Self–diffusiophoresis and biological motility

Gareth Alexander

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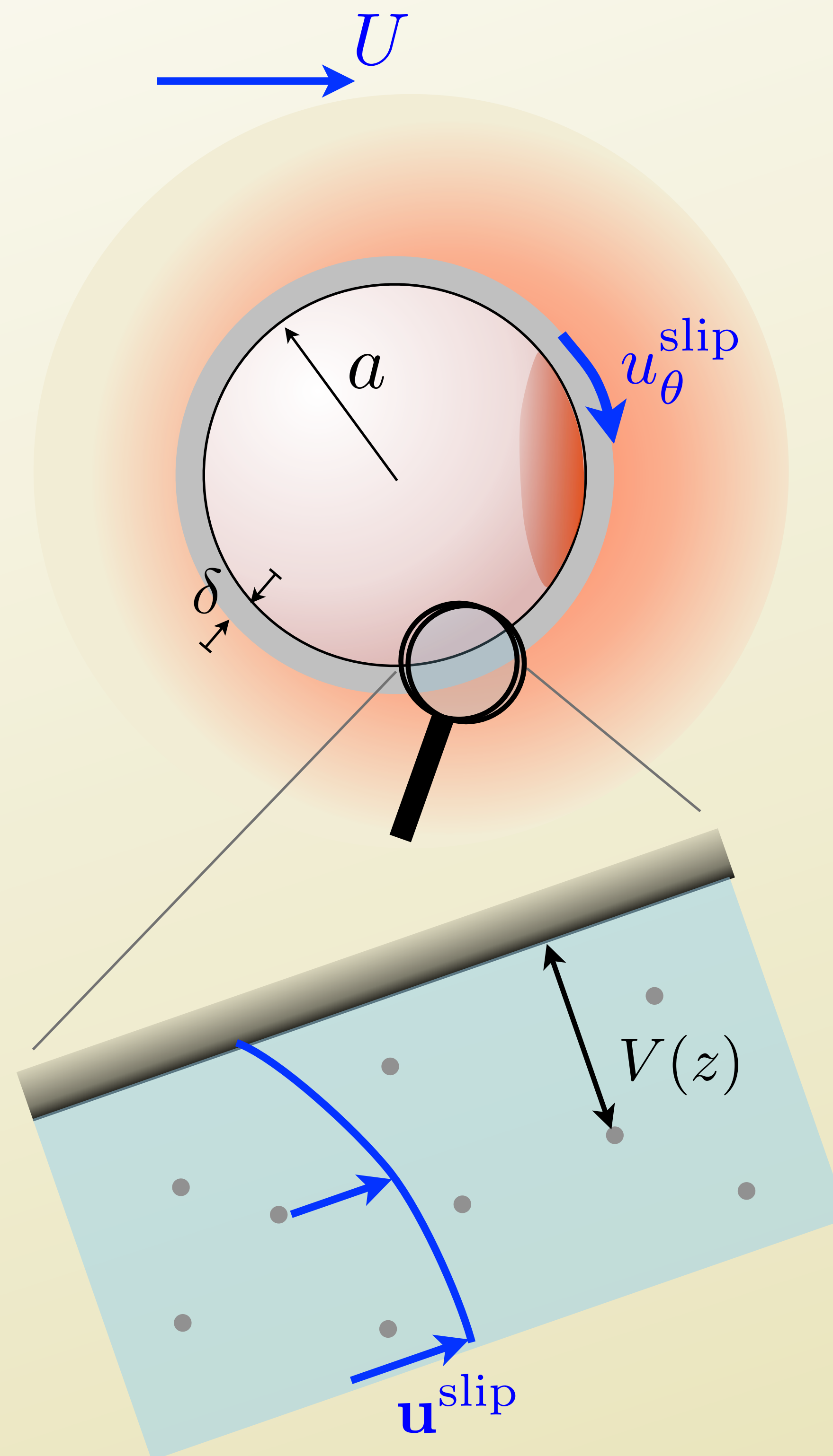

