Choice Deferral in Models of Preference Accumulation

Sudeep Bhatia (s.bhatia@warwick.ac.uk)
Behavioral Science Group, Warwick Business School, University of Warwick
Coventry, United Kingdom

Abstract

This paper models choice deferral using a sequential sampling and accumulation theory of preferential choice. It assumes that choice options are accepted or rejected if accumulators reach an upper or lower threshold, and that choice is deferred if these thresholds are not crossed by a fixed time. The proposed model can explain a wide range of findings regarding the determinants and consequences of choice deferral, including the relationship of deferral with choice option conflict, choice option desirability, choice option extremity, and the attraction and compromise effects.

Keywords: Choice deferral, Decision making, Sequential sampling, Associative accumulation, Context effects

Introduction

Cognitive models of preferential choice focus largely on predicting choice probabilities, decision times, and their dependence on task related factors such as decoy options, reference points, and anchors (Bhatia, 2013; Busemeyer & Townsend, 1993; Johnson & Busemeyer, 2005; Roe, Busemeyer & Townsend, 2001; Krajibich, Armeil & Rangel, 2010; Rangel & Hare, 2010; Trueblood, Brown, & Heathcote, in press; Usher & McClelland, 2004). Largely absent from this is a comprehensive approach to modeling choice deferral, that is, the decision not to make any choice whatsoever (Gourville & Soman, 2005; Dhar, 1997; Dhar & Simonson, 2003; Iyengar & Lepper, 2000; Tversky & Shafir, 1992). The failure to decide is a fundamental feature of everyday decision making and formally studying its occurrence can provide many important insights regarding the cognitive underpinnings of the choice process.

The past two decades have seen considerable empirical work outlining the determinants of choice deferral, and the consequences of allowing choice to be deferred. This work has established that the likelihood of choice deferral increases (paradoxically) with the size of the decision set: Adding choice options that are in conflict (i.e. are unalignable) with the options in the existing set, or that are equally desirable to the options in the existing set, can reduce the probability of making a choice (Dhar, 1997; Gourville & Soman, 2005; Tversky & Shafir, 1992). Additionally, the ability to defer choice has been shown to increase the choice probability of extreme options, compared to all-average options, and to increase the incidence of the attraction effect, but decrease the incidence of the compromise effect (Dhar & Simonson, 2003).

There has been some theoretical work attempting to explain deferral using decision field theory (DFT) (Busemeyer & Townsend, 1993, Roe et al., 2001). DFT is a dynamic model of preferential choice, which assumes that attribute values are sampled sequentially and stochastically over the time course of the decision process. The values of these attributes are accumulated into preferences, and decisions are made when preferences cross a fixed threshold. Recently Jessup, Veinott, Todd and Busemeyer (2009) have assumed that choice is deferred within a DFT model, if the decision is not made by a certain time. They have shown that this time constraint based extension to DFT can explain the observed increase in deferral with size of the choice set1.

While Jessup et al. provide a number of valuable insights regarding the psychological basis of choice deferral, they do not attempt to explain its relationship with option conflict and option desirability, or the effect of choice deferral on the choice probability of extreme options and on the incidence of the attraction and compromise effects.

In this paper we imbed Jessup et al.’s (2009) time constraint mechanism, in the associative accumulation model (AAM) (Bhatia, 2013). AAM is a sequential sampling and accumulation theory of preference, which assumes that the probability of sampling a particular attribute depends on the composition of the choice set. AAM is able to explain conflict and alignability effects, effects relating to extreme options, and predict the emergence of the attraction and compromise effects. For this reason, AAM is a suitable framework with which to study the many diverse findings regarding deferred choice. Using the choice options and parameter values assumed in Bhatia (2013) we show that AAM, when combined with a deferral based time constraint, provides a comprehensive explanation for all of the deferral effects described above.

Associative Accumulation Model

In multi-attribute choice tasks, decision makers are asked to choose between two or more options defined on a number of different attributes. An example of such a task is a choice between two cars, each of which differs on attributes such as mileage, cost, fuel efficiency and so on.

