Technology Shocks, Statistical Models, and The Great Moderation[1]

Cristina Fuentes-Albero
SAE, Granada

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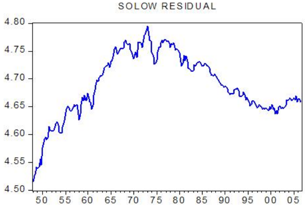
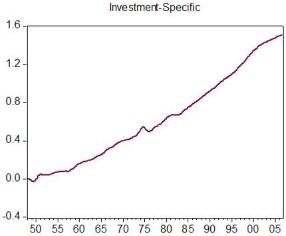
How should we model the stochastic process governing technological change?





- Trend-stationary vs Difference-stationary

Should we care about this?

- Trends implied by our modeling decision
  - Trend-stationary technology progress implies a deterministic trend (real variables are trend-stationary)
  - Difference-stationary technology progress implies a stochastic trend (growth rates of real variables are stationary)
- Persistence of technological improvements
  - Transitory versus permanent

How to select the statistical model?

- Unit Root Test
    - Lack of power
- Posterior Odds
- Analyzing the performance of a calibrated model under different stochastic specifications
    - The specification that better accounts for volatilities, correlations, and the Great Moderation.
    - I will also analyze the relative explicative power of the two technology shocks at hand.

The Model

- It is a simplified version of the one by Greenwood, Hercowitz, and Krusell (2000).
- There is a continuum of households solving

$$\begin{aligned} \max \quad & \boldsymbol{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(lnC_t - B \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right] \\ \text{s.t.} \quad & C_t + P_t^k X_t = W_t H_t + R_t^k P_t^k K_t \\ & (1+\eta) K_{t+1} = (1-\delta) K_t + X_t \end{aligned}$$

- There is a continuum of firms solving

$$\begin{aligned} \max \quad & \Pi_t = C_t + P_t^k X_t - W_t H_t - R_t P_t^k K_t \\ \text{s.t.} \quad & C_t + \frac{X_t}{V_t} = A_t K_t^{\alpha} H_t^{1-\alpha} \end{aligned}$$

Note that $I_t = P_t^k X_t = X_t / V_t$

Technological change

- $A_t = \frac{Y_t}{K_t^{\alpha} H_t^{1-\alpha}}$ represents the aggregate, sector-neutral, productivity level. It is represented by the Solow residual.
    - Capital stock series is in efficiency units (quality-adjusted)

- $V_t = 1/P_t^k$ represents the investment-specific technology level. P_t^k stands for the relative price of investment goods.

1. Linear trend with stationary errors

$$\begin{aligned} ln(A_t) &= \varphi_a + \gamma_a t + \varepsilon_{at} \\ ln(V_t) &= \varphi_v + \gamma_v t + \varepsilon_{vt} \end{aligned}$$

with

$$\begin{aligned} \varepsilon_{at} &= \rho_a \varepsilon_{at-1} + \xi_{at} \\ \varepsilon_{vt} &= \rho_{v1} \varepsilon_{vt-1} + \rho_{v2} \varepsilon_{vt-2} + \xi_{vt} \end{aligned}$$

where $\xi_a \sim \mathcal{N}(0, \sigma_a^2)$ and $\xi_v \sim \mathcal{N}(0, \sigma_v^2)$.

2. Random walk with drift

$$
\begin{aligned}
ln(A_t) &= ln(A_{t-1}) + \gamma_a + \varepsilon_{at} \\
&= ln(A_0) + \gamma_a t + \sum_{i=0}^{t} \varepsilon_{at-i} \\
ln(V_t) &= ln(V_{t-1}) + \gamma_v + \varepsilon_{vt} \\
&= ln(V_0) + \gamma_v t + \sum_{i=0}^{t} \varepsilon_{vt-i}
\end{aligned}
$$

where $\xi_a \sim \mathcal{N}(0, \sigma_a^2)$ and $\xi_v \sim \mathcal{N}(0, \sigma_v^2)$.

