Technology Shocks, Statistical Models, and The Great Moderation*

Cristina Fuentes-Albero†

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†fuentesa@sas.upenn.edu. Department of Economics, University of Pennsylvania, 3718 Locust Walk, 160 McNeil, Philadelphia, PA, 19104.

Abstract

In this paper we compare the cyclical features implied by an RBC model with two technology shocks under several statistical specifications for the stochastic processes governing technological change. We conclude that while a trend-stationary model accounts better for the observed volatilities, a difference-stationary model does a relatively better job of accounting for correlations. We analyze the relative importance of the two technology shocks and conclude that failing to include an investment specific shock will worsen more the performance of a level stationary model economy. We also explore some counterfactuals to assess the ability of our model to replicate the volatility slowdown of the mid 1980s. First, we conclude that the stochastic trend model outperforms the deterministic trend model in accounting for the Great Moderation. Finally, we obtain that even though the neutral technology shock is the main driving force in the volatility slowdown, allowing for a larger financial flexibility in the form of a smaller volatility for the investment-specific innovation improves the ability of our model to account for the magnitude of the Great Moderation.

Keywords: Business Cycle, Aggregate fluctuations, Technology Shocks, Unit Roots

JEL Classification: E32, O30, O41, C32

1 Introduction

Technology driven business cycles have been in the core of the Real Business Cycle literature from its origins. For example Prescott (1986) claims that technology shocks account for more than a half of the US business cycle fluctuations over the postwar period. In Cooley and Prescott (1995), technology shocks account for more than 75% of the volatility of output. Such an empirical success has been questioned by Galí (1999) and Galí and Rabanal (2004) among others. They claim that business cycle features are due mainly to non-technology factors. However, Greenwood, Hercowitz, and Krusell (1997) started a new wave of attention on technology-driven business cycles by allowing for not only a neutral technology shock, but also an investment-specific one.

In this paper we want to address the slowdown in volatility of macroeconomic variables in the US economy using a simple model inspired by Greenwood, Hercowitz, and Krusell (2000). We want to determine whether the slowdown in the volatility of the two shocks under analysis suffices to explain a significant part of the so called Great Moderation. Arias, Hansen, and Ohanian (2007) consider a basic RBC model à la Hansen (1985) with only one technology shock. They conclude that the slowdown in the volatility of productivity shocks can account for about a 50% decline in business cycle volatility. They also analyze a model based on Burnside and Eichenbaum (1996) with endogenous movements in TFP due to labor hoarding and capital utilization. They explore the explanatory power of different shocks and conclude that the most promising candidate for understanding the slowdown in volatility is a productivity-like shock.

We are interested in exploring the performance of our simple RBC model under three specifications for the technology processes. We will consider a general specification allowing for persistence but without imposing unit roots. Therefore, such a model will be trend stationary. Then, we will analyze two versions of a difference-stationary model. We want to determine which specification accounts better for the US business cycle features in the flavor of the analysis by Hansen (1997). He explores the specifications presented here in an economy with only one technology shock. He concludes that when technological progress is difference-stationary, the RBC model does a poor job accounting for features of observed business cycles.

We think it is challenging to analyze the explanatory power of those statistical models when the Great Moderation is at hand. In fact, we have found that while the deterministic trend model accounts better for observed volatilities, the stochastic trend models are preferable if we want to match correlations or address the slowdown in aggregate volatility observed in the mid 1980s.

Since Kim and Nelson (1999) and McConnell and Pérez-Quirós (2000) dated the starting of the Great Moderation[1], there has been a growing literature on explaining what is behind such a phenomenon. Kahn, McConnell, and Pérez-Quirós (2002) claim that the change in inventory behavior due to improvement in information technology can explain the output volatility slowdown. Stock and Watson (2002), Kim, Morley, and Piger (2004), Ahmed, Levin, and Wilson (2004), Leduc and Sill (2006), and Arias, Hansen, and Ohanian (2007) use different approaches to conclude that the Great Moderation can be explained by 'good luck' in the form of smaller shocks. Dynan, Elmendorf, and Sichel (2005), Campbell and Hercowitz (2005), Dynan, Elmendorf, and Sichel (2006), Guerron (2006), Jermann and Quadrini (2006), and Justiniano and Primiceri (2006) claim that financial innovations are one of the possible contributing sources to the macro stability observed since mid 1980s.

Our results suggest that 'good luck' in the form of smaller innovations to the technology processes can account for the bulk of the volatility slowdown in our model. Moreover, we find that while the neutral technology shock plays the main role in explaining the reduction in macro volatility, its performance improves when

[1]Stock and Watson (2002) came up with such an expression to refer to the slowdown in the volatility of macro variables in the US observed in the mid 1980s

the investment-technology shock is also at hand. Justiniano and Primiceri (2006) suggest to interpret investment-specific disturbances as proxy for investment financial frictions. Therefore, in our model economy, the Great Moderation is due not only to smaller shocks but also to lessened financial frictions.

The paper proceeds as follows. In section 2 we set up our baseline model. In section 3 we proceed with our calibration exercises. We will study the three statistical models under analysis. Section 4 presents several counterfactuals in order to analyze the Great Moderation in the framework defined by our model economy. Section 5 concludes.

2 The model

The model is a simplified version of the one proposed by Greenwood, Hercowitz, and Krusell (2000). In particular, we will abstract from different capital goods and degrees of capital utilization. We will preserve, however, the existence of both a neutral and an investment-specific technology shocks.

We will consider three statistical versions of the baseline model in order to assess which one accounts better for the US business cycle features. First, we will analyze a deterministic trend version of the model where the stochastic processes are trend stationary. Second, we will consider a stochastic trend model where the technology processes follow a unit root with drift. Finally, we will allow for some persistence to the innovation of the investment specific technology in a stochastic trend model. Therefore, in the first case we will study an economy where all shocks are temporary. In the second model, all shocks are permanent. In the last model, we will consider both permanent and transitory shocks. In particular, any neutral shock will be permanent, while any investment-specific shock will have both permanent and transitory effects.

Hansen (1997) performed a similar analysis to the one we propose here but considering a model with only a neutral technology shock. He concluded that the trend-stationary (but highly persistent) model does a better job accounting for the business cycle features than the difference-stationary one.

Since Nelson and Plosser (1982) there has been a large empirical literature about stochastic trends in macro variables. Unit roots and stationary processes differ in their implications at infinite time horizons, but for any given finite sample, there is a representative from either class of models that can account for all the observed features of the data[2]. In addition, the lack of power of univariate classical tests for unit roots is well known. Therefore, we have decided to choose among the three specifications described above using the following criterion: the most preferred statistical model will be the one able to account for a larger proportion of the US business cycle properties. Note that we will be performing this test not only over the whole sample, but also over the two subsamples of interest[3].

In this economy there is a continuum of households that maximize their lifetime utility given by

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U(C_t, H_t)\right] \tag{1}$$

with

$$U(C_t, H_t) = lnC_t - B\frac{H_t^{1+1/\nu}}{1+1/\nu} \tag{2}$$

where C_t stands for consumption, H_t for hours worked, ν for the short-run (Frisch) labor supply elasticity, and B is a preference weight.

We have chosen such a specification because we are not interested in exploring

[2] For a more detailed discussion on nonstationary time series see Hamilton (1994)

[3] We have performed ADF (Augmented Dickey-Fuller) tests on all of the variables of interest. We have run the test including a constant ie we were testing whether the series under analysis follow a random walk with drift. We were not able to reject the null of unit root for all the variables but (log) hours and (log) labor productivity. It is remarkable that for the neutral technology we reject the null at 5% but not at 1% for the whole sample and the first subsample. We cannot reject the hypothesis of stochastic trend for the second subsample.

the stationarity of hours issue[4]. Therefore, we will use a series for hours that is stationary in levels. It is well known that the log utility in consumption implies a constant long-run labor supply in response to a permanent change in technology. Hence, we do not have to worry about trending hours implied by our model even under the difference-stationary specification.

The representative household supplies labor at the competitive equilibrium wage W_t and rents capital K_t to the firms at rental rate R_t. The capital stock depreciates at rate δ. Therefore the representative household maximizes (1) subject to the following

$$C_t + P_t^k X_t = W_t H_t + R_t P_t^k K_t \quad (3)$$

$$(1+\eta) K_{t+1} = (1-\delta) K_t + X_t \quad (4)$$

where P_t^k is the (relative) price of investment (using the consumption good as a numeraire) and X_t stands for quality-adjusted investment. Note that while the budget constraint is expressed in consumption units, the capital accumulation equation is expressed in efficiency units. Population in this economy grows at rate $(1+\eta)$.

There is also a continuum of firms that rent capital and labor services from households and produce consumption and investment goods. The representative firm solves the following problem[5]:

$$\max \quad \Pi_t = C_t + P_t^k X_t - W_t H_t - R_t P_t^k K_t \quad (5)$$

$$\text{s.t.} \quad C_t + \frac{X_t}{V_t} = A_t K_t^\alpha H_t^{1-\alpha} \quad (6)$$

where A_t is the current level of (neutral) technology and V_t stands for the current

[4]See Chang, Doh, and Schorfheide (2007) for an interesting treatment of such an issue and Christiano, Eichenbaum, and Vigfusson (2003) for an analysis of the implications of different labor input measures in a SVAR framework.

[5]When we proceed with calibration we will introduce an additional parameter μ in the production function so that output at steady state is equal to 1

level of the investment-specific technology[6]. Firms will produce both consumption and investment goods only if $V_t = \frac{1}{P_t^k}$. Note that $I_t = P_t^k X_t$, therefore (6) is identical to the familiar resource constraint[7]

$$Y_t = C_t + I_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

Let us consider three statistical specifications for the stochastic processes governing the technology levels in this economy. In the deterministic trend model the technology processes are modeled as follows:

$$A_t = A_0 e^{\gamma_a t + \varepsilon_{at}}$$
$$V_t = V_0 e^{\gamma_v t + \varepsilon_{vt}}$$

where ε_{at} and ε_{at} are autoregressive processes. The explicit structure of the errors will be stated in section 3.

In the stochastic trend version of the model, the processes are given by

$$A_t = A_{t-1} e^{\gamma_a + \varepsilon_{at}}$$
$$V_t = V_{t-1} e^{\gamma_v + \varepsilon_{vt}}$$

ie the log technologies evolve according to a random walk with drift. In the baseline stochastic trend model, the errors are assumed to be white noise. In the stochastic trend model with persistence, the log of investment-specific technology level is assumed to follow a random walk with drift and moving average component.

Under all the specifications our model economy exhibits long-run growth. Therefore, we will transform our economy so that we can work with a detrended version of

[6]Note that a higher V implies a fall in the cost of producing a new unit of capital in terms of output. It could also be interpreted as an improvement in the quality of new capital produced with a given amount of resources.

[7]X_t refers to investment in efficiency units and I_t to investment in consumption units.

the original one. In our trend stationary model economy, the following variables are stationary[8]

$$\frac{Y_t}{q^t}, \quad \frac{C_t}{q^t}, \quad \frac{I_t}{q^t}, \quad \frac{W_t}{q^t}, \quad \frac{K_t}{(qv)^t}, \quad H_t, \quad R_t$$

where

$$q = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v}$$

and

$$v = e^{\gamma_v}$$

Let us denote a stationary variable Z by $\tilde{Z}$. Therefore, the stationary equilibrium conditions for this statistical version of the model are given by:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t \tag{7}$$

$$\tilde{Y}_t = A_0 e^{\varepsilon_{at}} \tilde{K}_t^{\alpha} H_t^{1-\alpha} \tag{8}$$

$$(1+\eta)qv\tilde{K}_{t+1} = (1-\delta)\tilde{K}_t + V_0 e^{\varepsilon_{vt}} \tilde{I}_t \tag{9}$$

$$1 = \beta E_t \left[\left(\frac{e^{\varepsilon_{vt} - \varepsilon_{vt+1}}}{qv} \right) \left(\frac{\tilde{C}_t}{\tilde{C}_{t+1}} \right) (1 - \delta + R_{t+1}) \right] \tag{10}$$

$$H_t = \left(\frac{1}{B} \frac{\tilde{W}_t}{\tilde{C}_t} \right)^{\nu} \tag{11}$$

$$R_t = \alpha V_0 e^{\varepsilon_{vt}} \frac{\tilde{Y}_t}{\tilde{K}_t} \tag{12}$$

$$\tilde{W}_t = (1-\alpha) \frac{\tilde{Y}_t}{H_t} \tag{13}$$

Given the detrended version of our economy we can solve for the steady state. Let us denote the steady state value of a variable Z by Z^*.

$$Y^* = C^* + I^* \tag{14}$$

$$Y^* = A_0 K^{*\alpha} H^{*(1-\alpha)} \tag{15}$$

$$(1+\eta)qvK^* = (1-\delta)K^* + V_0 I^* \tag{16}$$

[8]See the appendix for a detailed explanation on obtaining the growth rates for the economies under analysis

$$1 = \beta\left(\frac{1}{qv}\right)(1-\delta+R^*) \tag{17}$$

$$H^* = \left(\frac{1}{B}\frac{W^*}{C^*}\right)^{\nu} \tag{18}$$

$$R^* = \alpha V_0 \frac{Y^*}{K^*} \tag{19}$$

$$W^* = (1-\alpha)\frac{Y^*}{H^*} \tag{20}$$

Let us consider now the two difference-stationary models. Beveridge and Nelson (1981) showed in a model with only one shock that any of the trending variables of these kinds of models can be decomposed into a permanent component that is a random walk with drift (a stochastic trend) and a stationary stochastic process. In our case we have to take into account that the two stochastic processes have a unit root[9]. Hence, given such a statistical model, we have that the following variables are stationary

$$\frac{C_t}{Q_t}, \quad \frac{I_t}{Q_t}, \quad \frac{Y_t}{Q_t}, \quad H_t, \quad R_t, \quad \frac{K_{t+1}}{Q_t V_t}, \quad \frac{W_t}{Q_t}$$

where $Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$.