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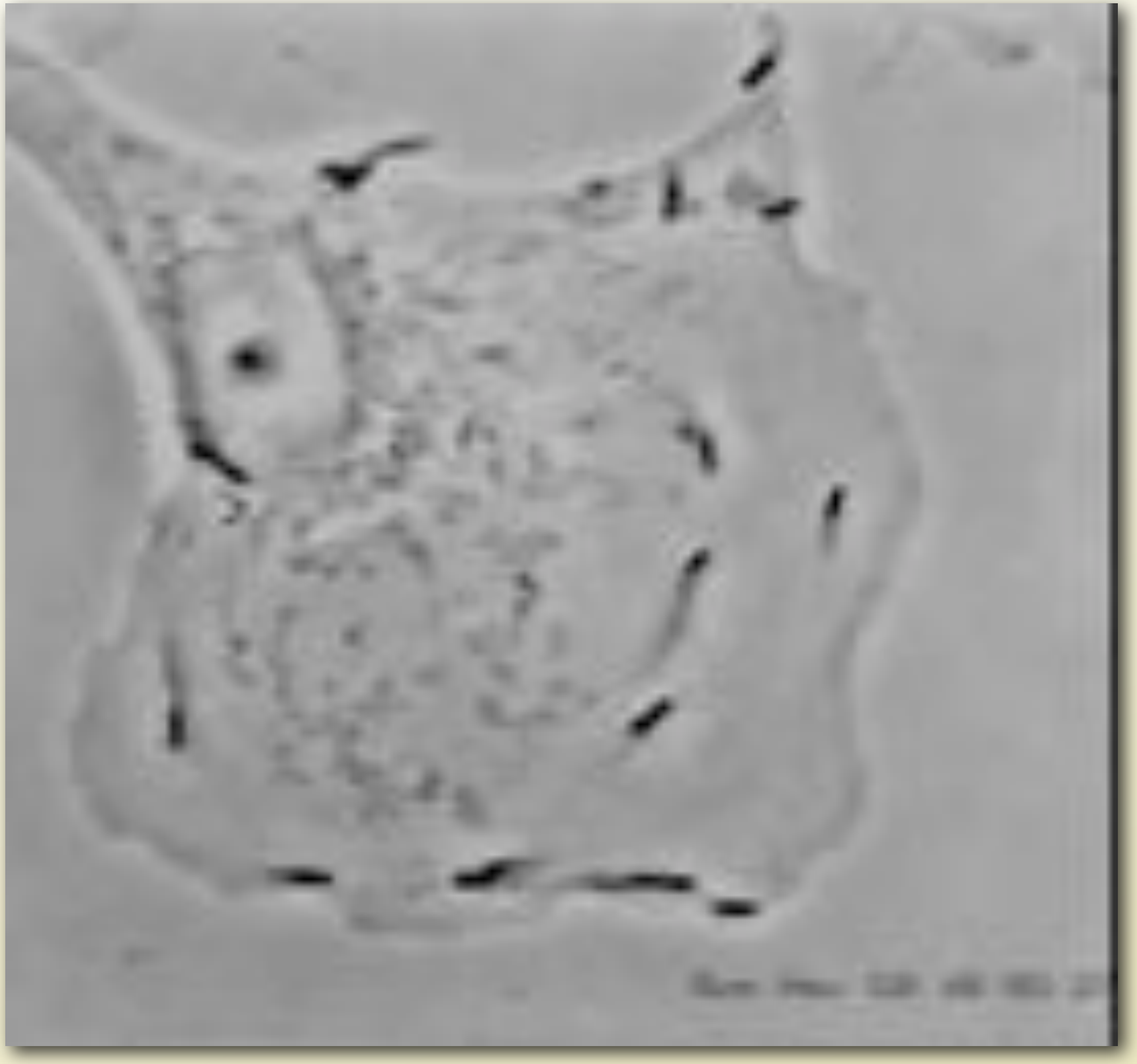
Informal Seminar, University of Oxford, 8th June 2011