3. Random walk with drift and moving average component

$$
\begin{aligned}
ln(A_t) &= ln(A_{t-1}) + \gamma_a + \varepsilon_{at} \\
ln(V_t) &= ln(V_{t-1}) + \gamma_v + \varepsilon_{vt}
\end{aligned}
$$

with

$$\varepsilon_{vt} = \rho \varepsilon_{vt-1} + \xi_t$$

where $\xi_a \sim \mathcal{N}(0, \sigma_a^2)$ and $\xi_v \sim \mathcal{N}(0, \sigma_v^2)$.

Statistical Models: some implications

- All the versions of our RBC model present long run growth. In particular,

$$g_j = g_A^{\frac{1}{1-\alpha}} g_V^{\frac{\alpha}{1-\alpha}} \quad j = Y, C, I, (Y/H)$$
$$g_K = g_Y \cdot g_V$$

- Deterministic Trend Model

$$g_j = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v} \qquad g_V = e^{\gamma_v} \qquad g_K = e^{\frac{1}{1-\alpha}(\gamma_a+\gamma_v)}$$

- Stochastic Trend Model

$$g_j = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}} \qquad g_K = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}$$

Deterministic Trend Model

Along a balanced growth path the following variables are stationary:

$$\tilde{Y}_t = \frac{Y_t}{q^t}, \tilde{C}_t = \frac{C_t}{q^t}, \tilde{I}_t = \frac{I_t}{q^t}$$

$$\tilde{W}_t = \frac{W_t}{q^t}, \tilde{K}_t = \frac{K_t}{(qv)^t}, H_t, R_t$$

where

$$q = e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v}, \qquad v = e^{\gamma_v}$$

Stationary equilibrium conditions

$$\begin{aligned}
\tilde{Y}_t &= \tilde{C}_t + \tilde{I}_t \\
\tilde{Y}_t &= A_0 e^{\varepsilon_{at}} \tilde{K}_t^{\alpha} H_t^{1-\alpha} \\
(1+\eta) q v \tilde{K}_{t+1} &= (1-\delta)\tilde{K}_t + V_0 e^{\varepsilon_{vt}} \tilde{I}_t \\
1 &= \beta E_t \left[\left(\frac{e^{\varepsilon_{vt} - \varepsilon_{vt+1}}}{qv} \right) \left(\frac{\tilde{C}_t}{\tilde{C}_{t+1}} \right) (1 - \delta + R_{t+1}) \right] \\
H_t &= \left(\frac{1}{B} \frac{\tilde{W}_t}{\tilde{C}_t} \right)^{\nu} \\
R_t &= \alpha V_0 e^{\varepsilon_{vt}} \frac{\tilde{Y}_t}{\tilde{K}_t}, \qquad \tilde{W}_t = (1-\alpha) \frac{\tilde{Y}_t}{H_t} \\
q &= e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v}, \qquad v = e^{\gamma_v}
\end{aligned}$$

Stochastic Trend Model

The following variables are stationary:

$$\tilde{Y}_t = \frac{Y_t}{Q_t}, \tilde{C}_t = \frac{C_t}{Q_t}, \tilde{I}_t = \frac{I_t}{Q_t}$$

$$\tilde{W}_t = \frac{W_t}{Q_t}, \tilde{K}_{t+1} = \frac{K_{t+1}}{Q_t V_t}, H_t, R_t$$

where

$$Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$$

Stationary equilibrium conditions

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t, \qquad \tilde{Y}_t = \left(\frac{1}{q_t v_t}\right)^{\alpha} \tilde{K}_t^{\alpha} H_t^{1-\alpha}$$

$$(1+\eta)\tilde{K}_{t+1} = (1-\delta)\left(\frac{1}{q_t v_t}\right)\tilde{K}_t + \tilde{I}_t$$

$$1 = \beta E_t\left[\left(\frac{1}{q_{t+1} v_{t+1}}\right)\left(\frac{\tilde{C}_t}{\tilde{C}_{t+1}}\right)(1-\delta+R_{t+1})\right]$$

$$H_t = \left(\frac{1}{B}\frac{\tilde{W}_t}{\tilde{C}_t}\right)^{\nu}$$

$$R_t = \alpha(q_t v_t)\frac{\tilde{Y}_t}{\tilde{K}_t}$$

$$\tilde{W}_t = (1-\alpha)\frac{\tilde{Y}_t}{H_t}$$

$$q_t = Q_t/Q_{t-1}, \qquad v_t = V_t/V_{t-1}, \qquad Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$$