1 Jessup et al. also consider a number of other methods to model choice deferral. Many of these alternate methods are unable to predict the increase in deferral probability with the size of the choice set. Additionally Busemeyer, Johnson & Jessup (2006) present an alternate model of deferral. This model assumes that the possibility of choice deferral is processed as just another choice option, and that choice is deferred when this option is chosen. Busemeyer et al.’s (2006) model can explain deferral’s relationship with the attraction and compromise effect, but cannot, however, capture the observed increase in deferral with set size (see Jessup et al., 2009).
The associative accumulation model (Bhatia, 2013) is a sequential sampling and accumulation theory of decision making that models behavior in multi-attribute choice tasks. AAM assumes that attributes are sampled stochastically, and accumulated over time, into preferences. Decisions are made when the preference for a choice option crosses an upper acceptance threshold, or a lower rejection threshold. Unlike alternate sequential sampling theories of preferential choice (Roe et al., 2001; Krajbich et al., 2010; Trueblood et al., in press; Usher & McClelland, 2004), AAM also assumes that associative relations between the available options and the attributes at play in the decision, determine the sampling probabilities of the attributes. For example, AAM assumes that decision makers are more likely to think about attributes associated with cars when they are deliberating between cars, than when they are deliberating between other options (such as computers or clothes).

While attending to attributes associated with the available options seems to be efficient, it can lead to certain types of irrationality. Particularly, adding or removing options from the available choice set can alter attribute sampling probabilities and subsequently reverse choice. This dependence between choice, and the options that are available in the decision, allows AAM to explain a large range of findings regarding choice set dependence, such as the attraction and compromise effects, alignability and conflict effects, less is more effects, and reference point effects (see Bhatia, 2013 for more details).

We can represent an available option as a vector of \( M \) attributes, \( x_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \). AAM assumes that the associative connection between a choice option, \( i \), and an attribute, \( j \), is simply equal to the amount of the attribute in the choice alternative, \( x_{ij} \). The probability of sampling an attribute is given by the relative strength of association of the attribute with the choice set. For an attribute \( j \) in a choice set with \( N \) available options, this sampling probability can be written as:

\[
w_j = \frac{\sum_{i=1}^{N} x_{ij} + a_0}{\sum_{i=1}^{M} \left( \sum_{j=1}^{N} x_{ij} + a_0 \right)}
\]

Here \( a_0 \) is a constant that determines the strength of the associative bias. As \( a_0 \) increases, the associative bias in AAM is reduced. At \( a_0 = \infty \), each attribute is equally likely to be sampled, and decisions are choice set independent. Overall, the above equation implies that an attribute is more likely to be sampled from if many options score highly on that attribute.

Once an attribute is sampled, AAM assumes that its value in every available option is calculated and added to the preferences for the options. The value of attribute \( j \) in alternative \( i \) can be written as \( V_j(x_{ij}) \), where \( V_j \) is a positive and increasing function if the attribute is desirable. Preferences are also subject to gradual leakage, captured by parameter \( d \), lateral inhibition, captured by parameter \( l \), and a zero mean noise with standard deviation \( \sigma \), captured by parameter \( e \). If attribute \( j \) is sampled at time \( t \), then the preference for option \( i \) can be written as:

\[
U_i(t) = d \cdot U_i(t-1) - l \cdot \sum_{j=1}^{M} U_j(t-1) + V_j(x_{ij}) + e_i(t-1)
\]

Finally, upper and lower thresholds determine both the option that is chosen, and the time at which the decision is made. If an option crosses the upper threshold \( Q \) then the option is accepted, and if an option crosses the lower threshold \( R \) then the option is eliminated from the decision. In choices where deferral is not allowed, the decision terminates once some option has crossed \( Q \) or all but one option have crossed \( R \).

What happens when the decision maker is allowed to defer choice? As in Jessup et al. (2009) we assume that choice is deferred if a time constraint \( T \) is crossed without the decision having been made at an earlier time period. Hence in choices with the possibility of deferral, some option is chosen if it crosses \( Q \) before \( T \), or if all other options cross \( R \) before \( T \), and choice is deferred if either of these two events do not happen before \( T \).