ACTIN BASED PROPULSION

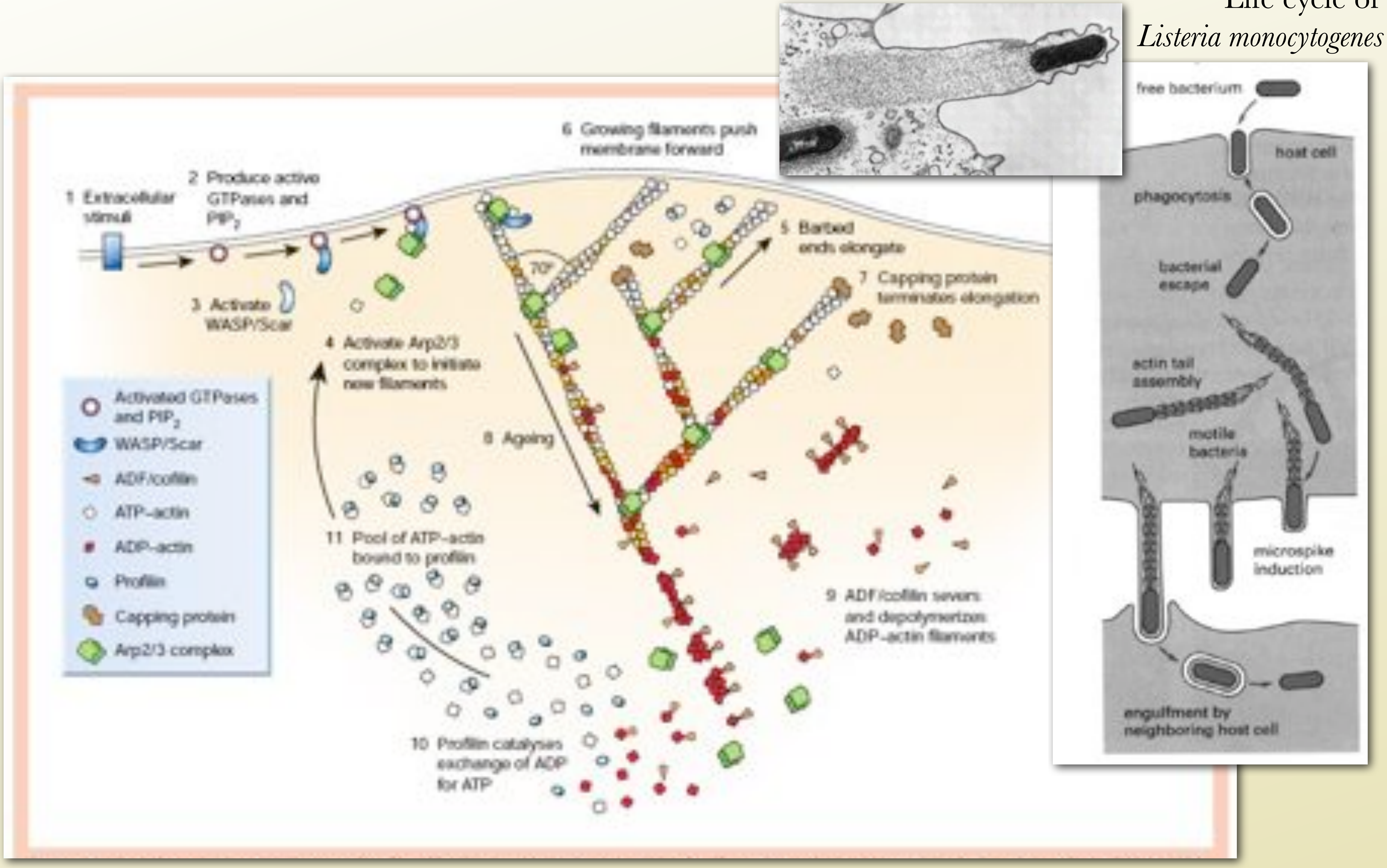
Listeria monocytogenes



Courtesy of Julie Theriot
http://cmgm.stanford.edu/theriot/movies.htm

ACTIN AND LISTERIA MOTILITY

Life cycle of
Listeria monocytogenes



Pollard et al *Ann. Rev. Biophys. Biomol. Struct.* **29**, 545–576 (2000)

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In–vitro realisations

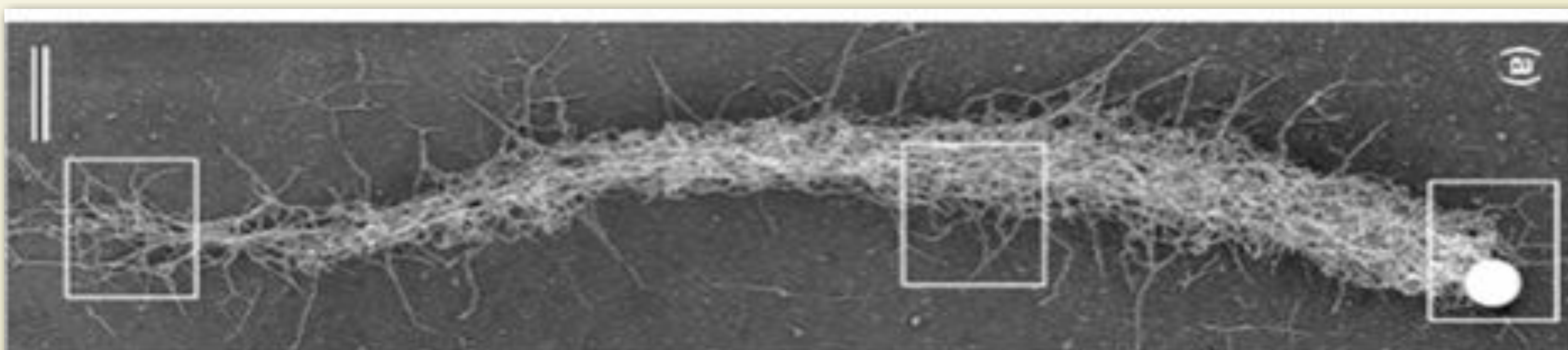
Courtesy of Julie Theriot
http://cmgm.stanford.edu/theriot/movies.htm

$\times 60$

sphere

"All" you need is

- Actin and buffer w/ATP
- Arp2/3 makes new growing ends
- Capping protein kills them off
- ADF/cofilin severs filaments
- Profilin converts ADP-G-actin to ATP-G-actin
- Bead coated with ActA, VCA, activates Arp2/3



Cameron et al *Curr. Biol.* **11**, 130 (2001)

how does self-assembly of actin into a branched structure lead to motility?