The stationary equilibrium conditions are:

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t \tag{21}$$

$$\tilde{Y}_t = \left(\frac{1}{q_t v_t}\right)^{\alpha} \tilde{K}_t^{\alpha} H_t^{1-\alpha} \tag{22}$$

$$(1+\eta)\tilde{K}_{t+1} = (1-\delta)\left(\frac{1}{q_t v_t}\right)\tilde{K}_t + \tilde{I}_t \tag{23}$$

$$1 = \beta E_t\left[\left(\frac{1}{q_{t+1}v_{t+1}}\right)\left(\frac{\tilde{C}_t}{\tilde{C}_{t+1}}\right)(1-\delta+R_{t+1})\right] \tag{24}$$

$$H_t = \left(\frac{1}{B}\frac{\tilde{W}_t}{\tilde{C}_t}\right)^{\nu} \tag{25}$$

$$R_t = \alpha(q_t v_t)\frac{\tilde{Y}_t}{\tilde{K}_t} \tag{26}$$

[9] For detrending issues there is no difference between having just a random walk with drift or a random walk with drift plus a moving average component.

$$\tilde{W}_t = (1-\alpha)\frac{\tilde{Y}_t}{H_t} \tag{27}$$

where

$$q_t = \frac{Q_t}{Q_{t-1}} = e^{\frac{1}{1-\alpha}(\gamma_a+\varepsilon_{at})+\frac{\alpha}{1-\alpha}(\gamma_v+\varepsilon_{vt})} \tag{28}$$

$$v_t = \frac{V_t}{V_{t-1}} = e^{\gamma_v+\varepsilon_{vt}} \tag{29}$$

Given that the stationary version of the difference-stationary model satisfies the usual assumptions, we can solve for the steady-state of this transformed economy. Then,

$$Y^* = C^* + I^* \tag{30}$$

$$Y^* = \left(\frac{1}{q^*v^*}\right)(K^*)^{\alpha}(H^*)^{1-\alpha} \tag{31}$$

$$(1+\eta)K^* = (1-\delta)\left(\frac{1}{q^*v^*}\right)K^* + I^* \tag{32}$$

$$1 = \beta\left(\frac{1}{q^*v^*}\right)(1-\delta+R^*) \tag{33}$$

$$H^* = \left(\frac{1}{B}\frac{W^*}{C^*}\right)^{\nu} \tag{34}$$

$$R^* = \alpha q^* v^* \frac{Y^*}{K^*} \tag{35}$$

$$W^* = (1-\alpha)\frac{Y^*}{H^*} \tag{36}$$

where

$$q^* = e^{\frac{1}{1-\alpha}\gamma_a+\frac{\alpha}{1-\alpha}\gamma_v}$$

$$v^* = e^{\gamma_v}$$

3 Calibration

3.1 Data Set

We use the data set constructed by Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko, and Santaeulàlia-Llopis (2007). They use data from NIPA-BEA, FAT-BEA, BLS, and Cummins and Violante (2002) to construct quarterly series of investment-specific technological change and neutral technological change. Basically, they construct a series for the relative price of investment (in terms of the consumption good) that spans from 1948.I to 2006.IV and then proceed with a growth accounting exercise to recover the neutral technological change series. For a detailed explanation please see Ríos-Rull et al. (2007).

While the investment-specific process is approximated by the inverse of the (relative) price of investment, the neutral technology process is associated with the Solow residual of the economy.

In the literature we find different ways of computing the quarterly Solow residual. Cooley and Prescott (1995) claim that as the BEA produces only annual estimates for the capital stock, any quarterly series will introduce additional noise in the measure of the Solow residual. Therefore, they propose a 'conservative' approach by omitting capital when computing the neutral technology process. This approach has been widely used in the literature, for a recent example see Arias, Hansen, and Ohanian (2007). Gomme and Rupert (2007) establish that another justification for omitting capital could be measurement errors. However, mismeasurement affects the level of the capital stock but not its time series properties. Thus, other approaches construct quarterly capital series by iterating on the law of motion for capital. Note that as Greenwood, Hercowitz, and Krusell (1997) point out, we have to be careful when constructing our capital stock series since it must be in efficiency units. In the data base, capital stock series is constructed recursively using the perpetual inventory method

$$K_{t+1} = (1-\delta)K_t + X_t$$

where X_t is the total nominal investment deflated by the quality-adjusted price of investment ie it stands for investment in efficiency units. δ is the average depreciation rate of the time-varying physical depreciation rates for total capital available from Cummins and Violante (2002). The initial capital stock in efficiency units is calibrated using the steady-state investment equation.

3.2 Deterministic Trend Model

We will consider the following statistical specification:

$$\begin{aligned} lnA_t &= lnA_0 + \gamma_a t + \varepsilon_{at} \\ lnV_t &= lnV_0 + \gamma_v t + \varepsilon_{vt} \end{aligned}$$

The econometric strategy is as follows:

1. Regress each technological change series on a constant and a linear time trend

$$lnA_t = \varphi_a + \gamma_a t + \varepsilon_{at} \quad (37)$$

$$lnV_t = \varphi_v + \gamma_v t + \varepsilon_{vt} \quad (38)$$

2. Generate the corresponding residual series $\{\hat{\varepsilon}_{at}\}$ and $\{\hat{\varepsilon}_{vt}\}$.

3. Estimate univariate autoregressive processes for those shocks

$$\varepsilon_{at} = \rho_a \varepsilon_{at-1} + \xi_{at} \quad (39)$$

$$\varepsilon_{vt} = \rho_{v1}\varepsilon_{vt-1} + \rho_{v2}\varepsilon_{vt-2} + \xi_{vt} \quad (40)$$

   where $\xi_a \sim \mathcal{N}(0, \sigma^2_{\xi_a})$ and $\xi_v \sim \mathcal{N}(0, \sigma^2_{\xi_v})$. The lag structure for the errors has been chosen following the Akaike Information and the Bayesian Information Criteria.

The estimated parameters are reported in *table 7*. We observe that in the post-1984 period there has been a 48% reduction in the volatility of the innovation to the neutral technology and a 40% reduction in the volatility of the innovation to the investment-specific technology. We will analyze in section 4 if such a reduction in innovations' volatilities suffices to explain the slowdown in the volatility of the macro variables of interest.

In our model the vector of parameters is given by

$$(\alpha, \gamma_a, \gamma_v, \beta, \delta, B, \nu, \eta, \mu, \varphi_a, \varphi_v, \rho_a, \rho_{v1}, \rho_{v2}, \sigma_{\xi_a}, \sigma_{\xi_v})$$

where μ is a scaling parameter. We can estimate $(\alpha, \gamma_a, \gamma_v, \eta, \varphi_a, \varphi_v, \rho_a, \rho_{v1}, \rho_{v2}, \sigma_{\xi_a}, \sigma_{\xi_v})$ from the data. In order to calibrate the remaining parameters we will consider the targets specified in *table 6*.

Given our specification we cannot calibrate both ν and B. In fact, our calibrated B will be conditional on the choice for the Frisch elasticity parameter. In the literature we find values for such a parameter in a wide range. To keep the analysis simple, we will simulate our model considering a grid for the labor supply elasticity. In particular, $\nu = \{0.5, 1, 1.5, 2\}$. The calibrated parameters are reported in *table 7*.

The ability of our model to account for the US business cycle features is sensitive to the value of the parameter governing the Frisch elasticity of labor supply. *Tables 10, 11, 12, and 13* in appendix A report our results for the grid over ν.

The deterministic trend model, however, is able to account for some relevant features of US business cycles irrespective of our choice for ν. In particular, the model accounts for the large fluctuations of investment compared to output and for the small fluctuations of capital and consumption compared to output.

The volatilities of investment (in efficiency units), output, capital (in efficiency units), and hours are increasing with the short-run elasticity of labor supply[10]. The

[10]See Appendix E for the results when $\nu = \infty$

standard deviation of hours implied by the model is smaller than the standard deviation of labor productivity which is at odds with the data. This is, however, a typical feature of RBC models with utility non-linear in hours. Hansen (1997)'s deterministic trend model was able to account for the pattern in the data by assuming that labor is indivisible and that agents trade employment lotteries[11].

The trend stationary model generates too much volatility in consumption in the first subsample for any value of the Frisch elasticity. For $\nu = \{1, 1.5, 2\}$, the model implies a capital volatility for the pre-1984 sample that is too large.

Finally, this statistical version of our baseline RBC model cannot generate enough correlation between output and consumption. It generates, however, a large correlation between labor productivity and output that is at odds with the data. Moreover, the model cannot account for the change in sign in such a correlation in the second sub-sample.

3.3 Stochastic Trend Model

3.3.1 Random Walk with Drift

Following King, Plosser, and Rebelo (1988) when addressing the difference stationary specification, we restrict our attention to the following class of parametric forms

$$\Phi(L)(1-L)log(X_t) = \gamma_x + \Theta(L)\varepsilon_{xt}$$

[11]The results under those assumptions for our model are reported in appendix E. We conclude that if the stochastic processes are trend stationary, a model à la Hansen overstates the volatilities of investment, output, capital, and hours. In such a setting, a model economy with only an investment-specific technology shock is able to replicate the volatility of hours. Finally, we also conclude that under a difference stationary framework our model economy is still not able to generate enough volatility for all the variables at hand.

where $\Phi(L)$ and $\Theta(L)$ are lag polynomials whose roots are outside the unit circle. The statistical model to be considered in this section is as follows

$$\begin{aligned} lnA_t &= lnA_{t-1} + \gamma_a + \varepsilon_{at} \\ lnV_t &= lnV_{t-1} + \gamma_v + \varepsilon_{vt} \end{aligned}$$

which can be rewritten as

$$\begin{aligned} lnA_t &= lnA_0 + \gamma_a t + \sum_{i=0}^{t} \varepsilon_{at-i} \\ lnV_t &= lnV_0 + \gamma_v t + \sum_{i=0}^{t} \varepsilon_{vt-i} \end{aligned}$$

Note that any shock to the stochastic trend at time t has a permanent effect in the log-level of the technology processes. Therefore, we are abstracting from transitory shocks in this specification which implies that we are just analyzing a lower bound of the effects of technology shocks.

Following Fisher (2006) and Fernández-Villaverde and Rubio-Ramírez (2006) we will assume

$$\begin{pmatrix} \varepsilon_{at} \\ \varepsilon_{vt} \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{D}\right] \tag{41}$$

where $\mathbf{D}$ is a diagonal matrix i.e.

$$\mathbf{D} = \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix}$$

Our estimates are reported in *table 8*. Under this specification we also observe a reduction in the volatility of the innovations to the technology shocks of about 48%.

In this version of the baseline RBC model, our calibration targets are identical to the ones in the previous subsection. The calibrated parameters are given in table *8*.

In *tables 14, 15, 16, and 17* of appendix A, we report the results for the different values of the Frisch elasticity. The results for the volatility of output, investment, capital, and hours are also sensitive to the value of such a parameter. This statistical specification accounts for the same qualitative features of the US business cycle as the deterministic trend version.

The difference-stationary model does not overpredict the volatilities of consumption and capital. In fact, this statistical version of the model generates lower volatilities for all the variables than the trend stationary one.

In addition, the stochastic trend model is successful in accounting for the correlation of consumption and output. But it shares with the deterministic trend model the remaining unmatched features.

3.3.2 Random Walk with Drift and Moving Average Component

Following Christiano (1988) we will allow for a moving average component in the unit root specification for the investment-specific technology process. Thus, (41) will be substituted by

$$lnV_t = lnV_{t-1} + \gamma_v + \rho\varepsilon_{vt-1} + \xi_t \tag{42}$$

However, we will not modify our statistical specification for the neutral technology process since there is no empirical evidence for the inclusion of a moving average component in such a representation.

