Notes on Econ 102 Sect. 2, Fall 2005

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Chapter 4 Growth Accounting
Evolved from notes written by Jesús Fernández-Villaverde

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4 Growth Accounting

- Output is produced by inputs capital K and labor L, in combination with the available technology A

- Want: decompose the growth rate of output into the growth rate of capital input, the growth rate of labor input and technological progress. This exercise is called growth accounting.

- Why?

Aggregate production function

- Maps inputs into output:

$$Y = F(A, K, L)$$

  A is called total factor productivity (TFP).

- Cobb-Douglas example:

$$Y = A \ K^{\alpha} \ L^{1-\alpha}$$

- Interpretation.

Discrete vs. Continuous Time

- In discrete time a variable is indexed by time: x_t.

- In continuous time a variable is a function of time: $x(t)$.

- We observe the world only in discrete time...

- but it is often much easier to work with continuous time!

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- Remember from calculus that validity of Taylor series expansion is local: g small!

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- When Δt is small (let's say a year):

$$g_x(t) = \frac{\dot{x}(t)}{x(t)} \simeq \frac{x(t+1) - x(t)}{x(t)} = g_x(t-1, t) \simeq \Delta \log x_t$$

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\end{aligned}$$

3 Growth rate of a ratio = the difference of the growth rates:

$$g_k(t) = g_K(t) - g_L(t)$$

4 A constant ratio that both variables grow at same rate:

$$g_k(t) = 0 \Rightarrow g_K(t) = g_L(t)$$

Growth Rates of Weighted Products. Suppose $Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$ What is $g_Y(t)$?

[1] Take logs $\quad \log(Y(t)) = \alpha \log(K(t)) + (1-\alpha)\log(L(t))$

[2] Differentiate

$$\begin{aligned} \frac{d\log(Y(t))}{dt} &= \alpha \frac{d\log(K(t))}{dt} + (1-\alpha)\frac{d\log(L(t))}{dt} \\ \frac{\dot{Y}(t)}{Y(t)} &= \alpha \frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)} \\ g_Y(t) &= \alpha \; g_K(t) + (1-\alpha) \; g_L(t) \end{aligned}$$

Growth rate = weighted sum, (weights equal to share parameters)

Growth Accounting

- Observations in discrete time.
- Production Function: $Y(t) = F(A(t), K(t), L(t))$
- Differentiating with respect to time and dividing by $Y(t)$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{F_A A(t)}{Y(t)}\frac{\dot{A}(t)}{A(t)} + \frac{F_k K(t)}{Y(t)}\frac{\dot{K}(t)}{K(t)} + \frac{F_L L(t)}{Y(t)}\frac{\dot{L}(t)}{L(t)}$$

- Useful benchmark: Cobb-Douglas $Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$.
- Why? (factor shares)
- Taking logs and differentiating with respect to time gives

$$g_Y(t) = g_A(t) + \alpha g_K(t) + (1-\alpha)g_L(t)$$

- g_A is called TFP growth or multifactor productivity growth.

The Cobb-Douglas Production Function and its properties

- Under competition factors of production are paid their marginal productivities.

$$w = \frac{\partial F(K,L)}{\partial L} = (1-\alpha) A K^{\alpha} L^{-\alpha} \qquad r = \frac{\partial F(K,L)}{\partial K} = \alpha\, A\, K^{\alpha-1} L^{1-\alpha}$$

- Labor and capital shares of output in any year t are

$$\frac{W_t\, L_t}{Y_t} = \frac{(1-\alpha) A_t K_t^{\alpha}\, L_t^{-\alpha} L_t}{Y_t} = 1-\alpha$$

$$\frac{r_t\, K_t}{Y_t} = \frac{\alpha\, A_t\, K_t^{\alpha-1}\, L_t^{1-\alpha}\, K_t}{Y_t} = \alpha$$

- Cobb-Douglas production functions are consistent with the Kaldor facts.

Doing the Accounting

- Pick α (capital share).

- Measure g_Y, g_K and g_L from the data.

- g_A is the residual $g_A(t) = g_Y(t) - \alpha\, g_K(t) - (1 - \alpha)\, g_L(t)$

- Therefore g_A is also called the Solow residual.

- Severe problems if missmeasurement (g_K is hard to measure).

Data for the US

- We pick $\alpha = \frac{1}{3}$

Per.	g_Y	αg_K	$(1-\alpha)g_L$	TFP (g_A)
$48-98$	2.5	0.8 (32%)	0.2 (8%)	1.4 (56%)
$48-73$	3.3	1.0 (30%)	0.2 (6%)	2.1 (64%)
$73-95$	1.5	0.7 (47%)	0.3 (20%)	0.6 (33%)
$95-98$	2.5	0.8 (32%)	0.3 (12%)	1.4 (56%)

- Key observation: Productivity Slowdown in the 70's
- Note: the late 90's look much better

Reasons for the Productivity Slowdown

1. Sharp increases in the price of oil in 70's
2. Structural changes: more services and less and less manufacturing goods produced
3. Slowdown in resources spent on R&D in the late 60's.
4. TFP was abnormally high in the 50's and 60's
5. Information technology (IT) revolution in the 70's

Growth Accounting for Other Countries

- One key question: was fast growth in East Asian growth miracles mostly due to technological progress or mostly due to capital accumulation?
- Why is this an important question?

Country	Per.	g_Y	α	αg_K	$(1-\alpha)g_L$	g_A
Germany	60-90	3.2	0.4	59%	−8%	49%
Italy	60-90	4.1	0.38	49%	3%	48%
UK	60-90	2.5	0.39	52%	−4%	52%
Argentina	40-80	3.6	0.54	43%	26%	31%
Brazil	40-80	6.4	0.45	51%	20%	29%
Chile	40-80	3.8	0.52	34%	26%	40%
Mexico	40-80	6.3	0.63	41%	23%	36%
Japan	60-90	6.8	0.42	57%	14%	29%
Hong Kong	66-90	7.3	0.37	42%	28%	30%
Singapore	66-90	8.5	0.53	73%	31%	−4%
South Korea	66-90	10.3	0.32	46%	42%	12%
Taiwan	66-90	9.1	0.29	40%	40%	20%