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In-vitro realisations

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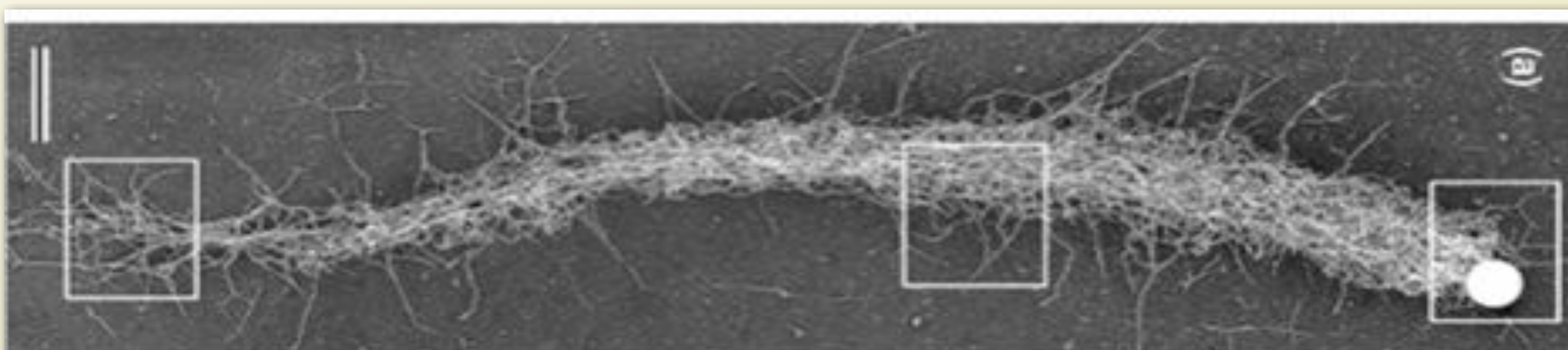
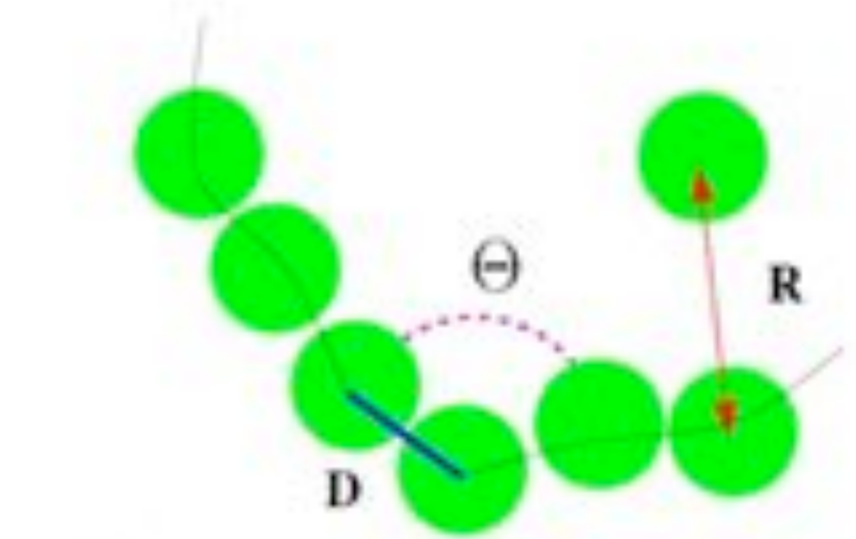


Cameron et al *Curr. Biol.* **11**, 130 (2001)

how does self-assembly of actin into a branched structure lead to motility?