Note that (42) allows for both temporary and permanent shocks. In particular, a fraction $1/(1-\rho)$ of any innovation to the investment-specific shock will be permanent. The remainder will be temporary.

Our estimation results are reported in *table 9.* We also observe here a reduction in the volatility of the innovations to the technology shocks of about 56% for the investment-specific technology and 48% for the neutral one.

The results over the grid for the elasticity of labor supply with respect to real wage are reported in tables *18, 19, 20, and 21* in appendix A. This version of the stochastic trend model shares all the 'virtues' of the baseline stochastic trend model and improves upon some of its shortcomings. For example, the volatility of hours is larger than in the baseline difference-stationary model.

3.4 Comparing Statistical Models

From our previous analysis we can conclude that irrespective of the value for ν, all the statistical models are able to qualitatively reproduce the slowdown in volatility. While the baseline difference-stationary model implies a reduction in the volatility of the variables at hand of about 52%, the trend-stationary model overpredicts the slowdown for all the variables but output. Even though the baseline stochastic trend model outperforms the other two statistical specifications, it over predicts the slowdown in capital, hours, and labor productivity. The model implies a 48% reduction while in the data we observe about a 35% slowdown.

To continue our analysis let us set the Frisch elasticity parameter equal to 1. We have chosen only one value in the grid for expositional purposes. *Table 1* reports how much volatility each model is able to account for. We observe that the trend-stationary model performs better than the difference-stationary models for the volatility of all variables but labor productivity. Notice that the stochastic trend model with a moving average component performs relatively better than the baseline stochastic trend model in the first sub-sample under analysis.

In *table 2* we report the variance decomposition for the different specifications under analysis. It is remarkable that for the deterministic trend model the investment-specific shock is the main contributor to the variance of consumption, capital, and hours. Therefore, we conclude that if we were interested in matching volatility levels using a simple level stationary RBC model, we should include not only the usual neutral productivity shock but also an investment specific disturbance. Note that for the stochastic trend versions of our model, the neutral shock accounts for the bulk of

Table 1: $\nu = 1$: $\sigma_{model}/\sigma_{data}$

	1948:1-2006:4			1948:1-1983:4			1984:1-2006:4		
	DT	ST	ST-MA	DT	ST	ST-MA	DT	ST	ST-MA
c	0.96	0.75	0.76	1.21	0.75	0.79	0.74	0.66	0.65
x	0.66	0.53	0.48	0.82	0.46	0.48	0.60	0.50	0.44
y	0.73	0.64	0.64	0.75	0.61	0.64	0.88	0.69	0.67
k	0.92	0.51	0.49	1.09	0.46	0.51	0.61	0.41	0.36
h	0.36	0.16	0.16	0.47	0.16	0.18	0.29	0.12	0.11
y/h	0.91	0.93	0.93	0.97	0.94	0.99	0.69	0.74	0.72

the variance for all variables. Therefore, failing to include an investment shock will not worsen the results as much as it would under a deterministic trend environment.

Table 2: Variance Decomposition. Whole sample: $\nu = 1$

	DT		ST		ST-MA	
	A	V	A	V	A	V
c	26	74	86	14	84	16
x	54	46	65	35	67	33
y	91	9	99	1	96	4
k	25	75	67	33	63	37
h	35	65	68	32	60	40
y/h	91	9	99	1	97	3

Figure 1 and 2 are the impulse response functions for the deterministic trend version and the baseline stochastic trend one. The responses to a neutral innovation only differ in the steady state to which each economy converges. Short run dynamics of consumption, hours, and labor productivity in response to an investment-specific shock are richer in a level stationary environment than in a difference stationary one. That would help to explain that the deterministic trend model accounts better for macro volatilities.

Let us now analyze the performance of the statistical specifications of our RBC model in terms of accounting for correlation with output. From *table 3* we can conclude that all versions do a similar job for all the variables of interest but consumption. While the stochastic trend versions of the baseline model are able to account fairly well for the correlation between consumption and output, the deterministic trend version falls too short. All the different specifications of the RBC model under analysis perform very poorly in matching the low correlation between output and labor productivity. Moreover, none of them is able to reproduce the change in sign we observe in the post-1984 period. Hansen (1997) concluded that the deterministic trend model

Table 3: $\nu = 1$: ρ_{model}/ρ_{data}

	1948:1-2006:4			1948:1-1983:4			1984:1-2006:4		
	DT	ST	ST-MA	DT	ST	ST-MA	DT	ST	ST-MA
c	0.33	1.12	1.09	0.05	1.14	1.09	0.25	1.11	1.16
x	1	0.93	0.96	0.97	0.95	0.98	1.02	0.95	1.01
y	1	1	1	1	1	1	1	1	1
k	0.92	0.92	0.92	0.85	0.82	0.87	1.15	1.22	1.22
h	0.90	0.95	0.93	0.87	0.97	0.94	0.94	0.96	1
y/h	8.70	8.91	8.91	3.39	4.26	4.26	-1.93	-2.13	-2.15

is the best one accounting for correlations of all the variables with output. Conversely, from our results we conclude that the stochastic trend model outperforms the deterministic trend one.

Given the counterintuitive result obtained for the correlation between consumption and output for the deterministic trend model, we have explored the cross-correlations with output for five lags and leads, and the correlations of other pairs of variables.

Tables 23 to 34 report the cross-correlations with output for lags and leads. We conclude that the results for all versions of the model are similar for all variables but consumption. Not only the deterministic trend under predicts the correlation between consumption and output for the current period, but also under predicts for all lags and leads. The stochastic versions of the model, however, account for the relative magnitude and signs at all lags and leads.

Table 4 reports the correlations for different pairs of variables. As expected, none of the versions of the model can capture any of the correlations with labor productivity. For all the other moments not involving consumption, the performance of all the statistical specifications is fairly uniform. Let us give a closer look to the correlations with consumption. First of all, the deterministic trend model predicts negative correlations between consumption and investment in efficiency units and hours which are at odds with the data. The stochastic specifications account correctly for the sign of the moments of interest. Secondly, we should stress out here that while the stochastic trend model with a moving average component can account for the relative magnitude of the increase in the correlation between consumption and investment, capital, and hours across subsamples, the baseline stochastic trend model fails to do so except for capital.

Given the above, we can conclude that choosing one specification over the others depends upon what we are attempting to explain. If we were interested in matching volatilities we would choose, as Hansen (1997), the deterministic trend model. However, we would need to include in our RBC model not only a neutral productivity shock, but also an investment-specific one. If we wanted to match correlations[12], we

[12]Let us use the term correlation in a broad sense ie it refers not only to the correlation with output, but also to the cross-correlations considering lags and leads, and the correlation for any other pair of variables

Table 4: Correlation ($nu = 1$)

	Data			DT			ST			ST-MA		
	Whole	Pre	Post	Whole	Pre	Post	Whole	Pre	Post	Whole	Pre	Post
c_x	0.61	0.58	0.75	-0.21	-0.46	-0.18	0.53	0.57	0.57	0.50	0.51	0.74
c_k	0.25	0.21	0.50	0.15	0.15	0.06	0.34	0.37	0.41	0.31	0.31	0.44
c_h	0.70	0.69	0.75	-0.41	-0.61	-0.37	0.50	0.54	0.54	0.40	0.40	0.67
c_lbp	0.03	0.14	-0.40	0.69	0.68	0.61	0.96	0.96	0.96	0.94	0.94	0.97
x_k	0.21	0.24	0.09	0.28	0.24	0.31	0.24	0.23	0.24	0.24	0.23	0.24
x_h	0.81	0.81	0.81	0.98	0.98	0.98	0.99	1	1	0.98	0.97	0.99
x_lbp	0.03	0.12	-0.39	0.56	0.34	0.68	0.75	0.77	0.77	0.75	0.76	0.87
k_h	0.54	0.59	0.35	0.22	0.18	0.29	0.32	0.34	0.37	0.20	0.20	0.17
k_lbp	-0.41	-0.42	-0.35	0.33	0.35	0.30	0.32	0.34	0.37	0.32	0.31	0.38
h_lbp	-0.40	-0.28	-0.80	0.38	0.16	0.51	0.73	0.75	0.75	0.67	0.68	0.83

would choose a stochastic trend model. Finally, if we wanted to match the magnitude of the volatility slowdown in the 1980s, we would also choose a stochastic trend model.

4 The Great Moderation

So far we have performed our analysis allowing for changes in all the structural parameters over the two subsamples of interest. In such a way we have shown that any of the statistical versions of our RBC model is able to account for a slowdown in macro volatilities. However, we are more interested in analyzing that part of the performance of our model due only to 'good luck'.

Thus, to better assess the relative importance of each technology shock in explaining the Great Moderation, we will perform some counterfactuals in the spirit of the ones performed by Arias, Hansen, and Ohanian (2007). In particular, we will proceed with three experiments in two scenarios. First, we will calibrate the parameters of the model to match the targets for the whole sample (i.e., we will fix them equal to the first column of tables 7, 8, and 9). Second, we will allow for time variation in the coefficients of the laws of motion for the technology processes.

In the **first counterfactual** we will analyze the explicative power of the neutral technology shock. To do so, we will set the volatility of the innovation to the investment-specific technology to match its volatility for the entire sample. The standard deviation of the neutral innovation will, however, change across subsamples. The **second counterfactual** is analogous to the first one but we focus on the investment-specific technology shock. Finally, in the **third counterfactual** we explore the explicative power of both shocks jointly by letting their standard deviations vary across subsamples.

The results under time invariant coefficients are reported in *tables 35, 36, and 37*. For the first experiment, we observe that while the stochastic trend models can reproduce a large fraction of the slowdown observed in the data, the trend-stationary model does only an acceptable job of accounting for the slowdown in output and labor productivity volatilities. Our main conclusion from this experiment is that smaller neutral technology innovations suffice to explain a large proportion of the aggregate stability observed in the mid 1980s if the model economy is difference stationary.

Table 36 presents the results of the second counterfactual. We conclude that the role of the investment-specific shock as a single actor is greatly reduced. For example, for the deterministic trend case we have that although the investment-specific shock is 62% as volatile in the second subperiod as the first, this has a very small effect on the volatility of output, investment, and labor productivity. However, we observe a reduction of about 22% in the volatility of consumption, capital, and hours. The relevance of the investment shock to explain the slowdown in real variables in our difference stationary economy is almost negligible.

The results for the third experiment are reported in *table 37.* Under this scenario we can quantify the relative importance of the interaction between the two shocks active in our model economy. Here all the models are able to imply volatility slowdowns relatively similar to the ones in the data.

Let us now perform the same counterfactuals but allowing for time variation not only in the volatilities of the innovation processes, but also in the laws of motion of the technology processes. Results are reported in *tables 38, 39, and 40.* The results are qualitatively similar to the ones explained previously. On the one hand, the investment shock in a difference stationary economy is not sufficient to induce a slowdown in macro volatilities of a similar magnitude to the ones observed in the data. The role of such a shock is larger for a level stationary economy. It is remarkable that the role of the investment shock is larger when the law of motion of the technology level is time-varying than when it is assumed to be fixed across subsamples. From the last experiment we can conclude again that the stochastic trend model accounts better for the magnitude of the slowdown than the deterministic trend one. In this environment, the slowdown implied by the level stationary model is not only larger than the one observed in the data, but also larger than the one implied by the model under time-invariant laws of motion.

We conclude that while the neutral shock is the main driving force in the slowdown in volatilities generated by our difference-stationary model, allowing for a larger financial flexibility in the form of milder investment-specific shocks substantially improves its ability to reproduce the magnitude of the observed slowdown. Such a financial flexibility plays an even larger role in a level stationary economy since not only enhances the slowdown due to the neutral shock, but also it is the main driving force in the slowdown of consumption, capital, and hours volatility. Therefore, the Great Moderation in our setting is not due only to 'good luck' but also to the interaction between the two technology shocks.

5 Conclusion

We find that the choice of the statistical model for the stochastic processes in an RBC model with two technology shocks is not a trivial one. In fact, one model would be preferred to the others depending on the features of the business cycle the researcher wants to match.

We conclude that even though the neutral technology shock is the main driving force in replicating the Great Moderation, having both technology shocks translate into a better accounting for such a macroeconomic phenomenon. Therefore, the cross effects seem to be relevant. However, a bivariate specification of the innovations to the technology processes does not translate into a significative improvement of the performance of the model under analysis(see appendix E).

We have shown that in a simple RBC model the two technology shocks can explain approximately 70% of the observed slowdown in volatilities of US macro variables in mid 1980s. The remaining 30% could be explained as suggested in the literature by a reduction in the standard deviation of other shocks eg preference shocks, by an improved financial environment, or by good policy. Discriminating among those alternatives requires a richer model which is beyond the scope of our analysis.