Some words on the database

- V. Ríos-Rull, F. Schorfheide, C. Fuentes-Albero, M. Kryshko, and R. Santaulàlia-Llopis (2007)
- Data from NIPA, FAT, BLS, and Cummins and Violante (2002)
- Sample 1948:1-2006:4
- They construct the following series:
  - Relative price of investment
  - Quality-adjusted investment and capital

Calibration Strategy

- A subset of the parameters can be directly estimated from the data
- The remaining parameters are calibrated using the following targets

Table: Calibration Targets

	1948:1-2006:4	1948:1-1983:4	1984:1-2006:4
H^*	0.31	0.31	0.31
Y^*	1	1	1
$(\frac{K/V}{Y})^*$	10.288	10.502	9.953
$(\frac{X}{K})^*$	0.0277	0.0276	0.0279
$(\frac{I}{Y})^*$	0.28	0.29	0.28

Calibration Strategy: A Problem

- Recall

$$U(C_t, H_t) = lnC_t - B\frac{H_t^{1+1/\nu}}{1+1/\nu}$$

- We cannot pin down both ν and B.
  - We will perform our analysis for a grid $\nu = \{0.5, 1, 1.5, 2\}$
  - Conditional on ν, we can obtain B from the labor supply equation evaluated at the steady-state.

Some on results

- All models are able to account for some relevant features of US business cycles irrespective of the choice for ν
  - large fluctuations of investment compared to output
  - small fluctuations of capital and consumption compared to output

- Volatility of investment, output, capital, and hours is increasing with ν.

- Volatility of hours implied by the model is smaller than the volatility of labor productivity which is at odds with the data.

- All models generate a large correlation between labor productivity and output that is at odds with the data.

Results for $\nu = 1$

Table: $\nu = 1$: 1948:1-2006:4

	$\sigma_{model}/\sigma_{data}$			ρ_{model}/ρ_{data}		
	DT	ST	ST-MA	DT	ST	ST-MA
c	0.96	0.75	0.76	0.33	1.12	1.09
x	0.66	0.53	0.48	1	0.93	0.96
y	0.73	0.64	0.64	1	1	1
k	0.92	0.51	0.49	0.92	0.92	0.92
h	0.36	0.16	0.16	0.90	0.95	0.93
y/h	0.91	0.93	0.93	8.70	8.91	8.91

Variance Decomposition

Table: Whole sample: $\nu = 1$

	DT		ST		ST-MA	
	A	V	A	V	A	V
c	26	74	86	14	84	16
x	54	46	65	35	67	33
y	91	9	99	1	96	4
k	25	75	67	33	63	37
h	35	65	68	32	60	40
y/h	91	9	99	1	97	3

Table: Correlation - whole sample($nu = 1$)

	Data	DT	ST	ST-MA
c_x	0.61	-0.21	0.53	0.50
c_k	0.25	0.15	0.34	0.31
c_h	0.70	-0.41	0.50	0.40
c_lbp	0.03	0.69	0.96	0.94
x_k	0.21	0.28	0.24	0.24
x_h	0.81	0.98	0.99	0.98
x_lbp	0.03	0.56	0.75	0.75
k_h	0.54	0.22	0.32	0.20
k_lbp	-0.41	0.33	0.32	0.32
h_lbp	-0.40	0.38	0.73	0.67

The Great Moderation: Empirical Evidence

	1950-2006		Pre-1984		Post-1984		Post/Pre	
	$\%\sigma_x$	ρ	$\%\sigma_x$	ρ	$\%\sigma_x$	ρ	$\%\sigma_x$	ρ
GNP	1.73	1	2.07	1	0.99	1	0.48	1
C	0.92	0.78	1.07	0.78	0.62	0.82	0.58	1.05
X(efficiency units)	5.77	0.89	6.85	0.89	3.47	0.92	0.51	1.03
I(consumption units)	5.44	0.91	6.40	0.91	3.42	0.92	0.53	1.01
K(efficiency units)	0.59	0.36	0.68	0.39	0.44	0.27	0.65	0.69
H	1.88	0.87	2.11	0.87	1.46	0.90	0.69	1.03
LBP	0.94	0.11	1.06	0.23	0.72	-0.46	0.68	-2
A	0.94	0.65	1.14	0.71	0.48	0.21	0.42	0.30
V	1.09	0.17	1.34	0.18	0.48	0.12	0.36	0.67

Source:Ríos-Rull et al. (2007) Data Set. We have HP-filtered the log of real variables. Standard deviations are in percentage terms.