Brownian dynamics simulations



$$\eta \frac{dX}{dt} = -\nabla\Phi_{H} - \nabla\Phi_{S} - \nabla\Phi_{B} + F_R$$

F_R = Random Force

$$\langle F_R \rangle = 0$$

$$\langle F_R(t) F_R(t') \rangle = 6 k_B T \eta \delta(t - t')$$

Φ_H = Hardcore interaction

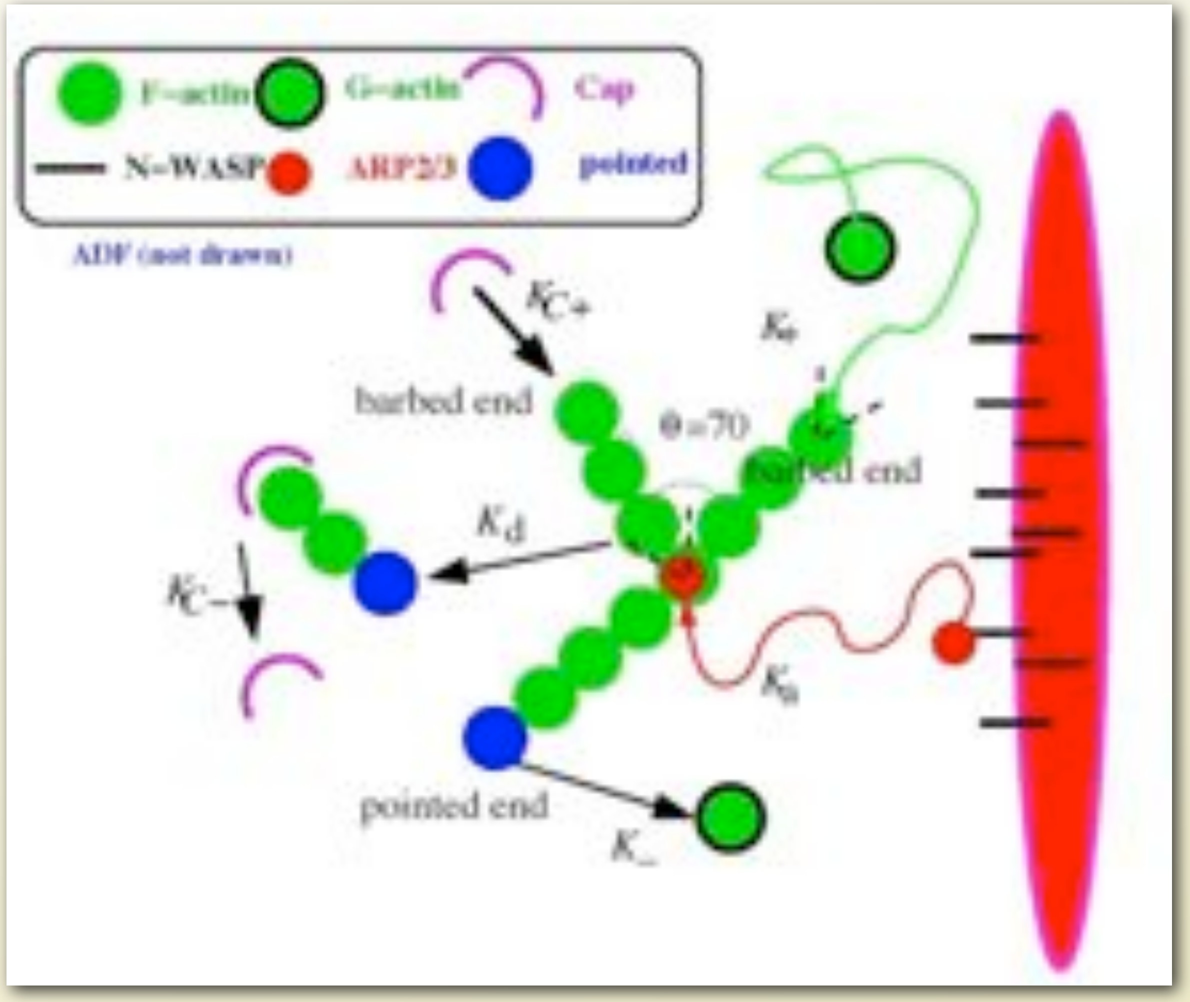
$$\Phi_H = \frac{1}{2} K_h (R - R_0)^2, \quad R < R_0$$

Φ_S = Bond Stretch interaction

$$\Phi_S = \frac{1}{2} K_s (D - D_0)^2$$

Φ_B = Bending interaction

$$\Phi_B = \frac{1}{2} K_b (\cos(\Theta) - \cos(\Theta_0))^2$$



- Polymerisation at + end (K_+)
- Depolymerisation at - end (K_-)
- Branching (K_a)
- Debranching (K_d)
- Capping

Lee & Liu *Biophys. J.* **95**, 4529–4539 (2008)
Lee & Liu *Biophys. J.* **97**, 1295–1304 (2009)

