Ambiguous Business Cycles

Cosmin Ilut *   Martin Schneider **

*Duke Univ.

**Stanford Univ.

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Motivation

- Role of uncertainty shocks in business cycles?
- usually: uncertainty = risk

$\Rightarrow$ agents confident in probability assessments

$\Rightarrow$ shocks to uncertainty = shocks to volatility

- this paper: uncertainty = risk + *ambiguity (Knightian uncertainty)*

$\Rightarrow$ allows for lack of confidence in prob assessments

$\Rightarrow$ shocks to uncertainty can be shocks to confidence

Overview

- Standard business cycle model with ambiguity aversion
  - recursive multiple priors preferences
  - ambiguity about mean aggregate productivity
  - $\Rightarrow$ 1st order effects of uncertainty
- Methodology
  - study uncertainty shocks with 1st order approximation
  - simple estimation strategy based on linearization
  - motivate & bound set of priors by concern w/ nonstationarity
- Properties
  - ambiguity shocks work like "unrealized" news shocks with bias.
  - in medium scale DSGE model estimated on US data, ambiguity shocks
    - ★ generate comovement and account for $> \frac{1}{2}$ of fluctuations in Y, C, I, H.
    - ★ imply countercyclical asset premia.

Literature

1. **Multiple Priors Utility**: Gilboa-Schmeidler (1989), Epstein and Wang (1994), Epstein and Schneider (2003).
2. **Business cycles with preference for robustness**: Hansen, Sargent and Tallarini (1999), Cagetti, Hansen, Sargent and Williams (2002), Smith and Bidder (2011).
3. **Signals**: Beaudry and Portier (2006), Christiano, Ilut, Motto and Rostagno (2008), Schmitt-Grohe and Uribe (2008), Jaimovich and Rebelo (2009), Lorenzoni (2009), Blanchard, L'Huillier and Lorenzoni (2010), Barsky and Sims (2010, 2011).
4. **Risk shocks**: Justiniano and Primiceri (2007), Fernandez-Villaverde and Rubio-Ramirez (2007, 2010), Bloom (2009), Bloom, Floetotto and Jaimovich (2009), Christiano, Motto and Rostagno (2010), Gourio (2010), Arellano, Bai and Kehoe (2010)

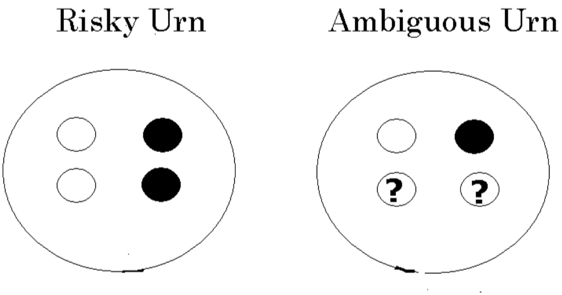
Ambiguity aversion & preferences

- S = state space
  - one element $s \in S$ realized every period
  - histories $s^t \in S^t$
- Consumption streams $C = (C_t(s^t))$
- Recursive multiple-priors utility (Epstein and Schneider (2003))

$$U_t\left(C; s^t\right) = u\left(C_t\left(s^t\right)\right) + \beta \min_{p \in \mathcal{P}_t(s^t)} E^p\left[U_{t+1}\left(C; s^{t+1}\right)\right],$$

- Primitives:
  - felicity u (possibly over multiple goods) & discount factor β
  - one-step-ahead belief sets $\mathcal{P}_t\left(s^t\right)$ – size captures (lack of) confidence
- Why this functional form?
  - preference for known odds over unknown odds (Ellsberg Paradox)
  - formally, weaken Independence Axiom

Ellsberg Paradox



- bet on black from risky urn $\succ$ bet on black from ambiguous urn
- bet on white from risky urn $\succ$ bet on white from ambiguous urn
- expected utility cannot capture choices, but $\min_{p \in \mathcal{P}} E^p[u(c)]$ can!