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A Tables

Table 5: The Great Moderation: Empirical Evidence

	1950-2006		Pre-1984		Post-1984		Post/Pre	
	$\%\sigma_x$	$\rho(GNP, x)$	$\%\sigma_x$	$\rho(GNP, x)$	$\%\sigma_x$	$\rho(GNP, x)$	$\%\sigma_x$	$\rho(GNP,$
GNP	1.73	1	2.07	1	0.99	1	0.48	1
Consumption	0.92	0.78	1.07	0.78	0.62	0.82	0.58	1.05
Investment(efficiency units)	5.77	0.89	6.85	0.89	3.47	0.92	0.51	1.03
Investment(consumption units)	5.44	0.91	6.40	0.91	3.42	0.92	0.53	1.01
Capital(efficiency units)	0.59	0.36	0.68	0.39	0.44	0.27	0.65	0.69
Hours	1.88	0.87	2.11	0.87	1.46	0.90	0.69	1.03
Labor productivity	0.94	0.11	1.06	0.23	0.72	-0.46	0.68	-2
Neutral Technology	0.94	0.65	1.14	0.71	0.48	0.21	0.42	0.30
Investment-specific Tech	1.09	0.17	1.34	0.18	0.48	0.12	0.36	0.67

Source:Ríos-Rull et al. (2007) Data Set. We have HP-filtered the log of real variables. Standard deviations are in percentage terms.

Table 6: Calibration Targets

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
H^*	0.31	0.31	0.31
Y^*	1	1	1
$(\frac{K/V}{Y})^*$	10.288	10.502	9.953
$(\frac{X}{K})^*$	0.0277	0.0276	0.0279
$(\frac{I}{Y})^*$	0.28	0.29	0.28

Table 7: Deterministic Trend: Calibrated Parameters

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
α	0.36	0.36	0.36
γ_a	0.000131	0.001413	-0.000824
γ_v	0.006760	0.005311	0.009438
β	0.995	0.992	0.998
δ	0.0135	0.0131	0.0121
η	0.0034	0.0037	0.0030
μ	0.0086	0.0091	0.0074
φ_a	4.67	4.59	4.83
φ_v	-0.16	-0.07	-0.65
ρ_a	0.98	0.97	0.96
ρ_{v1}	1.77	1.71	1.73
ρ_{v2}	-0.77	-0.76	-0.73
σ_{ξ_a}	0.0073	0.0086	0.0045
σ_{ξ_v}	0.0033	0.0037	0.0023
ν	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$
B	$\{30.02, 9.31, 6.30, 5.18\}$	$\{30.21, 9.36, 6.34, 5.21\}$	$\{29.73, 9.22, 6.24, 5.13\}$

Table 8: Baseline Stochastic Trend: Calibrated Parameters

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
α	0.36	0.36	0.36
γ_a	0.000619	0.001489	-0.000734
γ_v	0.00643	0.00493	0.008762
β	0.9894	0.9897	0.9889
δ	0.01348	0.01401	0.01267
η	0.0034	0.0037	0.0030
μ	0.92443	0.91670	0.93694
σ_a	0.0074	0.0087	0.0045
σ_v	0.0054	0.0060	0.0030
ν	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$
B	$\{30.02, 9.31, 6.30, 5.18\}$	$\{30.21, 9.36, 6.34, 5.21\}$	$\{29.73, 9.22, 6.24, 5.13\}$

Table 9: Stochastic Trend with a Moving Average Component: Calibrated Parameters

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
α	0.36	0.36	0.36
γ_a	0.000619	0.001489	-0.000734
γ_v	0.006404	0.004911	0.008739
β	0.9894	0.9897	0.9889
δ	0.01352	0.01404	0.01271
η	0.0034	0.0037	0.0030
μ	0.92439	0.91667	0.93692
ρ	0.618159	0.6359218	0.5071769
σ_a	0.0074	0.0087	0.0045
σ_v	0.0025	0.0028	0.0012
ν	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$	$\{0.5, 1, 1.5, 2\}$
B	$\{30.02, 9.31, 6.30, 5.18\}$	$\{30.21, 9.36, 6.34, 5.21\}$	$\{29.73, 9.22, 6.24, 5.13\}$

Table 10: **Results:**Deterministic Trend: $\nu = 0.5$

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	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.87	0.31	1.07	0.78	1.28	0.12	0.62	0.81	0.44	0.27	0.58	0.35
x	5.77	0.89	3.25	0.86	6.85	0.89	4.98	0.82	3.47	0.92	1.74	0.92	0.53	0.35
y	1.73	1	1.12	1	2.07	1	1.38	1	0.99	1	0.69	1	0.48	0.50
k	0.59	0.36	0.49	0.33	0.68	0.39	0.66	0.33	0.44	0.27	0.22	0.29	0.65	0.33
h	1.88	0.87	0.40	0.72	2.11	0.87	0.59	0.70	1.46	0.90	0.24	0.80	0.69	0.41
y/h	0.94	0.11	0.88	0.95	1.06	0.23	1.06	0.92	0.72	-0.46	0.52	0.96	0.68	0.49

Table 11: **Results:** Deterministic Trend: $\nu = 1$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.88	0.26	1.07	0.78	1.29	0.04	0.62	0.81	0.46	0.20	0.58	0.36
x	5.77	0.89	3.73	0.89	6.85	0.89	5.64	0.86	3.47	0.92	2.07	0.94	0.53	0.37
y	1.73	1	1.27	1	2.07	1	1.55	1	0.99	1	0.80	1	0.48	0.52
k	0.59	0.36	0.54	0.33	0.68	0.39	0.74	0.33	0.44	0.27	0.27	0.31	0.65	0.36
h	1.88	0.87	0.67	0.78	2.11	0.87	0.99	0.76	1.46	0.90	0.42	0.85	0.69	0.42
y/h	0.94	0.11	0.86	0.87	1.06	0.23	1.03	0.78	0.72	-0.46	0.50	0.89	0.68	0.44

Table 12: **Results:** Deterministic Trend: $\nu = 1.5$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.84	0.21	1.07	0.78	1.28	-0.01	0.62	0.81	0.46	0.14	0.58	0.36
x	5.77	0.89	3.95	0.91	6.85	0.89	6.17	0.89	3.47	0.92	2.26	0.95	0.53	0.37
y	1.73	1	1.33	1	2.07	1	1.69	1	0.99	1	0.86	1	0.48	0.51
k	0.59	0.36	0.56	0.34	0.68	0.39	0.82	0.35	0.44	0.28	0.27	0.34	0.65	0.34
h	1.88	0.87	0.85	0.82	2.11	0.87	1.28	0.80	1.46	0.90	0.55	0.87	0.69	0.43
y/h	0.94	0.11	0.80	0.78	1.06	0.23	1.02	0.65	0.72	-0.46	0.47	0.81	0.68	0.46

Table 13: **Results:** Deterministic Trend: $\nu = 2$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.86	0.19	1.07	0.78	1.30	-0.08	0.62	0.81	0.48	0.17	0.58	0.37
x	5.77	0.89	4.22	0.92	6.85	0.89	6.68	0.91	3.47	0.92	2.42	0.95	0.53	0.36
y	1.73	1	1.41	1	2.07	1	1.84	1	0.99	1	0.94	1	0.48	0.51
k	0.59	0.36	0.60	0.33	0.68	0.39	0.89	0.32	0.44	0.27	0.30	0.35	0.65	0.34
h	1.88	0.87	1.00	0.83	2.11	0.87	1.56	0.83	1.46	0.90	0.65	0.87	0.69	0.42
y/h	0.94	0.11	0.81	0.72	1.06	0.23	1.02	0.53	0.72	-0.46	0.48	0.76	0.68	0.47

Table 14: **Results:** Baseline Stochastic Trend: $\nu = 0.5$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	σ_{data}	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.68	0.89	1.07	0.78	0.82	0.90	0.62	0.81	0.41	0.90	0.58	0.50
x	5.77	0.89	2.67	0.84	6.85	0.89	3.01	0.84	3.47	0.92	1.57	0.85	0.53	0.52
y	1.73	1	1.06	1	2.07	1	1.26	1	0.99	1	0.63	1	0.48	0.50
k	0.59	0.36	0.26	0.33	0.68	0.39	0.30	0.32	0.44	0.27	0.16	0.31	0.65	0.53
h	1.88	0.87	0.18	0.83	2.11	0.87	0.21	0.83	1.46	0.90	0.11	0.83	0.69	0.52
y/h	0.94	0.11	0.91	0.99	1.06	0.23	1.09	0.99	0.72	-0.46	0.55	0.99	0.68	0.50

Table 15: **Results:** Baseline Stochastic Trend:$\nu = 1$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.69	0.87	1.07	0.78	0.80	0.89	0.62	0.81	0.41	0.90	0.58	0.51
x	5.77	0.89	2.91	0.85	6.85	0.89	3.15	0.86	3.47	0.92	1.73	0.87	0.53	0.54
y	1.73	1	1.11	1	2.07	1	1.27	1	0.99	1	0.68	1	0.48	0.54
k	0.59	0.36	0.30	0.33	0.68	0.39	0.31	0.32	0.44	0.27	0.18	0.33	0.65	0.58
h	1.88	0.87	0.30	0.83	2.11	0.87	0.33	0.84	1.46	0.90	0.18	0.86	0.69	0.55
y/h	0.94	0.11	0.87	0.98	1.06	0.23	1.00	0.98	0.72	-0.46	0.53	0.98	0.68	0.53

Table 16: **Results:** Baseline Stochastic Trend: $\nu = 1.5$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.70	0.89	1.07	0.78	0.83	0.89	0.62	0.81	0.42	0.89	0.58	0.51
x	5.77	0.89	2.93	0.87	6.85	0.89	3.45	0.88	3.47	0.92	1.85	0.88	0.53	0.54
y	1.73	1	1.15	1	2.07	1	1.37	1	0.99	1	0.71	1	0.48	0.52
k	0.59	0.36	0.29	0.31	0.68	0.39	0.35	0.33	0.44	0.27	0.19	0.34	0.65	0.54
h	1.88	0.87	0.37	0.85	2.11	0.87	0.44	0.86	1.46	0.90	0.23	0.86	0.69	0.52
y/h	0.94	0.11	0.85	0.97	1.06	0.23	1.02	0.98	0.72	-0.46	0.52	0.98	0.68	0.51

Table 17: Baseline Stochastic Trend:$\nu = 2$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.73	0.88	1.07	0.78	0.85	0.90	0.62	0.81	0.43	0.89	0.58	0.51
x	5.77	0.89	3.16	0.88	6.85	0.89	3.57	0.89	3.47	0.92	1.89	0.88	0.53	0.53
y	1.73	1	1.22	1	2.07	1	1.42	1	0.99	1	0.73	1	0.48	0.51
k	0.59	0.36	0.31	0.33	0.68	0.39	0.35	0.33	0.44	0.27	0.19	0.32	0.65	0.54
h	1.88	0.87	0.45	0.86	2.11	0.87	0.51	0.87	1.46	0.90	0.26	0.87	0.69	0.51
y/h	0.94	0.11	0.86	0.96	1.06	0.23	0.97	1.01	0.72	-0.46	0.52	0.97	0.68	0.51

Table 18: **Results:** Stochastic Trend with Moving Average Component:$\nu = 0.5$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.69	0.87	1.07	0.78	0.82	0.86	0.62	0.81	0.40	0.95	0.58	0.49
x	5.77	0.89	2.60	0.85	6.85	0.89	3.12	0.85	3.47	0.92	1.50	0.93	0.53	0.48
y	1.73	1	1.06	1	2.07	1	1.26	1	0.99	1	0.66	1	0.48	0.52
k	0.59	0.36	0.27	0.31	0.68	0.39	0.34	0.31	0.44	0.27	0.16	0.32	0.65	0.47
h	1.88	0.87	0.19	0.80	2.11	0.87	0.23	0.80	1.46	0.90	0.10	0.90	0.69	0.43
y/h	0.94	0.11	0.92	0.99	1.06	0.23	1.08	0.99	0.72	-0.46	0.57	1	0.68	0.53

Table 19: **Results:** Stochastic Trend with Moving Average Component: $\nu = 1$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.70	0.85	1.07	0.78	0.84	0.85	0.62	0.81	0.40	0.94	0.58	0.48
x	5.77	0.89	2.77	0.86	6.85	0.89	3.31	0.87	3.47	0.92	1.54	0.93	0.53	0.47
y	1.73	1	1.10	1	2.07	1	1.33	1	0.99	1	0.66	1	0.48	0.50
k	0.59	0.36	0.29	0.33	0.68	0.39	0.35	0.34	0.44	0.27	0.16	0.33	0.65	0.46
h	1.88	0.87	0.31	0.81	2.11	0.87	0.38	0.82	1.46	0.90	0.16	0.90	0.69	0.42
y/h	0.94	0.11	0.87	0.98	1.06	0.23	1.05	0.98	0.72	-0.46	0.52	0.99	0.68	0.50

