6 Introduction to Human Capital

- Education levels are very different across countries.

- Rich countries tend to have higher educational levels than poor countries.

- We have the intuition that education (learning skills) is an important factor in economic growth.

Production Function

- Cobb-Douglas aggregate production function:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$

- Again we have constant returns to scale.

- Human capital and labor enter with a different coefficient.

Inputs Accumulation

- Society accumulates human capital according to:

$$\dot{H} = s_h Y - \delta H$$

- Capital accumulation equation:

$$\dot{K} = s_k Y - \delta K$$

- Technological progress: $\frac{\dot{A}}{A} = g > 0$.

- Labor force grows at constant rate: $\frac{\dot{L}}{L} = n > 0$.

- Human Capital accumulates with schools. More schools more capital.

- But eventually there are decreasing returns to the combination of human and physical capital.

Alternative Specification (similar but different than Jones)

•Cobb-Douglas aggregate production function:

$$Y = K^{\alpha} (A\ H)^{1-\alpha-\beta}$$

•Human Capital Accumulates

$$\dot{H} = e^{\phi\, u}\, L$$

where u is fraction of time studying, and ϕ is a parameter.

•Then, again it is exactly like the basic model without human capital, at some point it cannot grow any more.

•In Jones there is no actual growth in H, it is just

$$H = e^{\phi\, u}\, L$$

It generates differences in levels.

Rewriting the Model in Efficiency Units

- Redefine the variables in efficiency units:

$$\tilde{x} \equiv \frac{X}{AL}$$

- Then, dividing the production function by AL:

$$\tilde{y} = \tilde{k}^{\alpha}\tilde{h}^{\beta}$$

- Decreasing returns to scale in per efficiency units.

Human Capital Accumulation

- The evolution of inputs is determined by:

$$\begin{aligned}\dot{\tilde{k}} &= s_k \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n+g+\delta)\tilde{k} \\ \dot{\tilde{h}} &= s_h \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n+g+\delta)\tilde{h}\end{aligned}$$

- System of two differential equations.

- Solving it analytically it is bit tricky so we will only look at the BGP.

Phase Diagram

- Solving the system analytically it is bit tricky.

- Alternatives:

  1. Use numerical methods.

  2. Linearize the system.

  3. Phase diagram.

Balanced Growth Path Analysis I

- To find the BGP equate both equations to zero:

$$
\begin{aligned}
s_k \tilde{k}^{*\alpha} \tilde{h}^{*\beta} - (n+g+\delta)\tilde{k}^* &= 0 \\
s_h \tilde{k}^{*\alpha} \tilde{h}^{*\beta} - (n+g+\delta)\tilde{h}^* &= 0
\end{aligned}
$$

- From first equation:

$$
\tilde{h}^* = \left(\frac{(n+g+\delta)}{s_k} \tilde{k}^{*1-\alpha} \right)^{\frac{1}{\beta}}
$$

Balanced Growth Path Analysis II

- Plugging it in the second equation

$$s_h \tilde{k}^{*\alpha} \frac{(n+g+\delta)}{s_k} \tilde{k}^{*1-\alpha} - (n+g+\delta) \left(\frac{n+g+\delta}{s_k} \tilde{k}^{*1-\alpha} \right)^{\frac{1}{\beta}} = 0 \Rightarrow$$

$$\frac{s_h}{s_k} \tilde{k}^{*} = \left(\frac{n+g+\delta}{s_k} \tilde{k}^{*1-\alpha} \right)^{\frac{1}{\beta}}$$

- Work with the expression.

Some Algebra

$$\frac{s_h}{s_k}\tilde{k}^* = \left(\frac{n+g+\delta}{s_k}\tilde{k}^{*1-\alpha}\right)^{\frac{1}{\beta}} \Rightarrow$$

$$\tilde{k}^{*1-\frac{1-\alpha}{\beta}} = \tilde{k}^{*-\frac{1-\alpha-\beta}{\beta}} = \frac{s_k}{s_h}\left(\frac{(n+g+\delta}{s_k}\right)^{\frac{1}{\beta}} \Rightarrow$$

$$\boxed{\tilde{k}^* = \left(\frac{s_k^{1-\beta}s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}}$$

$$\boxed{\tilde{h}^* = \left(\frac{s_k^{\alpha}s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}}$$

Evaluating the Model

- The central issue for growth is the value of β.

- If $\beta = 1/3$ we have that small differences in $\{s_h, s_k, \phi, n, g, \delta\}$ can account for (relatively) large differences in output per capita across countries.

- According to Jones differences in k account for a factor of 2 differences in output, differences in educational attainment account ro 2.2 (using educational attainment differentials and the return to schooling). The reminder is still 7 or 8 times that have to be imputed to differences in TFP.