A stylized business cycle model with ambiguity

- Representative agent with recursive multiple priors utility.
- Felicity from consumption, hours

$$u(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \beta N_t$$

- Output Y_t produced by

$$Y_t = Z_t N_{t-1}$$

- Labor chosen one period in advance
- Belief sets that enter utility
  - specify ambiguity about exogenous productivity
  - beliefs about endogenous variables derived from "structural knowledge" of economy
  - true TFP process: iid lognormal with $E[Z_t] = 1$, $var(\log Z_t) = \sigma_z^2$

Belief set: time variation in ambiguity

- Agents experience changes in confidence, described by process a_t

$\Rightarrow$ Representation of one-step-ahead belief set $\mathcal{P}_t$

$$\log Z_{t+1} = \mu_t - \frac{1}{2}\sigma_z^2 + \sigma_z \varepsilon_{z,t+1}$$
$$\mu_t \in [-a_t, a_t]$$

- Examples for evolution of (lack of) confidence a_t
  1. Linear, homoskedastic law of motion
     $$a_t = (1 - \rho_a)\,\bar{a} + \rho_a a_{t-1} + \varepsilon_{a,t}$$
     Interpretation: intangible information affects confidence
  2. Feedback from realized volatility
     $$a_t = \sqrt{2\eta}\sigma_{z,t}$$
     Interpretation: observed turbulence lowers confidence
     Follows if $\mathcal{P}_t$ is "constant entropy" ball around true DGP:
     $$\mu_t \in [-a_t, a_t] \Leftrightarrow R_t = \frac{\mu_t^2}{2\sigma_{z,t}^2} \leq \eta$$

Social planner problem

- Bellman equation

$$V(N, Z, a) = \max_{N'} \left[u(ZN, N') + \beta \min_{\mu \in [-a,a]} E^{\mu} V(N', Z', a') \right]$$

- Worst-case belief: future technology is low

$$\mu^* = -a$$

$\Rightarrow$ planner acts as if bad times ahead

- Interpretation: precautionary behavior
- First order effects of ambiguity.

Characterizing equilibrium

Two Steps

1. Solve planner problem under worst case belief $\mu^* = -a$
   Optimal hours from FOC

$$1 = E^{-a}\left[\beta\left(Z'N'\right)^{-\gamma}Z'\right]$$

2. Characterize variables under true shock process (in logs)

$$n_t = -\left(1/\gamma - 1\right)\left(a_t + \frac{1}{2}\gamma\sigma_z^2\right)$$
$$y_{t+1} = z_{t+1} + n_t$$

$\Rightarrow$ Worst case belief reflected in action n_t, but not in shock realization z_{t+1}

Properties of equilibrium

- Dynamics

$$y_{t+1} = z_{t+1} + n_t$$
$$n_t = -\left(1/\gamma - 1\right)\left(a_t + \frac{1}{2}\gamma\sigma_z^2\right)$$
$$a_t = \left(1 - \rho_a\right)\bar{a} + \rho_a a_{t-1} + \varepsilon_{a,t}$$

  - first order effects of uncertainty on output, even as $\sigma_z^2 \to 0$
  - if substitution effect is strong enough $\left(1/\gamma > 1\right)$:

1. loss of confidence generates a recession
2. increase in confidence leads to an expansion

  - TFP is not unusual in either cases
  - hours do not forecast TFP if $cov\left(a_t, z_{t+1}\right) = 0$

- Asset prices reflect time varying ambiguity premia
  - price of 1-step-ahead consumption claim $= \frac{E_t C_{t+1}}{R_t^f} \exp\left(-a_t - \gamma\sigma_z^2\right)$

Comparison of shocks

- Ambiguity shocks $n_t = -(1/\gamma - 1)(a_t + \frac{1}{2}\gamma\sigma_z^2)$
  - loss of confidence generates recession if $1/\gamma > 1$
  - hours do not forecast TFP: regress z_{t+1} on n_t to get slope 0
  - time varying ambiguity premium on consumption claim $= a_t$
- News & noise shocks $n_t = (1/\gamma - 1)\left(\pi s_t - \frac{1}{2}\gamma(1-\pi)\sigma_z^2\right)$
  - signal about productivity $s_t = z_{t+1} + \sigma_s \epsilon_{s,t}$ with $\pi := \sigma_z^2/(\sigma_z^2 + \sigma_s^2)$
  - bad signal (news or noise) generates recession if $1/\gamma > 1$
  - hours forecast TFP: regress z_{t+1} on n_t to get slope $\gamma/(1-\gamma)$
  - constant risk premium on consumption claim
- Volatility shocks $n_t = -(1/\gamma - 1)\frac{1}{2}\gamma\sigma_{z,t}^2$
  - volatility process $\sigma_{z,t}^2 = var_t(z_{t+1})$, mean adjusts so $E_t Z_{t+1} = 1$
  - volatility increase generates recession if $1/\gamma > 1$
  - hours do not forecast TFP (but forecast turbulence)
  - time varying risk premium on consumption claim $= \gamma\sigma_{z,t}^2$