According to this framework, the probability of deferring choice can be changed by altering the speed at which preferences accumulate upwards (towards \( Q \)) or downwards (towards \( R \)) in the decision task, with faster upward or downward accumulation leading to reduced deferral. Additionally, allowing for the possibility of choice deferral can disproportionally alter the choice probabilities of available options, if the preferences for these options increase at different rates. The next two sections will show how the time dynamics of preference accumulation in AAM relate to these two determinants of deferral. Simulations will use model parameters specified in Bhatia (2013). Particularly, we will set \( d = 0.8 \), \( \sigma = 0.05 \), \( a_0 = 10 \), \( U_i(0) = 0 \) for all \( i \), \( V_j(x_{ij}) = x_{ij}^{0.5} \) for all \( j \). Additionally, we will assume that \( Q = 10 \), \( R = -10 \) and \( l = 0.01 \) (for simplicity, the simulations in Bhatia (2013) did not involve decision thresholds or inhibition, but these variables are necessary to study deferral). Finally, we will set the deferral time constraint \( T = 10 \), to be used when deferral is available in the choice task. Each simulation will be repeated 10,000 times, and displayed responses will be averaged over these trials. Choice options used in Bhatia (2013) will be the basis of these simulations. These will be described in the coming sections.

**Determinants of Choice Deferral**

**Conflict**

Choice options are considered to be alignable if they overlap significantly on most of their attributes. Conflicting options, in contrast, are those that do not overlap on their attributes. Tversky and Shafir (1992) have found that increasing the conflict (i.e. reducing the alignability) between the options in a choice set increases the probability of choice deferral. Particularly, with the choice options represented in Figure 1, Tversky and Shafir found that the
probability of deferral in the set \( \{x_1, x_3\} \), is lower than in the set \( \{x_1, x_4\} \) (see also Figure 1 in Tversky & Shafir, 1992).

AAM equipped with a deferral time constraint can explain this effect. Recall that AAM assumes that attribute attention is proportional to the association of the attributes with the available options. Attributes that are present in multiple options are also associated with multiple options, and thus receive a higher attentional weight (see Bhatia, 2013 for a discussion). In choices between two low-conflict options, as in the set \( \{x_1, x_3\} \) in Figure 1, the common attribute, attribute 1, is highly likely to be sampled. This means that the preference for option \( x_1 \) – the most desirable alternative – increases and crosses a threshold quickly, and that choice is subsequently unlikely to be deferred.

When a low-conflict option is replaced with an equally desirable high-conflict option, however, there is dispersion in the sampling probabilities of the underlying attributes, as attributes associated with the novel, high-conflict option are now more likely to be sampled. Thus, in our example, if \( x_1 \) is replaced with \( x_4 \), decision makers are more likely to sample attribute 2. This reduces the rate of accumulation for all choice options (including the most desirable option, \( x_1 \)) increasing the probability that thresholds are not crossed by the deferral time constraint.

Consider, for example, the choice options used in Bhatia (2013): \( x_1 = (7, 3) \), \( x_3 = (6.5, 2.5) \) and \( x_4 = (2.5, 6.5) \). When we implement our model with the parameters listed in the previous section, we find that the sampling probability of attribute 1 in the set \( \{x_1, x_3\} \) is 0.60, whereas the sampling probability of attribute 1 in the set \( \{x_1, x_4\} \) is 0.50. Subsequently the expected increase in the preference for \( x_1 \), in each time period, in the set \( \{x_1, x_1\} \), is 2.28, whereas the equivalent increase in preference in the set \( \{x_1, x_4\} \) is 2.18. As a result of this \( x_1 \) is less likely to cross the threshold before the time constraint in the set \( \{x_1, x_3\} \) compared to the set \( \{x_1, x_1\} \), and choice is deferred 81.9% of the time in the set \( \{x_1, x_3\} \) but 91.6% of the time in the set \( \{x_1, x_4\} \).

![Figure 1. Choice options commonly used to study choice deferral.](image)

Related empirical results have been documented by Gourville and Soman (2005). Unlike Tversky and Shafir (1992), however, Gourville and Soman use options defined on more than two attributes, with each attribute being a binary variable (attribute present or not present) rather than a continuous variable. Nonetheless, the mechanism used to explain Tversky and Shafir’s (1992) results also explains Gourville and Soman’s (2005) results. In settings where available options have multiple common attributes, these attributes are especially likely to be sampled, increasing the rate of preferences accumulation for all available options, and reducing choice deferral. The opposite happens when the available options have mostly unique attributes.

