ANSWER KEY TO QUIZ 3: ECON 102, SECTION 1

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Q1.

A balanced growth path is a situation in which output per worker, capital per worker and consumption per worker grow at constant (potentially different) rates, $g_y = g_A$ .Steady state is a balanced growth path with zero growth rates.

Q2.

1) y and k grow at the same constant and positive rate g over time.

$$y^* = A(t)[\frac{s}{n+g+\delta}]^{\frac{a}{1-a}}$$

$$k^* = A(t)[\frac{s}{n+g+\delta}]^{\frac{1}{1-a}}$$

→ the model satisfies fact 1)

2) K/Y is relatively constant over time.

On the BGP: $\frac{K}{Y} = \frac{s}{n+g+\delta}$ is constant.

3) r is relatively constant over time.

$r = MP_K = A(t)\alpha K^{\alpha-1}L^{1-\alpha} = \alpha\frac{Y}{K}$ is constant.

4) Capital and labor shares are roughly constant over time.

$$\frac{rK}{Y} = \frac{A(t)\alpha K^{\alpha-1}L^{1-\alpha}K}{A(t)K^{\alpha}L^{1-\alpha}} = \alpha$$

$$\frac{wL}{Y} = \frac{A(t)(1-\alpha)K^{\alpha}L^{-\alpha}L}{A(t)K^{\alpha}L^{1-\alpha}} = 1-\alpha$$

Q3.

$$k^*_A = (\frac{s_A}{n_A+\delta_A})^{\frac{1}{1-a}} = (\frac{0.2}{0.01+0.07})^{\frac{1}{1-a}} = \left(\frac{5}{2}\right)^{\frac{1}{1-a}} > \left(\frac{20}{9}\right)^{\frac{1}{1-a}} = (\frac{0.2}{0.02+0.07})^{\frac{1}{1-a}} = (\frac{s_B}{n_B+\delta_B})^{\frac{1}{1-a}} = k^*_B$$

$$y^*_A = (k^*_A)^{\alpha} > (k^*_B)^{\alpha} = y^*_B$$

Q4.

If hurricane wipes out half of the capital, the capital per worker is divided by two instantaneously. The economy will gradually shift back to its BGP after the hurricane.

Q5.

In the steady state, $\dot{k}^* = 0 \Rightarrow 0 = s(k^*)^{\alpha} - \delta k^* \Rightarrow k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$

Q6.

False. The Solow model without technological progress predicts zero growth rate of k and y in the steady state and a changing growth rate of k and y when economy moves to its steady state.

Note: True for K and Y is an acceptable answer. In the steady state they grow at rate n, population growth rate.

Q7.

Other things being equal, the poorer country will grow faster. Eventually both countries will reach their steady states.

Q8.

1) $\dot{H} = s_H Y - \delta H$

2) $\dot{H} = e^{\varphi u} L$, where φ is a parameter, u is the fraction of time studying.

Q9.

1) $r = \alpha \frac{Y}{K} \Rightarrow \frac{r_1}{r_2} = \frac{y_1 / k_1}{y_2 / k_2} = 10\frac{k_2}{k_1}$, $\frac{y_1}{y_2} = \frac{Ak_1{}^{\alpha}}{Ak_2{}^{\alpha}} = \left(\frac{k_1}{k_2}\right)^{\frac{1}{3}} \Rightarrow \frac{k_1}{k_2} = 1000 \Rightarrow \frac{r_1}{r_2} = 0.01$

2) This may be surprising because capital should flow to the poorer country.

3) If H is larger in the richer country it is possible to have $\frac{y_1}{y_2} = 10$ and $r_1 = r_2$

Q10.

True. OECD countries starting further below the BGP grow faster than OECD countries closer to the BGP.