General framework: Rep. agent & Markov uncertainty

- Notation (as before $s =$ exogenous state)
  - $X =$ endogenous states (e.g. capital)
  - $A =$ agent actions (e.g. consumption, investment)
  - $Y =$ other endogenous variables (e.g. prices)
- Recursive equilibrium
  - Functions for actions A, other endog vars Y, value V s.t., for all (X, s):

$$V(X,s) = \max_{A \in B(Y,X,s)} \left\{ u(c(A)) + \beta \min_{p \in \mathcal{P}(s)} E^p \left[V(X', s') \right] \right\}$$
$$s.t. \; X' = T(X, A, Y, s, s')$$

  - endog var determination: $G(A, Y, X, s) = 0$
  - true exogenous Markov state process $p^*(s) \in \mathcal{P}(s)$
- Analysis again in 2 steps
  - find recursive equilibrium
  - characterize variables under "true" state process

Characterizing equilibrium: a guess-and-verify approach

- basic idea
  1. guess the worst case belief p^0
  2. find recursive equilibrium under expected utility & belief p^0
  3. compute value function under worst case belief, say V^0
  4. verify that the guess p^0 indeed achieves the minimum
- "essentially linear" economies & productivity shocks
  - environment T, B, G s.t. 1st order approx. ok *under expected utility*
  - ambiguity is about mean of innovations to s
- simplification in essentially linear case
  - in step 1, guess that worst case mean is linear in state variables
  - step 2 uses loglinear approximation around "zero risk" steady state (sets risk to zero, but retains worst case mean)
  - step 4 then checks monotonicity of value function

An estimated DSGE model with ambiguity

- Similar to CEE (2005), SW (2007)

1. Intermediate goods producers
   - Price setting monopolist; competitive in the factor markets
     - ★ mark-up shocks.
2. Final goods producers.
   - Combines intermediate goods to produce a homogenous good.
3. Households: ambiguity-averse
   - Own capital stock, consume, monopolistically supply specialized labor
   - investment adjustment costs, internal habit in consumption.
     - ★ efficiency of investment and price of investment shocks.
4. "Employment agencies"
   - aggregate specialized labor into homogenous labor.
5. Government
   - Taylor-type interest rule: reacts to inflation, output gap and growth.
     - ★ government spending and monetary policy shocks.

Technology

- The intermediate good j is produced using the function:

$$Y_{j,t} = Z_t K_{j,t}^{\alpha} \left(\epsilon_t L_{j,t}\right)^{1-\alpha} - F_t$$

- Final goods:

$$Y_t = \left[\int_0^1 Y_{j,t}{}^{\frac{1}{\lambda_{f,t}}} dj\right]^{\lambda_{f,t}}$$

- Capital accumulation:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\zeta_t \frac{I_t}{I_{t-1}}\right)\right] I_t.$$

- Beliefs about technology:

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t + \sigma_u \varepsilon_{z,t+1}$$
$$\mu_t \in [-a_t, a_t]$$
$$a_t - \bar{a} = \rho_a \left(a_{t-1} - \bar{a}\right) + \sigma_a \varepsilon_{a,t}$$

Solution method: Steps

- Let x_t and s_t denote endog and exog vars

1. Find deterministic "distorted" steady state x_o and s_o based on

$$\log Z_o = \rho_z \log Z_o - \bar{a} + 0$$

2. Linearize around distorted SS: Find A, B :

$$x_t - x_o = A(x_{t-1} - x_o) + B(s_t - s_o)$$

$$s_t = \begin{bmatrix} s_t^* \\ a_t \\ z_t \end{bmatrix} = \underbrace{\begin{bmatrix} \rho & 0 & 0 \\ 0 & 0 & \rho_a \\ 0 & \rho_z & -1 \end{bmatrix}}_{P} \begin{bmatrix} s_{t-1}^* \\ z_{t-1} \\ a_{t-1} \end{bmatrix} + \Xi_t$$

$$\Xi_t = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^a \\ \varepsilon_t^z \end{bmatrix} \sim N(0, \Sigma)$$

$$\widetilde{E}_{t-1} z_t = \rho_z z_{t-1} - a_{t-1}$$

3. True dynamics:
   - True DGP:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z = \widetilde{E}_{t-1} z_t + \varepsilon_t^z + a_{t-1}$$

   - Endogenous variables:

$$x_t - x_o = A(x_{t-1} - x_o) + B(s_t - s_o)$$

$$s_t - s_o = P\left(s_{t-1} - s_o\right) + \widehat{\Xi}_t,\ \widehat{\Xi}_t = \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^a \\ \varepsilon_t^z \end{bmatrix} + \begin{bmatrix} 0_{(n-2)\times 1} \\ 0 \\ a_{t-1} \end{bmatrix}$$