Table 20: **Results:** Stochastic Trend with Moving Average Component: $\nu = 1.5$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.72	0.85	1.07	0.78	0.87	0.84	0.62	0.81	0.42	0.94	0.58	0.48
x	5.77	0.89	3.06	0.88	6.85	0.89	3.55	0.87	3.47	0.92	1.64	0.93	0.53	0.46
y	1.73	1	1.18	1	2.07	1	1.40	1	0.99	1	0.71	1	0.48	0.51
k	0.59	0.36	0.32	0.32	0.68	0.39	0.38	0.33	0.44	0.27	0.17	0.32	0.65	0.45
h	1.88	0.87	0.42	0.83	2.11	0.87	0.49	0.81	1.46	0.90	0.21	0.91	0.69	0.43
y/h	0.94	0.11	0.87	0.96	1.06	0.23	1.04	0.96	0.72	-0.46	0.53	0.99	0.68	0.51

Table 21: **Results:** Stochastic Trend with Moving Average Component: $\nu = 2$

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.72	0.84	1.07	0.78	0.87	0.84	0.62	0.81	0.42	0.93	0.58	0.48
x	5.77	0.89	3.14	0.88	6.85	0.89	3.59	0.88	3.47	0.92	1.74	0.94	0.53	0.48
y	1.73	1	1.20	1	2.07	1	1.42	1	0.99	1	0.73	1	0.48	0.51
k	0.59	0.36	0.33	0.33	0.68	0.39	0.37	0.35	0.44	0.27	0.18	0.34	0.65	0.49
h	1.88	0.87	0.48	0.84	2.11	0.87	0.57	0.83	1.46	0.90	0.25	0.91	0.69	0.44
y/h	0.94	0.11	0.85	0.95	1.06	0.23	1.01	0.95	0.72	-0.46	0.52	0.98	0.68	0.51

Table 22: Autocorrelation ($nu = 1$)

	Data			DT			ST			ST-MA		
	Whole	Pre	Post	Whole	Pre	Post	Whole	Pre	Post	Whole	Pre	Post
y	0.84	0.84	0.88	0.76	0.76	0.73	0.73	0.73	0.72	0.71	0.73	0.72
c	0.81	0.81	0.83	0.86	0.92	0.72	0.74	0.75	0.74	0.73	0.74	0.74
x	0.82	0.81	0.90	0.81	0.85	0.72	0.72	0.72	0.70	0.75	0.76	0.74
k	0.93	0.93	0.93	0.97	0.97	0.97	0.96	0.96	0.96	0.96	0.97	0.96
h	0.90	0.89	0.94	0.83	0.88	0.71	0.71	0.72	0.70	0.70	0.71	0.72
lbp	0.71	0.71	0.74	0.76	0.78	0.74	0.73	0.74	0.73	0.72	0.73	0.72

Table 23: Cross-correlation output with x: DATA. Whole sample

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	-0.14	0.05	0.31	0.60	0.84	1	0.84	0.60	0.31	0.05	-0.14
V	-0.30	-0.22	-0.12	-0.01	0.09	0.17	0.25	0.29	0.30	0.30	0.29
A	0.19	0.31	0.46	0.60	0.65	0.65	0.32	0.003	-0.27	-0.43	-0.49
C	-0.04	0.16	0.37	0.59	0.75	0.78	0.66	0.47	0.27	0.09	-0.06
X	-0.04	0.13	0.36	0.60	0.79	0.89	0.71	0.46	0.17	-0.07	-0.22
I	0.02	0.18	0.41	0.64	0.82	0.91	0.71	0.43	0.12	-0.13	-0.29
K	-0.51	-0.44	-0.31	-0.12	0.11	0.36	0.55	0.66	0.67	0.61	0.51
H	-0.26	-0.09	0.14	0.42	0.69	0.87	0.87	0.75	0.54	0.30	0.10
LBP	0.26	0.28	0.29	0.28	0.19	0.11	-0.19	-0.39	-0.51	-0.51	-0.45

Table 24: Cross-correlation output with x: Deterministic Trend $nu = 1$. Whole Sample

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.01	0.15	0.32	0.52	0.75	1	0.75	0.52	0.32	0.15	0.01
C	-0.22	-0.18	-0.12	-0.02	0.10	0.26	0.24	0.22	0.21	0.20	0.18
X	0.11	0.24	0.38	0.54	0.71	0.89	0.65	0.42	0.23	0.06	-0.07
K	-0.28	-0.21	-0.11	0.01	0.16	0.54	0.46	0.53	0.55	0.55	0.51
H	0.16	0.26	0.38	0.51	0.64	0.67	0.55	0.35	0.17	0.02	-0.11
LBP	-0.11	0.02	0.18	0.37	0.60	0.86	0.67	0.50	0.35	0.21	0.10

Table 25: Cross-correlation output with x: Stochastic Trend $nu = 1$. Whole Sample

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.03	0.15	0.30	0.50	0.73	1	0.73	0.50	0.30	0.15	0.03
C	-0.06	0.04	0.19	0.37	0.60	0.87	0.68	0.51	0.37	0.25	0.15
X	0.09	0.19	0.32	0.47	0.65	0.85	0.59	0.37	0.19	0.05	-0.06
K	-0.36	-0.30	-0.21	-0.08	0.10	0.33	0.48	0.57	0.60	0.60	0.57
H	0.13	0.23	0.34	0.49	0.65	0.83	0.55	0.32	0.13	-0.01	-0.12
LBP	-0.01	0.11	0.27	0.46	0.70	0.98	0.73	0.52	0.34	0.20	0.08

Table 26: Cross-correlation output with x: Stochastic Trend-MA $nu = 1$. Whole Sample

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	-0.00	0.13	0.30	0.50	0.72	1	0.72	0.50	0.30	0.13	-0.00
C	-0.09	0.03	0.18	0.36	0.58	0.85	0.67	0.52	0.38	0.24	0.13
X	0.06	0.18	0.33	0.48	0.66	0.86	0.57	0.35	0.16	0.00	-0.12
K	-0.37	-0.31	-0.21	-0.08	0.10	0.33	0.48	0.56	0.58	0.57	0.52
H	0.08	0.19	0.32	0.47	0.62	0.81	0.52	0.29	0.10	-0.05	-0.16
LBP	-0.04	0.09	0.26	0.46	0.69	0.98	0.73	0.52	0.34	0.18	0.05

Table 27: Cross-correlation output with x: DATA. 1948:1-1983:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	-0.18	0.00	0.28	0.58	0.84	1	0.84	0.58	0.28	0.00	-0.18
V	-0.30	-0.22	-0.12	-0.01	0.84	0.18	0.25	0.29	0.31	0.31	0.29
A	0.20	0.33	0.50	0.65	0.84	0.71	0.36	0.03	-0.25	-0.44	-0.50
C	-0.07	0.14	0.36	0.58	0.84	0.78	0.64	0.43	0.21	0.02	-0.14
X	-0.09	0.08	0.32	0.57	0.84	0.89	0.70	0.43	0.13	-0.11	-0.26
I	-0.03	0.14	0.37	0.61	0.81	0.91	0.70	0.40	0.07	-0.18	-0.33
K	-0.53	-0.47	-0.33	-0.12	0.12	0.39	0.58	0.68	0.68	0.59	0.46
H	-0.36	-0.20	0.05	0.36	0.66	0.87	0.88	0.75	0.52	0.27	0.06
LBP	0.34	0.39	0.43	0.42	0.33	0.23	-0.11	-0.35	-0.50	-0.54	-0.49

Table 28: Cross-correlation output with x: Deterministic Trend $nu = 1$. 1948:1-1983:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.02	0.16	0.33	0.53	0.75	1	0.75	0.53	0.33	0.16	0.02
C	-0.26	-0.26	-0.24	-0.19	-0.10	0.04	0.05	0.08	0.13	0.17	0.21
X	0.15	0.28	0.41	0.56	0.72	0.86	0.64	0.42	0.23	0.06	-0.09
K	-0.35	-0.27	-0.17	-0.03	0.14	0.33	0.47	0.55	0.59	0.59	0.55
H	0.19	0.30	0.42	0.54	0.66	0.76	0.56	0.36	0.18	0.02	-0.12
LBP	-0.15	-0.04	0.10	0.28	0.51	0.78	0.60	0.45	0.33	0.23	0.15

Table 29: Cross-correlation output with x: Stochastic Trend $nu = 1$. 1948:1-1983:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	-0.00.13	0.28	0.48	0.71	1	0.71	0.48	0.28	0.13	-0.00	
C	-0.09	0.03	0.18	0.37	0.61	0.89	0.69	0.51	0.35	0.22	0.11
X	0.07	0.17	0.30	0.45	0.64	0.87	0.58	0.35	0.17	0.03	-0.08
K	-0.38	-0.32	-0.23	-0.10	0.08	0.32	0.47	0.55	0.59	0.58	0.54
H	0.11	0.21	0.32	0.47	0.64	0.84	0.54	0.31	0.12	-0.02	-0.13
LBP	-0.04	0.09	0.25	0.45	0.69	0.98	0.72	0.50	0.32	0.17	0.04

Table 30: Cross-correlation output with x: Stochastic Trend-MA $nu = 1$. 1948:1-1983:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.01	0.14	0.31	0.50	0.73	1	0.73	0.50	0.31	0.14	0.01
C	-0.10	0.02	0.17	0.37	0.59	0.85	0.68	0.52	0.37	0.24	0.14
X	0.11	0.21	0.35	0.50	0.68	0.87	0.60	0.37	0.18	0.03	-0.09
K	-0.36	-0.29	-0.20	-0.06	0.12	0.34	0.49	0.57	0.60	0.59	0.55
H	0.13	0.23	0.35	0.48	0.64	0.82	0.55	0.32	0.13	-0.02	-0.13
LBP	-0.03	0.10	0.27	0.47	0.70	0.98	0.74	0.53	0.34	0.19	0.06

Table 31: Cross-correlation output with x: DATA. 1984:1-2006:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.27	0.45	0.61	0.78	0.89	1	0.88	0.78	0.61	0.45	0.27
V	-0.23	-0.15	-0.08	-0.01	0.07	0.12	0.20	0.24	-0.33	0.25	0.25
A	0.20	0.17	0.13	0.18	0.15	0.21	-0.06	-0.18	-0.33	-0.36	-0.42
C	0.10	0.27	0.44	0.61	0.72	0.82	0.78	0.73	0.62	0.47	0.32
X	0.39	0.53	0.66	0.79	0.88	0.92	0.82	0.69	0.52	0.35	0.16
I	0.43	0.56	0.68	0.80	0.88	0.92	0.80	0.67	0.49	0.32	0.13
K	-0.38	-0.27	-0.17	-0.04	0.08	0.27	0.39	0.51	0.60	0.65	0.66
H	0.25	0.44	0.61	0.74	0.84	0.90	0.89	0.83	0.71	0.55	0.39
LBP	-0.14	-0.27	-0.40	-0.44	-0.50	-0.46	-0.61	-0.62	-0.61	-0.51	-0.43

Table 32: Cross-correlation output with x: Deterministic Trend $nu = 1$. 1984:1-2006:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.03	0.16	0.31	0.50	0.73	1	0.73	0.50	0.31	0.16	0.03
C	-0.12	-0.09	-0.04	0.02	0.11	0.20	0.22	0.22	0.22	0.20	0.16
X	0.07	0.19	0.33	0.50	0.70	0.94	0.66	0.43	0.24	0.09	-0.03
K	-0.36	-0.29	-0.20	-0.07	0.10	0.31	0.45	0.53	0.56	0.56	0.53
H	0.09	0.20	0.32	0.47	0.64	0.85	0.58	0.36	0.18	0.04	-0.06
LBP	-0.03	0.09	0.23	0.41	0.64	0.89	0.69	0.50	0.35	0.21	0.10

Table 33: Cross-correlation output with x: Stochastic Trend $nu = 1$. 1984:1-2006:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	0.00	0.13	0.30	0.49	0.73	1	0.73	0.49	0.30	0.13	0.00
C	-0.09	0.03	0.19	0.38	0.61	0.90	0.70	0.53	0.38	0.24	0.12
X	0.07	0.18	0.32	0.47	0.66	0.87	0.60	0.37	0.19	0.04	-0.08
K	-0.39	-0.33	-0.23	-0.09	0.09	0.33	0.48	0.57	0.60	0.60	0.56
H	0.11	0.22	0.35	0.49	0.67	0.86	0.56	0.32	0.13	-0.02	-0.14
LBP	-0.03	0.10	0.26	0.46	0.70	0.98	0.73	0.52	0.34	0.18	0.05