Model implied slowdown

	$\sigma_{post}/\sigma_{pre}$			
	Data	DT	ST	ST-MA
c	0.58	0.36	0.51	0.49
x	0.53	0.37	0.54	0.48
y	0.48	0.52	0.54	0.52
k	0.65	0.36	0.58	0.47
h	0.69	0.42	0.55	0.43
y/h	0.68	0.44	0.53	0.53

Innovations to Technological Change

Table: Innovations Slowdown

	DT Post/Pre	ST Post/Pre	ST-MA Post/Pre
N-Shock	0.52	0.52	0.52
I-Shock	0.62	0.50	0.43

- We have allowed parameter variation across subsamples.
- To quantify the relative importance of technology shocks as explicative source of the volatility slowdown of real variables we will consider some counterfactuals.

Counterfactuals

- Arias, Hansen, and Ohanian (ET, 2007).
- Fix model parameters to match the targets for the whole sample.
- Counterfactuals:
  1. Set parameters of the laws of motion for technological change equal to the ones estimated considering the whole sample.
     - Variance Decomposition for each subsample
     - Set the standard deviation of one of the innovations to match its volatility for the entire sample and let the other innovation to have subsample-specific volatility.
     - Let both innovation volatilities to vary across subsamples
  2. Let parameters of the laws of motion vary over time.

Counterfactual 1

- σ_v is set to match its volatility for the entire sample

Table: Experiment 1

	$\sigma_{post}/\sigma_{pre}$ Data	DT	ST	ST-MA
c	0.58	0.88	0.59	0.57
x	0.51	0.73	0.70	0.66
y	0.48	0.57	0.54	0.51
k	0.65	0.86	0.71	0.68
h	0.69	0.83	0.69	0.71
y/h	0.68	0.56	0.54	0.50

Counterfactual 2

- σ_a is set to match its volatility for the entire sample

Table: Experiment 2

	$\sigma_{post}/\sigma_{pre}$			
	Data	DT	ST	ST-MA
c	0.58	0.74	0.93	0.94
x	0.51	0.86	0.83	0.86
y	0.48	0.96	0.99	1.01
k	0.65	0.78	0.83	0.81
h	0.69	0.78	0.84	0.79
y/h	0.68	0.95	0.98	1.01

Counterfactual 3

- σ_a and σ_v vary across subsamples

Table: Experiment 3

	$\sigma_{post}/\sigma_{pre}$			
	Data	DT	ST	ST-MA
c	0.58	0.58	0.53	0.50
x	0.51	0.56	0.53	0.49
y	0.48	0.53	0.54	0.50
k	0.65	0.59	0.53	0.47
h	0.69	0.59	0.53	0.49
y/h	0.68	0.53	0.54	0.51

- Fix all the parameters to match the whole sample moments but allow for time-varying laws of motion for technological change.

Table: Experiment 3

	$\sigma_{post}/\sigma_{pre}$			
	Data	DT	ST	ST-MA
c	0.58	0.35	0.56	0.49
x	0.51	0.43	0.39	0.47
y	0.48	0.50	0.56	0.52
k	0.65	0.48	0.60	0.50
h	0.69	0.40	0.53	0.46
y/h	0.68	0.49	0.56	0.52

- Deterministic Trend Models account better for volatilities
- Stochastic Trend Models account better for correlations
- The role of investment-specific technological change in explaining volatilities is larger in deterministic trend models than in stochastic trend ones.
- The Great Moderation
  - If we allow for time-varying coefficients, stochastic trend models outperform the deterministic trend one.
  - If we only allow for time-varying innovation volatilities, all models perform similarly.
  - If we allow for time-varying laws of motion, stochastic trend versions of our model outperform.
  - None of the specifications is able to account for the milder slowdown of hours, labor productivity, and capital.