As a demonstration, let us randomly generate choice options \( x_1 \) and \( x_2 \), and explore the relationship between the conflict between these options and the deferral probability from the set \( \{x_1, x_2\} \). We consider a four attribute setting in which the probability of each option having any given attribute is 0.5. This probability is independent across attributes and across alternatives. Additionally, \( x_{ij} = 10 \) specifies that option \( x_i \) contains attribute \( j \), and \( x_{ij} = 0 \) specifies that option \( x_i \) does not contain attribute \( j \) (with identical values for option \( x_{i'} \)). We randomly generate 100 such choice sets, and simulate our model on these sets 10,000 times to generate a deferral probability. Conflict is specified using the negative cosine similarity of the available options, which is equal to \(-x_i \cdot x_j/||x_i|| \cdot ||x_j||\).

Regressing simulated deferral probability on this metric of conflict (assuming a linear model, censored at 0 and 1), we find that the effect on conflict is significantly positive (\( \beta = 0.57, p < 0.01 \)), with higher conflict leading to higher deferral. This validates the insight presented in this section.

Desirability

An alternative cause of choice deferral is explored by Dhar (1997), who finds that the addition of desirable options to a choice set consisting of a single option, can lead to an increased deferral probability, independently of the conflict between the choice options. When the sets are represented as in Figure 1, this implies that the probability of deferral in the set \( \{x_1\} \) is lower than in the choice set \( \{x_1, x_2\} \).

![Figure 2. The difference in deferral probability between the set \( \{x_1, x_1\} \) and the set \( \{x_1\} \), as a function of lateral inhibition.](image)

This is explained by AAM using lateral inhibition. With inhibition, strong preferences for one option reduce the preferences for other options. When there are multiple desirable options in the choice set, this inhibitory effect is fairly strong, and the speed of preference accumulation for
all options is low. This lead to deferral as choice thresholds are unlikely to be crossed by the time constraint $T$. However, when there is only one available option, inhibition does not play any role in the decision process. Hence the speed of accumulation for the available option is high, and a threshold is more likely to be crossed before $T$, resulting in low deferral. Indeed, with the choice options used in Bhatia (2013), $x_1 = (7, 3)$ and $x_2 = (3, 7)$, we find that choice is deferred 85.3% of the time in the set $\{x_1, x_2\}$, but only 75.4% of the time in the set $\{x_1\}$.

To further explore the relationship of lateral inhibition with desirability and choice deferral, consider Figure 2. Figure 2 presents the difference in deferral probability between the set $\{x_1, x_2\}$ and the set $\{x_1\}$, as a function of lateral inhibition. We set $x_1 = (7, 3)$ and $x_2 = (3, 7)$, and inhibition is varied in increments of 0.001 from 0 to 0.05. The desirability deferral effect emerges for positive values of the deferral probability difference. Observe that the probability of deferral is increasing in lateral inhibition, and the desirability effect is obtained for all $l > 0.005$. For $l < 0.005$, the addition of $x_2$ to the choice set reduces deferral probability. This happens because two noisy accumulators are more likely to randomly cross a threshold than one noisy accumulator, when inhibition is too weak to significantly reduce their rate of accumulation.

Consequences of Choice Deferral

Extreme options

The above section discussed how the probability of deferral is affected by the composition of the choice set. Here we study how allowing for the possibility of deferral can alter the choice probabilities of the options in the choice set. This has been empirically examined by Dhar and Simonson (2003) who found that allowing for deferral disproportionally reduces the choice probability of an all-average option, compared to an extreme option. When choices are represented as in Figure 1, this implies that the probability of choosing the all-average option $x_6$ over the extreme option $x_5$ is lower when decision makers are allowed to defer choice, relative to when deferral is not a possibility.