Table 34: Cross-correlation output with x: Stochastic Trend $nu = 1$. 1984:1-2006:4

	x(-5)	x(-4)	x(-3)	x(-2)	x(-1)	x(0)	x(1)	x(2)	x(3)	x(4)	x(5)
Y	-0.01	0.12	0.28	0.48	0.72	1	0.72	0.48	0.28	0.12	-0.01
C	-0.08	0.04	0.20	0.39	0.64	0.94	0.72	0.53	0.36	0.22	0.10
X	0.05	0.16	0.31	0.49	0.69	0.93	0.63	0.39	0.19	0.03	-0.08
K	-0.42	-0.36	-0.26	-0.12	0.07	0.33	0.49	0.59	0.62	0.61	0.57
H	0.09	0.20	0.33	0.50	0.69	0.90	0.59	0.33	0.13	-0.03	-0.14
LBP	-0.04	0.09	0.25	0.45	0.70	0.99	0.73	0.50	0.31	0.16	0.04

Table 35: **The Great Moderation:** Time-invariant coefficients-Experiment 1

	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	0.90	0.79	0.88	0.79	0.47	0.59	0.82	0.47	0.57	0.58
x	4.11	3.01	0.73	3.13	2.19	0.70	3.21	2.11	0.66	0.53
y	1.48	0.84	0.57	1.28	0.70	0.54	1.34	0.68	0.51	0.48
k	0.57	0.49	0.86	0.31	0.22	0.71	0.34	0.23	0.68	0.65
h	0.71	0.59	0.83	0.33	0.23	0.69	0.35	0.25	0.71	0.69
y/h	1	0.56	0.56	1.01	0.55	0.54	1.05	0.53	0.50	0.68

Table 36: **The Great Moderation:** Time-invariant coefficients-Experiment 2

	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	0.93	0.69	0.74	0.70	0.65	0.93	0.71	0.67	0.94	0.58
x	3.78	3.25	0.86	2.93	2.49	0.83	2.90	2.46	0.86	0.53
y	1.28	1.23	0.96	1.10	1.09	0.99	1.11	1.12	1.01	0.48
k	0.55	0.43	0.78	0.29	0.24	0.83	0.31	0.25	0.81	0.65
h	0.69	0.54	0.78	0.31	0.26	0.84	0.33	0.26	0.79	0.69
y/h	0.88	0.84	0.95	0.87	0.86	0.98	0.87	0.88	1.01	0.68

Table 37: **The Great Moderation:** Time-invariant coefficients-Experiment 3

	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	0.98	0.57	0.58	0.80	0.43	0.53	0.84	0.42	0.50	0.58
x	4.20	2.37	0.56	3.23	1.71	0.53	3.35	1.65	0.49	0.53
y	1.47	0.78	0.53	1.29	0.69	0.54	1.34	0.68	0.50	0.48
k	0.61	0.36	0.59	0.32	0.17	0.53	0.36	0.17	0.47	0.65
h	0.75	0.44	0.59	0.34	0.18	0.53	0.37	0.18	0.49	0.69
y/h	0.99	0.52	0.53	1.02	0.55	0.54	1.06	0.54	0.51	0.68

Table 38: **The Great Moderation:** Time-varying coefficients-Experiment 1

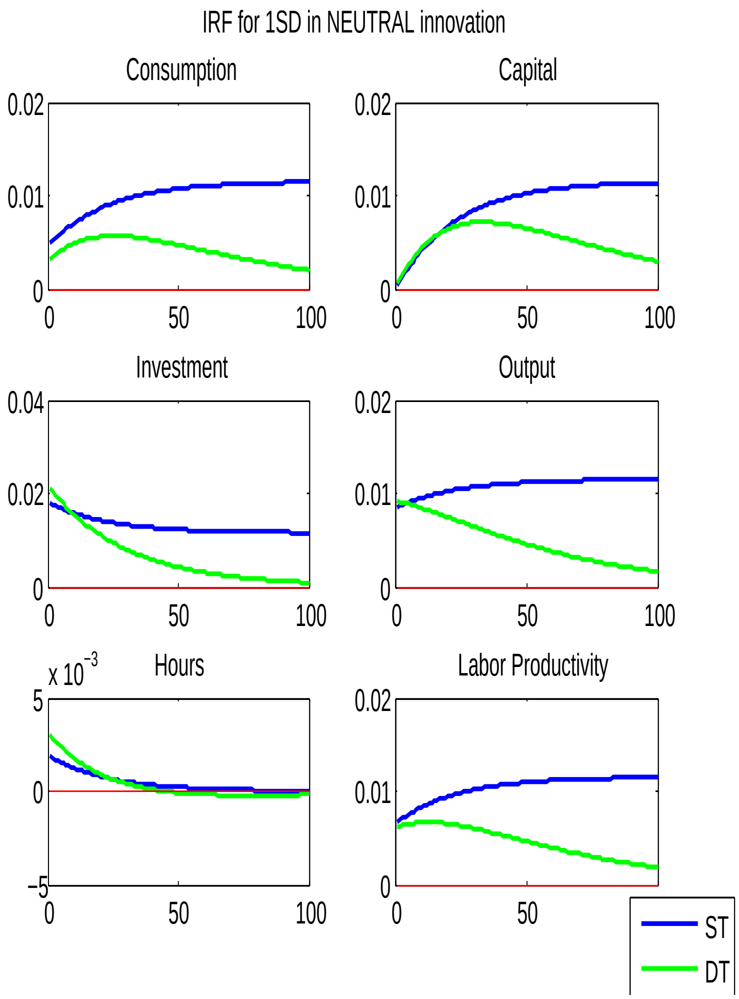
	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	1.23	0.64	0.52	0.79	0.49	0.62	0.80	0.44	0.55	0.58
x	5.19	2.71	0.52	3.20	2.12	0.66	3.26	1.84	0.56	0.53
y	1.55	0.83	0.54	1.30	0.70	0.54	1.30	0.67	0.52	0.48
k	0.65	0.40	0.62	0.30	0.22	0.73	0.33	0.21	0.64	0.65
h	0.98	0.51	0.52	0.33	0.22	0.67	0.36	0.20	0.56	0.69
y/h	1.01	0.53	0.52	1.02	0.56	0.55	1.02	0.53	0.52	0.68

Table 39: **The Great Moderation:** Time-varying coefficients-Experiment 2

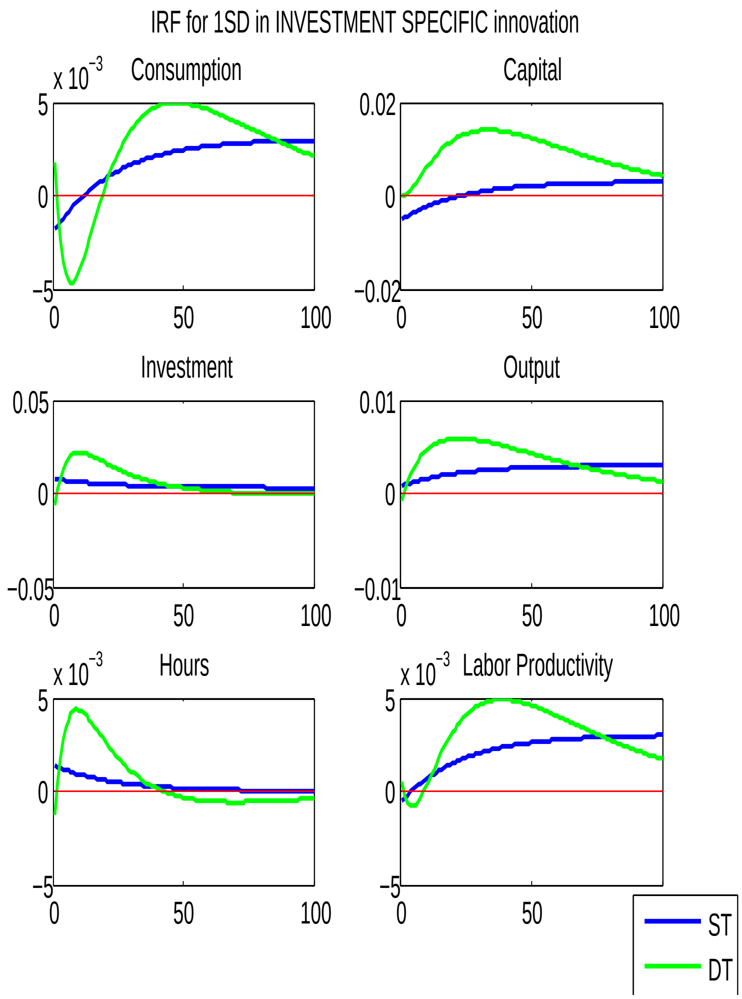
	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	1.29	0.55	0.43	0.71	0.66	0.93	0.73	0.65	0.89	0.58
x	4.94	3.24	0.66	2.89	2.49	0.86	3.02	2.34	0.77	0.53
y	1.33	1.25	0.94	1.11	1.11	1	1.12	1.10	0.98	0.48
k	0.64	0.39	0.61	0.28	0.26	0.93	0.31	0.24	0.77	0.65
h	0.97	0.53	0.55	0.30	0.26	0.87	0.34	0.25	0.74	0.69
y/h	0.88	0.80	0.91	0.88	0.88	1	0.89	0.87	0.98	0.68

Table 40: **The Great Moderation:** Time-varying coefficients-Experiment 3

	DT			ST			ST-MA			Data
	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\%\sigma_{pre}$	$\%\sigma_{post}$	$\sigma_{post}/\sigma_{pre}$	$\sigma_{post}/\sigma_{pre}$
c	1.30	0.46	0.35	0.79	0.44	0.56	0.83	0.41	0.49	0.58
x	5.30	2.26	0.43	3.27	1.72	0.39	3.32	1.56	0.47	0.53
y	1.58	0.79	0.50	1.28	0.71	0.56	1.31	0.68	0.52	0.48
k	0.65	0.31	0.48	0.30	0.18	0.60	0.34	0.17	0.50	0.65
h	1.00	0.40	0.40	0.34	0.18	0.53	0.37	0.17	0.46	0.69
y/h	1.04	0.51	0.49	1.01	0.57	0.56	1.03	0.54	0.52	0.68



IRF for 1SD in NEUTRAL innovation
Consumption
Capital
Investment
Output
Hours
x 10^-3
Labor Productivity
0.02
0.01
0
0.04
5
-5
50
100
ST
DT


IRF for 1SD in INVESTMENT SPECIFIC innovation
x 10^-3
Consumption
5
0
−5
0
50
100
Capital
0.02
0
−0.02
0
50
100
Investment
0.05
0
−0.05
0
50
100
Output
0.01
0
−0.01
0
50
100
x 10^-3
Hours
5
0
−5
0
50
100
x 10^-3
Labor Productivity
5
0
−5
0
50
ST
DT

B Balanced Growth Path

From the feasibility constraint we can conclude that output, consumption, and investment grow at the same rate

$$
\begin{aligned}
Y_t &= C_t + I_t \\
\frac{Y_t}{Y_{t-1}} &= \frac{C_t}{C_{t-1}}\frac{C_{t-1}}{Y_{t-1}} + \frac{I_t}{I_{t-1}}\frac{I_{t-1}}{Y_{t-1}} \\
g_Y &= g_C\frac{C_{t-1}}{Y_{t-1}} + g_I\frac{I_{t-1}}{Y_{t-1}}
\end{aligned}
$$

Therefore, g_Y is constant if and only if $g_Y = g_C = g_I$.