4. Zero risk steady state
   - In Steady State:

$$\widehat{\Xi} = \begin{bmatrix} 0_{(n-1)\times 1} \\ \bar{a} \end{bmatrix}$$

   - Steady state exogenous vars s^{SS}:

$$s^{SS} - s_o = P(s^{SS} - s_o) + (\widehat{\Xi} - \Xi_o)$$

   - Then solve for endogenous vars x^{SS}:

$$x^{SS} - x_o = A(x^{SS} - x_o) + B(s^{SS} - s_o)$$

A bound on ambiguity

- Beliefs about technology

$$\log Z_{t+1} = \rho_z \log Z_t + \mu_t + \sigma_u \varepsilon_{z,t+1}$$
$$\mu_t \in [-a_t, a_t]$$
$$a_t - \bar{a} = \rho_a (a_{t-1} - \bar{a}) + \sigma_a \varepsilon_{a,t}$$

  - agents view innovations as risky $(\sigma_u \varepsilon_{z,t+1})$ and ambiguous (μ_t)
  - they know empirical moments of true $\{\mu_t^*\}$, but not the exact sequence
  - empirical moments say $\log Z_{t+1} - \rho_z \log Z_t \sim i.i.\mathcal{N}\left(0, \sigma_z^2\right)$; $\sigma_z^2 > \sigma_u^2$
  - agents respond to uncertainty about μ_t^* as if minimizing over $[-a_t, a_t]$

- Constrain a_t to lie in a maximal interval $[-a^{\max}, a^{\max}]$
  - require that "boundary" beliefs $\pm a_{\max}$ imply "good enough" forecasts:
  - there exists σ_u^2 s.t. for <u>every</u> potential true DGP $\{\mu_t^*\}$, $a_{\max}$ or $-a_{\max}$ is best forecasting rule at least α of the time
  - $a_{\max}$ is best forecasting rule at date t if true mean $\mu_t^* > a_{\max}$
  - for example, $\alpha = 5\%$ implies $a_{\max} = 2\sigma_z$ = bound used in estimation

Parametrization

- Recall

$$a_t - \bar{a} = \rho_a \left(a_{t-1} - \bar{a}\right) + \sigma_a \varepsilon_{a,t}$$

- Restrictions: process a_t s.t.

  1. a_t is positive:

  $$\bar{a} - m\frac{\sigma_a}{\sqrt{1-\rho_a^2}} \geq 0$$

  2. a_t is bounded by the discipline of the non-stationary argument

  $$\bar{a} + m\frac{\sigma_a}{\sqrt{1-\rho_a^2}} \leq 2\sigma_z$$

- Scale

$$\bar{a} = n\sigma_z, \;\; n \in [0,1]$$

- Directly estimate n, ρ_a.

Estimation

- Linearization $\rightarrow$ estimation using standard Kalman filter methods.
- Data: US 1984Q1-2010Q1: Output, consumption, investment, price of investment growth, hours, FFR, inflation.
- Law of motion for a_t is estimated
- Estimates:
  - productivity dynamics

$$\rho_z = 0.95, \sigma_z = 0.0045$$

  - confidence dynamics

$$\bar{a} = 0.0043, \rho_a = 0.96, \sigma_a = 0.00041$$

  - other parameters broadly consistent with previous studies.