AAM captures this effect using both the associative relationship between the choice options and the attributes, and the stochastic sequential sampling of attributes. Attention towards an attribute is proportional to its association with the choice set. Subsequently, the extreme option’s primary attribute (attribute 1 in Figure 1), which is also associated with the all-average option, is more likely to be sampled compared to the second attribute (attribute 2 in Figure 1), which is associated with the all-average option but not with the extreme option. This creates a bias in favor of the extreme option (see Bhatia, 2013, for an extensive discussion of this bias). Now, stochastic sequential sampling introduces time dependence in the accumulation of preference. At earlier time periods, when few attributes have been sampled, preferences are more sensitive to the attributes that are sampled, and the attentional bias favoring the extreme option is particularly strong. Hence the extreme option is highly likely to be chosen early on in the decision process. As time progresses preferences asymptote towards the total weighted value of their attributes. Due to concave valuation functions, all-average options have higher total attribute values, and are subsequently more likely to be selected later on in the decision process.

Now, allowing for deferral creates a bias in favor of the options that are highly preferred early on in the choice process. This happens as these options are likely to cross the choice thresholds prior to the deferral time constraint. Choice options that only increase in preference as time progresses, on the other hand, are unlikely to be selected in the presence of the deferral. As a result of this, we observe a lower choice probability for all-average options in the presence of deferral, compared to when deferral is not allowed. Setting $x_5 = (5, 5)$ and $x_6 = (10, 0)$, as in Bhatia (2013), we find that $x_5$ is selected 37.1% of the time and $x_6$ is selected 62.9% of the time in the absence of deferral. In contrast, $x_6$ is selected 100% of the time whenever a choice is made, in the presence of deferral.

![Figure 3. Relative choice probability of choosing all-average option ($x_6$) and probability of deferral, as a function of deferral time constraint ($T$).](image)

To further explore the relationship between deferral and decision time, consider Figure 3. The vertical axis in Figure 3 presents the relative choice proportion of $x_6 = (5, 5)$ compared $x_5 = (10, 0)$, $P[x_6] / (P[x_6] + P[x_5])$, where $P[x_i]$ is the proportion of times that $x_i$ is chosen in the simulations. The vertical axis also presents the probability of deferral, which is $1 - (P[x_5] + P[x_6])$. The horizontal axis represents the deferral time constraint $T$. As $T$ is increased, decision makers have more time to make their choice, and the relative choice proportion of $x_5$ increases. After $T = 15$, we find that this proportion stabilizes at around 0.4, and the probability of deferral similarly stabilizes at 0 (and choice probabilities with deferral allowed are the same as choice probabilities in the absence of deferral). Note that choice proportions for $T < 5$ are not displayed, as neither of the options are chosen for these values of $T$, choice is deferred 100% of the time, and the relative choice proportion is not defined (this is also the case for $T < 8$ in Figures 4 and 5).

Context Effects
Another effect of allowing deferral relates to the attraction and compromise effect. The attraction effect is the finding that the relative choice probability of an option increases with the introduction of a novel option (a decoy), that is, but not its competitor, dominates (i.e., is better than on all attributes) (Huber, Payne & Puto, 1982). Similarly, the compromise effect refers to the finding that the relative choice probability of an option increases with the addition of a decoy that makes the option appear as a compromise (Simonson, 1989). In Figure 1, the attraction effect is described by the higher choice probability of $x_1$ relative to $x_2$ from the set $\{x_1, x_2, x_3\}$ compared to the set $\{x_1, x_2\}$, whereas the compromise effect is described by the higher choice probability of $x_1$ relative to $x_2$ from the set $\{x_1, x_2, x_6\}$ compared to the set $\{x_1, x_2\}$.

Dhar and Simonson (2003) found that allowing for the possibility of deferral increases the attraction effect but reduces the compromise effect. Particularly, the relative choice probability of $x_1$ vs. $x_2$ from the set $\{x_1, x_2, x_3\}$ minus the same probability from the set $\{x_1, x_2\}$ is higher in the presence of deferral, whereas the relative choice probability of $x_1$ vs. $x_2$ from the set $\{x_1, x_2, x_6\}$ minus the same probability from the set $\{x_1, x_2\}$ is lower in the presence of deferral.

With the compromise effect, however, it is the extreme novel option that is strongest on the most sampled attribute in the presence of the decoy (attribute 1). This extreme option disproportionality competes with the compromise option, reducing its choice probability. As in with the all-average and extreme options in the above section, this competitive effect happens only at early time periods. As a result of this, the compromise effect is weakened at early time periods, and is thus less likely to emerge when deferral is a possibility.