Let us consider now the investment equation

$$
\begin{aligned}
(1+\eta)K_{t+1} &= (1-\delta)K_t + V_t I_t \\
(1+\eta)\frac{K_{t+1}}{K_t} &= (1-\delta) + \frac{V_t I_t}{K_t} \\
(1+\eta)g_K &= (1-\delta) + \frac{V_t I_t}{K_t}
\end{aligned}
$$

g_K is constant if and only if (VI) grows at the same rate as K which requires

$$g_K = g_I g_V$$

Let us analyze the production function

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} H_t^{1-\alpha} \\
g_Y &= g_A g_K^{\alpha} g_H^{1-\alpha}
\end{aligned}
$$

As we are considering hours are stationary, we have that $g_H = 1$. Hence,

$$
\begin{aligned}
g_Y &= g_A g_K^{\alpha} \\
&= g_A (g_Y g_V)^{\alpha}
\end{aligned}
$$

Therefore,

$$g_Y = g_A^{\frac{1}{1-\alpha}} g_V^{\frac{\alpha}{1-\alpha}}$$

Let us consider the deterministic trend model. We will have that

$$g_Y = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v} \quad (43)$$

$$g_V = e^{\gamma_v} \quad (44)$$

which implies

$$g_K = e^{\frac{1}{1-\alpha}(\gamma_a+\gamma_v)} \quad (45)$$

Let us consider the stochastic trend model. Then, we will have

$$g_Y = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}} \quad (46)$$

$$(47)$$

and

$$g_K = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}} \quad (48)$$

C Log-linearization around the steady state

C.1 Deterministic Trend Model

Let us define $\hat{x}_t = ln(\tilde{X}_t/X^*)$, then the log-linearized system of equations is given by:

$$\hat{y}_t = \frac{C^*}{Y^*}\hat{c}_t + \frac{I^*}{Y^*}\hat{i}_t \quad (49)$$

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\hat{h}_t + \varepsilon_{at} \quad (50)$$

$$qv\hat{k}_{t+1} = (1-\delta)\hat{k}_t + V_0\frac{I^*}{K^*}[e^{\varepsilon_{vt}}(1+\hat{i}_t) - 1] \quad (51)$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t-1} + \varepsilon_{vt} - \varepsilon_{vt+1} + \left(\frac{R^*}{1-\delta+R^*}\right)\hat{r}_{t+1}\right] \quad (52)$$

$$\hat{h}_t = \nu(\hat{w}_t - \hat{c}_t) \quad (53)$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \varepsilon_{vt} \quad (54)$$

$$\hat{w}_t = \hat{y}_t - \hat{h}_t \quad (55)$$

C.2 Stochastic Trend Model

Let us define $\hat{x}_t = ln(\tilde{X}_t/X^*)$, then the log-linearized system of equations is given by:

$$\hat{y}_t = \hat{c}_t\frac{C^*}{Y^*} + \hat{i}_t\frac{I^*}{Y^*} \quad (56)$$

$$\hat{y}_t = -\alpha(\hat{q}_t + \hat{v}_t) + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \quad (57)$$

$$\hat{k}_{t+1} = (1-\delta)\left(\frac{1}{q^*v^*}\right)\left[\hat{k}_t - (\hat{q}_t + \hat{v}_t)\right] + \hat{i}_t\frac{I^*}{K^*} \quad (58)$$

$$0 = E_t\left[\hat{c}_t - \hat{c}_{t-1} - (\hat{q}_{t+1} + \hat{v}_{t+1}) + \hat{r}^k_{t+1}\right] \quad (59)$$

$$\hat{r^k_t} = \left(\frac{R^*}{R^{k*}}\right)\hat{r}_t \quad (60)$$

$$\hat{h}_t = \nu(\hat{w}_t - \hat{c}_t) \quad (61)$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \hat{q}_t + \hat{v}_t \quad (62)$$

$$\hat{w}_t = \hat{y}_t - \hat{h}_t \quad (63)$$

$$\hat{q}_t = \frac{1}{1-\alpha}\varepsilon_{at} + \frac{\alpha}{1-\alpha}\varepsilon_{vt} \quad (64)$$

$$\hat{v}_t = \varepsilon_{vt} \quad (65)$$

The above is a system of 11 equations and 11 unknowns: $\{\hat{c}_t, \hat{i}_t, \hat{y}_t, \hat{k}_t, \hat{h}_t, \hat{b}_t, \hat{r}_t, \hat{r}^k_t, \hat{q}_t, \hat{v}_t, \}$.

To proceed with estimation we need to also consider the following conditions:

$$q^* = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v}$$

$$v^* = e^{\gamma_v}$$

$$R^* = \frac{q^*v^*}{\beta} - (1-\delta)$$

$$R^{k*} = (1-\delta) + R^* = \frac{q^*v^*}{\beta}$$

$$\frac{I^*}{K^*} = 1 - (1-\delta)\frac{1}{q^*v^*}$$

$$\frac{K^*}{Y^*} = \frac{1}{\alpha q^*v^*}R^*$$

$$\frac{C^*}{Y^*} = \left(\frac{1-\delta}{q^*v^*} - 1\right)\frac{K^*}{Y^*} + 1$$

$$\frac{I^*}{Y^*} = 1 - \frac{C^*}{Y^*}$$

$$H^* = \left(\frac{1}{B^*}\right)^{\frac{\nu}{1+\nu}}\left[(1-\alpha)\frac{Y^*}{C^*}\right]^{\frac{\nu}{1+\nu}}$$

D Stochastic Trend Model: Closed form solution

In the economy under analysis both welfare theorems hold, therefore we can solve the planner's problem which is given by

$$
\begin{aligned}
max_{C_t,H_t} U &= \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(lnC_t - B\frac{H_t^{1+1/\nu}}{1+1/\nu}\right)\right] \\
s.t. & \\
C_t + I_t &= A_t K_t^{\alpha} H_t^{1-\alpha} \\
K_{t+1} &= (1-\delta)K_t + V_t I_t \\
A_t &= A_{t-1}e^{\gamma_a+\epsilon_{at}} \\
V_t &= V_{t-1}e^{\gamma_v+\epsilon_{vt}} \\
A_0, V_0, K_0 & \qquad given
\end{aligned}
$$

We will proceed first by combining the resource constraint and the law of motion for capital. Thus, our equilibrium conditions are given by the Euler equation, the labor supply, and the new resources constraint. Secondly, as our economy is non-stationary we need to transform it to be able to solve our model. All variables but hours and capital grow at rate $Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$. Capital grows at rate $Q_t V_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}$ and hours are stationary. Therefore, the equilibrium conditions for the transformed model economy are given by[14]:

$$
\begin{aligned}
1 &= \beta\mathbb{E}_t\left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}}\left((1-\delta)e^{-\frac{1}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at+1}+\epsilon_{vt+1})} + \alpha e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at+1}+\epsilon_{vt+1})}\tilde{K}_{t+1}^{\alpha-1}H_{t+1}^{1-\alpha}\right)\right] \\
BH_t^{1/\nu} &= (1-\alpha)\frac{\tilde{Y}_t/H_t}{\tilde{C}_t} \\
\tilde{C}_t + \tilde{K}_{t+1} &= (1-\delta)e^{-\frac{1}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})}\tilde{K}_t + e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})}\tilde{K}_t^{\alpha}H_t^{1-\alpha}
\end{aligned}
$$

[14] Maybe it is more intuitive to write the Euler equation as

$$
1 = \beta\mathbb{E}_t\left[e^{-\frac{1}{1-\alpha}(\gamma_a+\epsilon_{at+1}+\alpha(\gamma_v+\epsilon_{vt+1}))}\frac{\tilde{C}_t}{\tilde{C}_{t+1}}\left((1-\delta)e^{-\gamma_v-\epsilon_{vt+1}} + \alpha e^{\gamma_a+\epsilon_{at+1}}\tilde{K}_{t+1}^{\alpha-1}H_{t+1}^{1-\alpha}\right)\right]
$$

Let us assume there is full depreciation (i.e. $\delta = 1$). Thus, the above reduces to:

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\frac{\tilde{C}_t}{\tilde{C}_{t+1}} \left(\alpha e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at+1}+\epsilon_{vt+1})} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} \right) \right] && (66) \\
B H_t^{1/\nu} &= (1-\alpha) \frac{\tilde{Y}_t / H_t}{\tilde{C}_t} && (67) \\
\tilde{C}_t + \tilde{K}_{t+1} &= e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})} \tilde{K}_t^{\alpha} H_t^{1-\alpha} && (68)
\end{aligned}
$$

D.1 Baseline System: An Exact Solution

Our guess for policy function for capital will be

$$\tilde{K}_{t+1} = \alpha\beta e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})} \tilde{K}_t^{\alpha} H_t^{1-\alpha} \tag{69}$$

Then, from (68) we have that the policy function for consumption is given by:

$$\tilde{C}_t = e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})} \tilde{K}_t^{\alpha} H_t^{1-\alpha} (1-\alpha\beta) \tag{70}$$

Let us plug (69) and (70) in (66)

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[e^{-\frac{\alpha}{1-\alpha}(\epsilon_{at}+\epsilon_{vt}-\epsilon_{at+1}-\epsilon_{vt+1})} \frac{\tilde{K}_t^{\alpha} H_t^{1-\alpha}}{\tilde{K}_{t+1}^{\alpha} H_{t+1}^{1-\alpha}} \alpha e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at+1}+\epsilon_{vt+1})} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha} \right] \\
&= \alpha\beta \mathbb{E}_t \left[\frac{1}{\tilde{K}_{t+1}} e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})} \tilde{K}_t^{\alpha} H_t^{1-\alpha} \right]
\end{aligned}
$$

which implies

$$\tilde{K}_{t+1} = \alpha\beta e^{-\frac{\alpha}{1-\alpha}(\gamma_a+\gamma_v+\epsilon_{at}+\epsilon_{vt})} \tilde{K}_t^{\alpha} H_t^{1-\alpha}$$

since $\tilde{K}_{t+1}$ is a choice variable at time t not an unknown variable dated at time $t+1$. Therefore, as our guess satisfies the equilibrium conditions, we can ensure the policy function for capital is of the form given by (69). Consequently, the guess for the consumption policy rule is also part of the solution to our model. Note that we constructed such a guess by using (69) and the resources constraint. Note that both policy rules depend on model parameters, current realizations of shocks (we assume current shocks are observed before current decisions are taken), capital at time t which is a predetermined variable (chosen at time $t-1$), and current labor decision. Therefore, to completely characterize the policy rules of interest it remains to provide the labor supply policy function. To do so let us consider (67) and plug (70) so that:

$$\begin{aligned} BH_t^{1/\nu} &= \frac{\tilde{W}_t}{\tilde{C}_t} \\ &= \frac{(1-\alpha)\tilde{Y}_t/H_t}{(1-\alpha\beta)\tilde{Y}_t} \end{aligned}$$

Thus,

$$H_t = \left(\frac{1-\alpha}{B(1-\alpha\beta)}\right)^{\frac{\nu}{1-\nu}} \tag{71}$$

which is a constant.

Therefore, by substituting (71) in (69) and (70) we have our policy rules as functions of model parameters and current state variables.

D.2 Log-linearized system: An Approximate Solution

Let us consider the following log-linearized system (under the assumption of full depreciation).

$$\hat{y}_t = \frac{C^*}{Y^*}\hat{c}_t + \frac{I^*}{Y^*}\hat{i}_t \tag{72}$$

$$\hat{y} = -\frac{\alpha}{1-\alpha}(\epsilon_{at} + \epsilon_{vt}) + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \tag{73}$$

$$\hat{k}_{t+1} = \frac{I^*}{K^*}\hat{i}_t = \frac{I^*/Y^*}{K^*/Y^*}\hat{i}_t \tag{74}$$

$$0 = \mathbb{E}\left[\hat{c}_t - \hat{c}_{t+1} - \frac{1}{1-\alpha}(\epsilon_{at+1} + \epsilon_{vt+1}) + r_{t+1}\right] \tag{75}$$

$$\hat{h}_t = \nu(\hat{w}_t - \hat{c}_t) \tag{76}$$

$$\hat{r}_t = \hat{y}_t - \hat{k}_t + \frac{1}{1-\alpha}(\epsilon_{at} + \epsilon_{vt}) \tag{77}$$

$$\hat{w}_t = \hat{y}_t - \hat{h}_t \tag{78}$$

Plugging (74) into (72) we obtain

$$\hat{y}_t = \frac{C^*}{Y^*}\hat{c}_t + \frac{K^*}{Y^*}\hat{k}_{t+1} \tag{79}$$

Substitute (77) in (75)

$$0 = \mathbb{E}\left[\hat{c}_t - \hat{c}_{t+1}\hat{y}_{t+1} - \hat{k}_{t+1}\right] \tag{80}$$

Our guesses for policy rules are given by:

$$\begin{aligned} \hat{k}_{t+1} &= -\frac{\alpha}{1-\alpha}(\epsilon_{at} + \epsilon_{vt}) + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \quad (81)\\ \hat{c}_t &= \frac{1}{C^*/Y^*}\left(1 - \frac{K^*}{Y^*}\right)\hat{k}_{t+1} \quad (82) \end{aligned}$$

Plugging (81) and (82) into (75)

$$0 = \mathbb{E}[\frac{1}{C^*/Y^*}\left(1 - \frac{K^*}{Y^*}\right)\left(-\frac{\alpha}{1-\alpha}(\epsilon_{at} + \epsilon_{vt} - \epsilon_{at+1} - \epsilon_{vt+1}) + \alpha(\hat{k}_t - \hat{k}_{t+1}) + (1-\alpha)(\hat{h}_t - \hat{h}_{t+1})\right)$$
$$-\frac{\alpha}{1-\alpha}(\epsilon_{at+1} + \epsilon_{vt+1}) + \alpha\hat{k}_{t+1} + (1-\alpha)\hat{h}_{t+1} + \frac{\alpha}{1-\alpha}(\epsilon_{at} + \epsilon_{vt} - \alpha\hat{k}_t - (1-\alpha)\hat{h}_t)]$$

$$0 = \mathbb{E}\left[\left(1 - \frac{K^*}{Y^*}\right)\hat{y}_t - \frac{C^*}{Y^*}\hat{y}_t - \left(1 - \frac{K^*}{Y^*}\right)\hat{y}_{t+1} + \frac{C^*}{Y^*}\hat{y}_{t+1}\right]$$

where

$$\left(1 - \frac{K^*}{Y^*}\right)\hat{y}_t - \frac{C^*}{Y^*}\hat{y}_t - \left(1 - \frac{K^*}{Y^*}\right)\hat{y}_{t+1} + \frac{C^*}{Y^*}\hat{y}_{t+1} = 0 \tag{83}$$

since

$$1 = \frac{C^*}{Y^*} + \frac{K^*}{Y^*} \tag{84}$$

Therefore, we can conclude that the policy functions for capital and consumption are given by (81) and (82) respectively.