Brownian dynamics simulations

- 2D projection of 3D simulation
- Motion allowed in $\pm z$ direction only

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Lee & Liu *Biophys. J.* **95**, 4529–4539 (2008)
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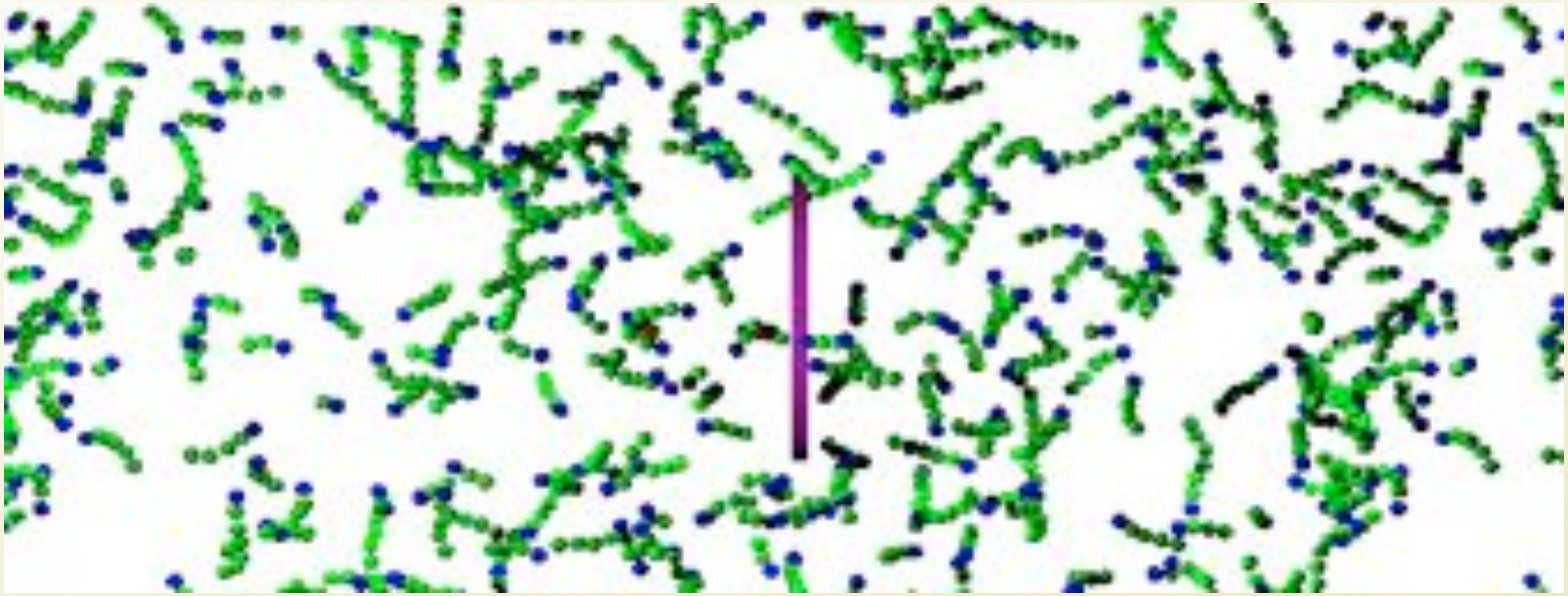
Brownian dynamics simulations