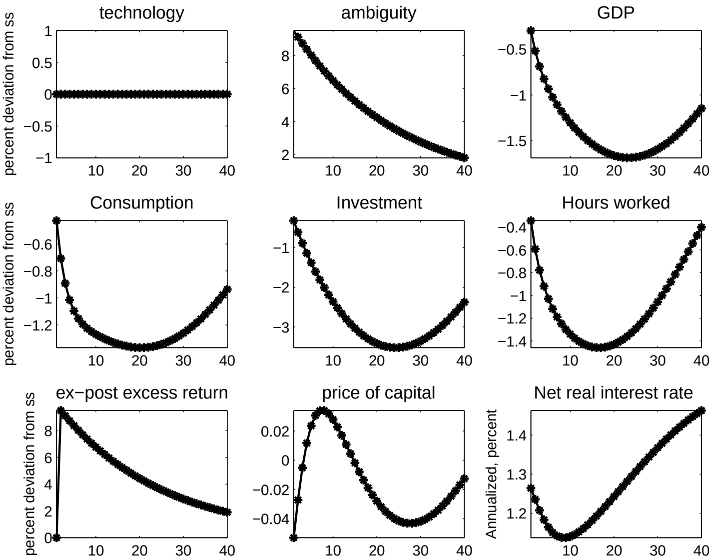
Role of ambiguity in fluctuations

- Variance decompositions: business cycle frequency

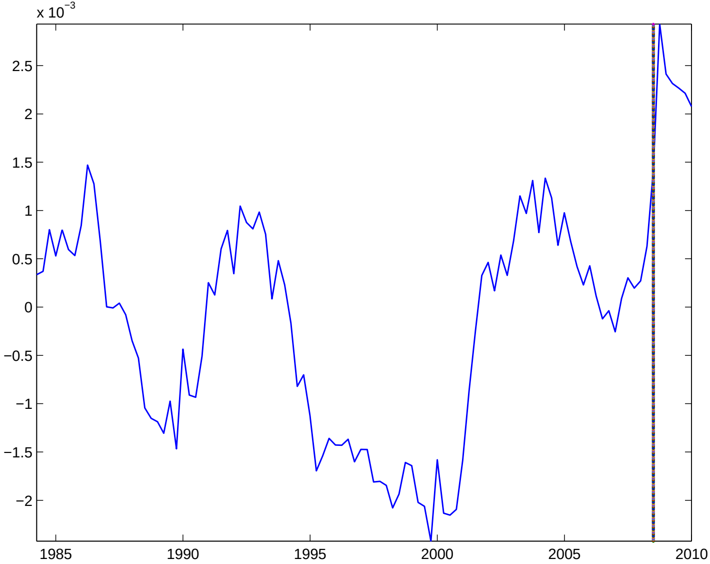
Shock/Var.	Output	Consumption	Investment	Hours
Ambiguity	27	51	14	30
Stationary techn.	11	13	9	2
Efficiency of invest.	33	7	53	32
Stochastic Growth	7	7	6	13
Price mark-up	12	12	12	13

  - any other shocks $< 5\%$ for above variables.
  - long-run theoretical decomposition: $\varepsilon_{a,t}$ about 50% of fluctuations.

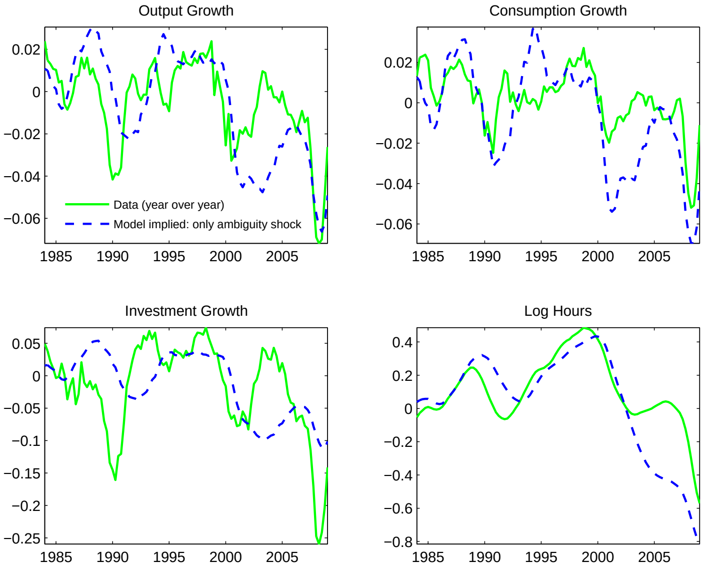
Dynamics: loss of confidence



Estimated ambiguity path



Historical shock decomposition



Welfare cost of fluctuations through ambiguity

Setting $\sigma_z = 0$ vs estimate $\Longrightarrow \bar{a} = 0$ vs estimate

- Welfare: $\overline{V} \equiv$ Value function under "zero risk steady state" (with estimated $\bar{a}$)
  - Welfare cost of fluctuations, as % of $C_{SS}(\bar{a} = 0)$, due to:

1. ambiguity:

$$\lambda^{ambig} = \left[\overline{V} - V^{SS}(\bar{a} = 0)\right](1 - \beta)\beta^{-1} = 13\%$$

2. risk (known probability distributions):

$$\lambda^{risk} = V_{\sigma\sigma}(1 - \beta)\beta^{-1} = 0.01\%$$

- ★ $V_{\sigma\sigma}$: effect of fluctuations in $\varepsilon_{z,t+1}$ in a second order approx. of $V(.)$.

- Other vars: Output, Capital, Consumption, Hours lower by 15%

Conclusion

- Standard business cycle model with ambiguity aversion:
  - recursive multiple priors preferences.
  - ambiguity about mean productivity.
  - discipline from modeling concern with nonstationarity
- With ambiguity, uncertainty shocks have 1st order effects:
  - can apply standard linearization techniques for solution and estimation
  - work like "unrealized" news shocks with bias
  - potentially large role in business cycle
- Next
  - characterize further essentially linear settings