We need to provide also a policy rule for hours. Intuitively, the policy rule should be equal to zero. Remember that in the exact solution hours were constant over time. Therefore, the deviation from steady state should be zero. We show below that given (82), $\hat{h}_t = 0$ for all t.

$$\begin{aligned} \hat{h}_t &= \nu\hat{w}_t - \nu\hat{c}_t = \nu(\hat{y}_t - \hat{h}_t) - \nu\hat{c}_t \\ \hat{h}_t &= \frac{\nu}{1-\nu}(\hat{y}_t - \hat{c}_t) \\ &= \frac{1}{C^*/Y^*}\left[\frac{C^*}{Y^*} + \frac{K^*}{Y^*} - 1\right]\hat{k}_{t+1} \\ &= 0 \end{aligned}$$

E Extensions

E.1 Hansen-Rogerson Preferences

So far our analysis have only considered the intensive margin of the labor input. Here we will assume another specification for household's preferences so that we will analyze the extensive margin of the labor input. To do so let us assume the following:

1. Labor is indivisible.
2. Agents can trade employment lotteries.
3. Households have a constant relative risk-aversion utility function with a coefficient of risk-aversion equal to 1.

Therefore, (2) will be substituted by

$$U(C_t, H_t) = lnC_t - BH_t \tag{85}$$

which implies that the short-run Frisch elasticity of labor supply is infinite.

It is obvious that the (detrended) equilibrium conditions under all the statistical models are identical but the one associated with the labor supply. In particular, (11) and (25) will be substituted by

$$\frac{\tilde{W}_t}{\tilde{C}_t} = B \tag{86}$$

We need to recalibrate only the parameter linked to the weight of hours in the utility function i.e. B. In fact, we have that

$$B = (1-\alpha)\frac{Y^*}{C^*}\frac{1}{H^*} \tag{87}$$

We will stochastic simulate our model only for the deterministic trend case and the baseline stochastic trend one. We will perform our analysis only for the whole sample.

We will first allow for the presence of both technology shocks. Then, we will perform two counterfactuals in order to asses the relative importance of each technology shock in accounting for the business cycles features observed in the US data. On the one hand, we will shut down the investment specific shock and investigate the volatilities implied by our model. On the other hand, we will shut down the neutral shock and perform the same analysis.

Table 41: Deterministic Trend Model: 1948:1-2006:4

	Data		Both shocks			Only N-shock			Only I-shock		
	$\%\sigma_z$	ρ_{zy}	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$
c	0.92	0.78	0.89	0.08	0.97	0.55	0.94	0.60	0.70	-0.84	0.76
i	5.44	0.91	6.99	0.93	1.28	4.01	0.99	0.73	5.72	0.97	1.05
y	1.73	1	2.07	1	1.20	1.68	1	0.97	1.21	1	0.70
k	0.59	0.36	0.81	0.35	1.37	0.39	0.33	0.66	0.71	0.44	1.20
h	1.88	0.87	2.19	0.91	1.16	1.18	0.99	0.63	1.84	0.98	0.98
y/h	0.94	0.11	0.89	0.08	0.95	0.55	0.94	0.59	0.70	-0.84	0.74

From table E.1 we conclude that the volatilities of all the variables at hand are larger than in the divisible labor economy. Moreover, the Hansen-Rogerson economy overstates the volatilities of investment, output, capital, and hours when both shocks are at hand. It is remarkable that, as in Hansen (1997), the volatility of hours is larger than the volatility of labor productivity.

The model performs better, in terms of accounting for volatilities, when there is only an investment-specific shock than when there is only a neutral one. In fact, a model with Hansen-Rogerson preferences and only an I-shock is able to replicate almost perfectly the standard deviation of hours.

Table 42: Stochastic Trend Model: 1948:1-2006:4

	Data		Both shocks			Only N-shock			Only I-shock		
	$\%\sigma_z$	ρ_{zy}	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$	$\%\sigma_z$	ρ_{zy}	$\sigma_{model}/\sigma_{data}$
c	0.92	0.78	0.75	0.87	0.82	0.72	0.98	0.78	0.21	-0.81	0.23
i	5.44	0.91	3.80	0.91	0.70	3.08	0.99	0.57	2.23	0.99	0.41
y	1.73	1	1.41	1	0.82	1.38	1	0.81	0.32	1	0.18
k	0.59	0.36	0.37	0.33	0.63	0.30	0.35	0.51	0.22	0.38	0.37
h	1.88	0.87	0.85	0.90	0.45	0.69	0.98	0.37	0.50	0.97	0.27
y/h	0.94	0.11	0.75	0.87	0.80	0.72	0.98	0.77	0.21	-0.81	0.22

From the above table we conclude that a stochastic trend model is not able to generate enough volatility in this scenario either. Under this specification, the neutral shock is the one able to account for the bulk of the volatility for all the variables at hand.

E.2 Multivariate Analysis

We have performed univariate analysis of the error structure associated to the different specifications for the technology processes. We are interested here in exploring a multivariate error structure in order to analyze the interaction between both innovations.

E.2.1 Deterministic Trend Model

We will consider the following specification

$$lnA_t = \varphi_a + \gamma_a t + \varepsilon_{at} \quad (88)$$

$$lnV_t = \varphi_v + \gamma_v t + \varepsilon_{vt} \quad (89)$$

and we assume

$$\begin{pmatrix} \varepsilon_{at} \\ \varepsilon_{vt} \end{pmatrix} = \Gamma_1 \begin{pmatrix} \varepsilon_{at-1} \\ \varepsilon_{vt-1} \end{pmatrix} + \Gamma_2 \begin{pmatrix} \varepsilon_{at-2} \\ \varepsilon_{vt-2} \end{pmatrix} + \begin{pmatrix} \xi_{at} \\ \xi_{vt} \end{pmatrix} \tag{90}$$

where

$$\begin{pmatrix} \xi_{at} \\ \xi_{vt} \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \Sigma_{\xi}) \tag{91}$$

We will restrict our attention to the performance of the model under a unit Frisch elasticity. Our estimates are reported in the following table. All the vector autoregressive processes estimated satisfy the stability condition i.e. there is no root that lies outside the unit circle.

The results obtained from the stochastic simulation of our model economy are summarized in the table 44.

Table 43: Deterministic Trend: Estimated Parameters

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
γ_a	0.000131	0.001413	-0.000824
γ_v	0.006760	0.005311	0.009438
φ_a	4.67	4.59	4.83
φ_v	-0.16	-0.07	-0.65
Γ_1	$\begin{pmatrix} 0.999477^* & -0.200311^* \\ 0.072667^* & 1.791754^* \end{pmatrix}$	$\begin{pmatrix} 1.032293^* & -0.161686 \\ 0.051353^* & 1.735752^* \end{pmatrix}$	$\begin{pmatrix} 0.753032^* & -0.140866 \\ 0.075780 & 1.635421^* \end{pmatrix}$
Γ_2	$\begin{pmatrix} -0.022612 & 0.197971^* \\ -0.081605^* & -0.805428^* \end{pmatrix}$	$\begin{pmatrix} -0.075576 & 0.122759 \\ -0.055663^* & -0.784986^* \end{pmatrix}$	$\begin{pmatrix} 0.201175^* & 0.170665 \\ -0.127270^* & -0.644031^* \end{pmatrix}$
Σ_ξ	$\begin{pmatrix} 5.22 \cdot 10^{-5} & -6.80 \cdot 10^{-6} \\ \cdot & 1.11 \cdot 10^{-5} \end{pmatrix}$	$\begin{pmatrix} 7.28 \cdot 10^{-5} & -9.98 \cdot 10^{-6} \\ \cdot & 1.40 \cdot 10^{-5} \end{pmatrix}$	$\begin{pmatrix} 1.98 \cdot 10^{-5} & -2.89 \cdot 10^{-6} \\ \cdot & 4.73 \cdot 10^{-6} \end{pmatrix}$

Table 44: Deterministic Trend: Multivariate Analysis Results

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.97	0.33	1.07	0.78	1.37	0.24	0.62	0.81	0.51	-0.23	0.58	0.37
i	5.44	0.91	4.26	0.71	6.40	0.91	5.99	0.69	3.42	0.92	2.87	0.88	0.53	0.48
y	1.73	1	1.26	1	2.07	1	1.49	1	0.99	1	0.88	1	0.48	0.59
k	0.59	0.36	0.51	0.34	0.68	0.39	0.63	0.15	0.44	0.27	0.42	0.51	0.65	0.67
h	1.88	0.87	0.66	0.71	2.11	0.87	0.89	0.65	1.46	0.90	0.56	0.89	0.69	0.63
y/h	0.94	0.11	0.92	0.86	1.06	0.23	1.13	0.80	0.72	-0.46	0.46	0.84	0.68	0.41

Let us compare the previous table with table 11. The direction of change for the volatilities of the different variables of interest is not unilateral. For example, while the volatility of consumption is larger in the multivariate setting, the volatility of capital is lower. The performance of the deterministic trend model, however, improves in accounting for the volatility slowdown of investment, capital, and hours.

E.2.2 Stochastic Trend Model

Let us consider the following

$$lnA_t = lnA_{t-1} + \gamma_a t + \varepsilon_{at} \tag{92}$$

$$lnV_t = lnV_{t-1} + \gamma_v t + \varepsilon_{vt} \tag{93}$$

and we assume

$$\begin{pmatrix} \varepsilon_{at} \\ \varepsilon_{vt} \end{pmatrix} = \Gamma_1 \begin{pmatrix} \varepsilon_{at-1} \\ \varepsilon_{vt-1} \end{pmatrix} + \begin{pmatrix} \xi_{at} \\ \xi_{vt} \end{pmatrix} \tag{94}$$

where

$$\begin{pmatrix} \xi_{at} \\ \xi_{vt} \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \Sigma_\xi) \tag{95}$$

The results from the estimation of the above specification are reported in table 45. The moments implied by this specification are in table 46.

Table 45: Stochastic Trend: Estimated Parameters

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
γ_a	0.000619	0.001489	-0.000734
γ_v	0.00643	0.00493	0.008762
Γ	$\begin{pmatrix} 0.024185 & -0.205103^{*} \\ 0.070240^{*} & 0.805801^{*} \end{pmatrix}$	$\begin{pmatrix} 0.055551 & -0.178899 \\ 0.065821 & 0.782526^{*} \end{pmatrix}$	$\begin{pmatrix} -0.191304 & 0.017916 \\ 0.105430^{*} & 0.732534^{*} \end{pmatrix}$
Σ_ξ	$\begin{pmatrix} 5.30 \cdot 10^{-5} & -6.96 \cdot 10^{-6} \\ \cdot & 1.14 \cdot 10^{-5} \end{pmatrix}$	$\begin{pmatrix} 7.35 \cdot 10^{-5} & -9.31 \cdot 10^{-6} \\ \cdot & 1.54 \cdot 10^{-5} \end{pmatrix}$	$\begin{pmatrix} 1.98 \cdot 10^{-5} & -2.561 \cdot 10^{-6} \\ \cdot & 4.91 \cdot 10^{-6} \end{pmatrix}$

Table 46: Stochastic Trend: Multivariate Analysis Results

	1948:1-2006:4				1948:1-1983:4				1984:1-2006:4				$\sigma_{post}/\sigma_{pre}$	
	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	$\%\sigma_{data}$	ρ_{data}	$\%\sigma_{model}$	ρ_{model}	Data	Model
c	0.92	0.78	0.68	0.95	1.07	0.78	0.78	0.96	0.62	0.81	0.41	0.95	0.58	0.53
i	5.44	0.91	2.17	0.91	6.40	0.91	2.47	0.91	3.42	0.92	1.34	0.90	0.53	0.54
y	1.73	1	1.05	1	2.07	1	1.22	1	0.99	1	0.63	1	0.48	0.52
k	0.59	0.36	0.22	0.32	0.68	0.39	0.25	0.31	0.44	0.27	0.13	0.32	0.65	0.52
h	1.88	0.87	0.23	0.89	2.11	0.87	0.26	0.90	1.46	0.90	0.14	0.88	0.69	0.54
y/h	0.94	0.11	0.86	0.99	1.06	0.23	0.99	0.99	0.72	-0.46	0.51	0.99	0.68	0.52

Comparing the previous table with table 15 we conclude that the multivariate specification implies even lower volatilities for all the variables at hand for all the periods. The performance in terms of replicating the magnitude of the Great Moderation, however, does not change significantly.

From this analysis, we conclude that there is no a significative gain from using a multivariate specification for the innovations.