A standard RBC model with ambiguity

- Representative agent with recursive multiple priors utility.
- Felicity:

$$u(C_t, H_t) = \frac{C_t^{1-\chi}}{1-\chi} - \psi_L \frac{H_t^{1+\sigma_L}}{1+\sigma_L}$$

- Technology: output Y_t is produced by

$$Y_t = Z_t K_t^{\alpha} H_t^{1-\alpha}$$

- Capital accumulation:

$$K_{t+1} - (1-\delta) K_t = I_t$$

- Resource constraint:

$$C_t + I_t = Y_t$$

Structure of beliefs

- Specify ambiguity about exogenous productivity
- Beliefs about endogenous variables derived from "structural knowledge" of economy
- Representation of belief set $\mathcal{P}_t$

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t + u_{t+1}$$
$$\mu_t \in [-a_t, a_t]$$

- "True" productivity process: $\mu_t = 0$.
- Interval for μ_t = lack of confidence in prob. assessments
- Process a_t = time varying ambiguity
- Example 1: homoskedastic Z_t & exogenous time-varying ambiguity

$$u_{t+1} = \sigma_z \varepsilon_{z,t+1}$$
$$a_t = (1 - \rho_v)\,\bar{a} + \rho_v a_{t-1} + \sigma_a \varepsilon_{a,t}$$

Volatility shocks & changes in ambiguity

- Example 2: heteroskedastic Z_t & ambiguity increases with volatility

$$u_{t+1} = \sigma_{z,t}\varepsilon_{z,t+1}$$
$$\sigma_{z,t} = (1-\rho_\sigma)\overline{\sigma}_z + \rho_\sigma \sigma_{z,t-1} + \sigma_\sigma \varepsilon_{\sigma,t}$$
$$a_t = \sqrt{2\eta}\sigma_{z,t}$$

- Interpretation: sufficiently small relative entropy between truth & belief in $\mathcal{P}_t$

Relative entropy

$$\mu_t \in [-a_t, a_t] : R = \frac{\mu_t^2}{2\sigma_{z,t}^2} \leq \eta$$

Social planner problem

- Bellman equation

$$V(K,Z,a) = \max_{\{C,K',H\}} \left\{ u(C,H) + \beta \min_{\mu \in [-a,a]} E^{\mu} V(K',Z',a') \right\} \quad (1)$$
$$s.t.\ C = ZK^{\alpha}H^{1-\alpha} + (1-\delta)K - K'$$

- Worst-case belief: future technology is low!

$$\mu^* = -a$$

$\Rightarrow$ planner acts as if bad times ahead & today a pleasant surprise!

- Interpretation: precautionary behavior (not irrational pessimism!)
- First order effects of ambiguity:
  - if risk $var(\varepsilon) \approx 0$, optimal policies still reflect $\mu^* = -a$
  - effect vanishes only in the limit at zero risk

Characterizing equilibrium

Two Steps

1. Solve planner problem under worst case belief
   $\Rightarrow$ optimal policies $C(K, Z, a), K'(K, Z, a), H(K, Z, a)$
2. Characterize variables under "true" shock process

$$\log Z_{t+1} = \rho_z \log Z_t + u_{t+1}$$

Approximation, ignoring risk effects

- compute "small risk" steady state
  - find policies assuming $var(\varepsilon) \approx 0$
    (reflect first order effects of ambiguity, but risk neutrality)
  - consider steady state given those policies
- linearize policies around steady state

"Small risk" steady state

- Start from planner's optimal policies C, K', H, computed assuming small risk $var(\varepsilon) \approx 0$
- Set exogenous state variables to constants

$$Z_t = 1, \quad a_t = \bar{a}$$

- Steady state capital stock $\bar{K}$ solves

$$\bar{K} = K'\left(\bar{K}, 1, \bar{a}\right)$$

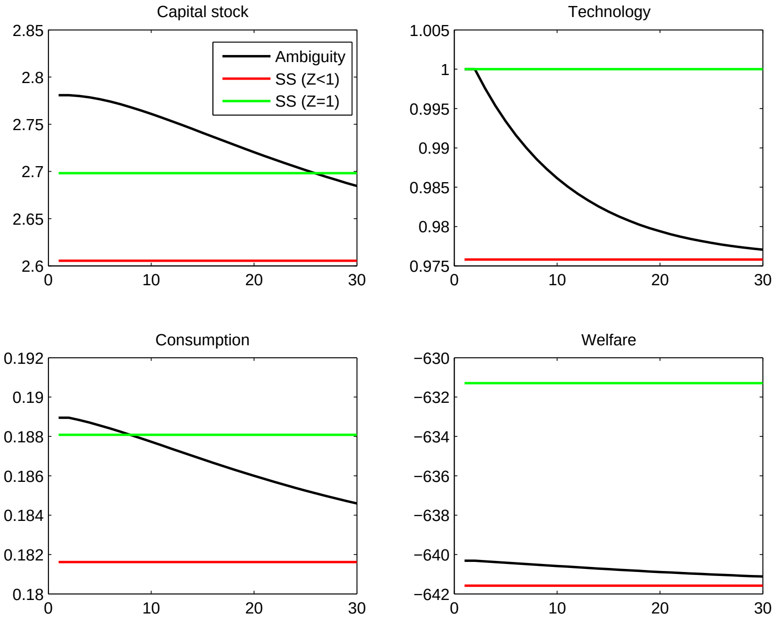
- Mechanics:
  - optimal policy K' as if convergence to steady state with

$$Z = e^{-\bar{a}/(1-\rho_z)} < 1$$

  - current productivity $Z_t = 1$ a "pleasant surprise"

  $\Rightarrow$ $\bar{K}$ high, possibly higher than steady state with $Z_t = 1$
- Interpretation: precautionary savings due to ambiguity

Intuition for zero risk steady state



Dynamics

- Use loglinear approximation to policies C, K', H
  - e.g. around "small risk" steady state
  - with little ambiguity, approx point not important for dynamics
- Policies reflect "worst case" productivity dynamics

$$\log Z_{t+1} = \rho_z \log Z_t - a_t + u_{t+1}$$

- To characterize model, use linearized policies & true dynamics

$$\log Z_{t+1} = \rho_z \log Z_t + u_{t+1}$$

- Works like expected utility models with "news shocks" that do not materialize.