Indeed, taking the choice options used in Bhatia (2013), $x_1 = (7, 3), x_2 = (3, 7), x_3 = (6.5, 2.5)$ and $x_6 = (10, 0)$, we find that the relative choice probability of $x_1$ over $x_2$, $P[x_1]/(P[x_1] + P[x_2])$, from the set $\{x_1, x_2, x_3\}$, is 0.71 in the presence of deferral, but only 0.59 in the absence of deferral. This shows that the attraction effect increases in the presence of deferral. In contrast the relative choice probability of $x_1$ over $x_2$, $P[x_1]/(P[x_1] + P[x_2])$ from the set $\{x_1, x_2, x_6\}$, is 0 in the presence of deferral, but 0.6 in the absence of deferral. Thus the compromise effect decreases (and in fact vanishes) in the presence of deferral. Note the choice shares of $x_1$ and $x_2$ from the set $\{x_1, x_2\}$, are equal both with and without deferral, as these options are identical on symmetric attributes.

![Figure 4. Relative choice probability x1 and probability of deferral, as a function of deferral time constraint (T), for attraction effect.](image1)

![Figure 5. Relative choice probability x1 and probability of deferral, as a function of deferral time constraint (T), for compromise effect.](image2)

Figures 4 and 5 describe this finding in more detail. These figures model the effect of the deferral time constraint on $P[x_1]/(P[x_1] + P[x_2])$ in the presence of the decoy, for the attraction and compromise effects respectively. As the choice probability of $x_1$ in the absence of the decoy is 0.5 (the two core options are symmetric on identical attributes), values on the vertical axis that are higher than 0.5 correspond to decoy effects.

Note that we find that the strength of the attraction decoy effect decreases and the strength of the compromise decoy effect increases with increase in the deferral time constraint. While the attraction effect is always predicted to emerge (regardless of the value of this constraint), the compromise effect actually reverses when the time constraint is especially low. Ultimately both the attraction and compromise effects emerge for a large enough deferral.
time constraint. For these values, the probability of deferral is zero and corresponding choice probabilities displayed in Figures 4 and 5 are equal to choice probabilities in the absence of deferral.

Discussion and Conclusion

Sequential sampling and accumulation theories of preferential choice assume that attributes are attended to at random, and accumulated over time, into preferences. The associative accumulation model adds to this approach by assuming that attribute attention is a function of the association of the attribute with the available choice options. In this paper, we extended AAM by assuming that choice is deferred if a threshold is not crossed by a prefixed time constraint, a mechanism initially proposed by Jessup et al. (2009). We show that this mechanism provides a compressive explanation for a large range of behavioral findings regarding the causes and consequences of choice deferral.

The proposed mechanism is able to explain these findings, due to the relationship between deferral and decision time. Deferral, for example, depends on the speed of preference accumulation. Increasing the conflict in a choice set or adding options that are highly desirable to a choice set reduces the speed of preference accumulation, and thus increases the probability of deferral. Likewise, the possibility of deferral creates a bias in favor of choice options that are preferable at the start of the decision process. Allowing for deferral can systematically increase the choice probabilities of these options, generating a higher choice probability for extreme options compared to all-average options, an increased incidence of the attraction effect, and a decreased incidence of the compromise effect.

The proposed model also makes a number of novel predictions. For example, it predicts that deliberation times should be lower in the presence of deferral than in the absence of deferral, and that the time taken to defer choice should be higher than the time taken to choose one of the available options, in the presence of deferral. Additional, novel assumptions can be derived by examining the relationship between the various effects and decision times.

Previous work has attempted to explain deferral using conflict, preference uncertainty, and increased difficulty, but it is not clear how these mechanisms can explain all of the effects discussed in this paper (much less the entire range of behavioral effects explained by AAM and related theories). The time constraint mechanism, in contrast, is a parsimonious and intuitive, but analytically sound way to model deferral. It draws upon the strengths of sequential sampling models, which are particularly useful for studying the time dependencies in the decision process. Future work should try to incorporate other findings in preferential choice within this framework, so as to generate a more comprehensive cognitive theory with which decision making can be studied.

References


