-cycle model to analyze this.

Consumption in both periods is higher with social security than without if and only if $\tilde{y}_{1}>y_{1}$, i.e. if and only if $\frac{(1+g)(1+n)}{1+r}>1$. People are better off with social security if

$$
(1+g)(1+n)>1+r
$$

Intuition: If people save by themselves for retirement, return on their savings equals $1+r$. If they save via a social security system, return equals $(1+n)(1+g)$

Numbers: $n=1 \%, g=2 \%, r=7 \%$ (average return on stock market for last 100 years)

Reform of the social security system desired.

Problem: one missing generation: at the introduction of the system there was one generation that received social security but never paid taxes.

Dilemma:

1. Currently young pay double, or
2. Default on the promises for the old, or
3. Increase government debt, financed by higher taxes in the future, i.e. by currently young and future generations

## Application of the Theory: Ricardian Equivalence

- What are the effects of government deficits in the economy?
- A first answer: none (Ricardo (1817) and Barro (1974)).
- How can this be?
- The answer outside our small model is tricky.
- Lump-sum taxes.
- Government budget constraints:

$$
\begin{gathered}
G_{1}=T_{1}+B \\
G_{2}+(1+r) B=T_{2}
\end{gathered}
$$

- Consolidating:

$$
G_{1}+\frac{G_{2}}{1+r}=T_{1}+\frac{T_{2}}{1+r}
$$

- Note that $r$ is constant (you should not worry too much about this).


## Household's Problem

Original problem

$$
\begin{gathered}
\max _{c_{1}, c_{2}} U\left(c_{1}\right)+\beta U\left(c_{2}\right) \\
\text { s.t. } c_{1}+\frac{c_{2}}{1+r}+T_{1}+\frac{T_{2}}{1+r}=I
\end{gathered}
$$

- Now suppose that the government changes timing of taxes $T_{1}^{\prime}, T_{2}^{\prime}$ and government consumption $G_{1}^{\prime}, G_{2}^{\prime}$. Then the problem of the household is:

$$
\begin{gathered}
\max _{c_{1}, c_{2}} U\left(c_{1}\right)+\beta U\left(c_{2}\right) \\
\text { s.t. } c_{1}+\frac{c_{2}}{1+r}+T_{1}^{\prime}+\frac{T_{2}^{\prime}}{1+r}=I
\end{gathered}
$$

- Since these new taxes must satisfy:

$$
T_{1}+\frac{T_{2}}{1+r}=T_{1}^{\prime}+\frac{T_{2}^{\prime}}{1+r}
$$

problem of the consumer is equivalent!!!

## Empirical Evidence

- Taxes in the world are not lump-sum.
- Does the Ricardian Equivalence hold?
- Important debate.