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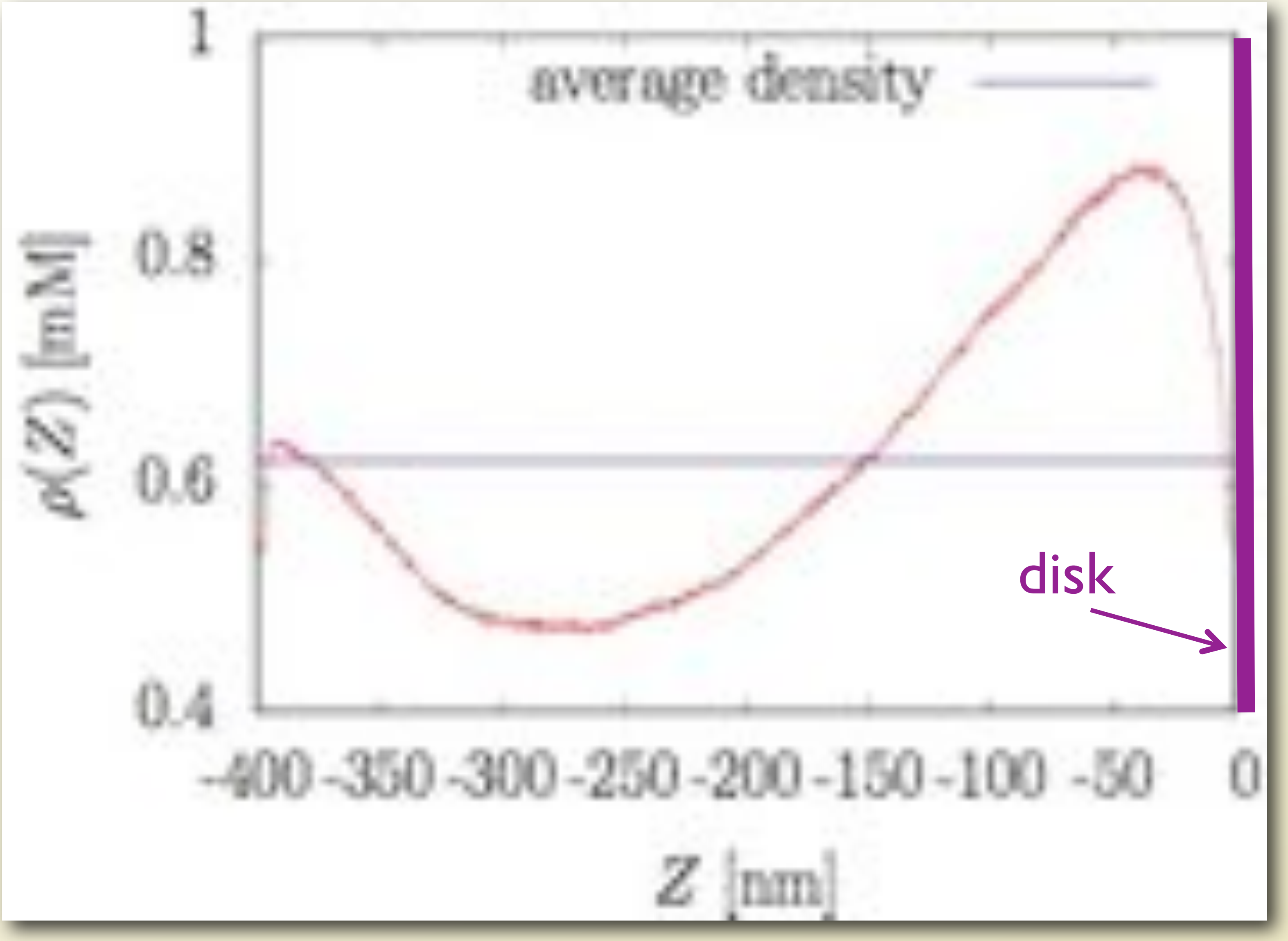
LEE & LIU *Biophys. J.* **95**, 4529–4539 (2008)
LEE & LIU *Biophys. J.* **97**, 1295–1304 (2009)

Actin concentration gradient

Courtesy of J. Theriot



- Disk activates Arp2/3, which recruits F-actin
- Concentration of F-actin is high behind the disk compared to average
- If the disk repels actin then it will move forwards to avoid F-actin
- In real systems the concentration gradient is even bigger; mechanism should still apply

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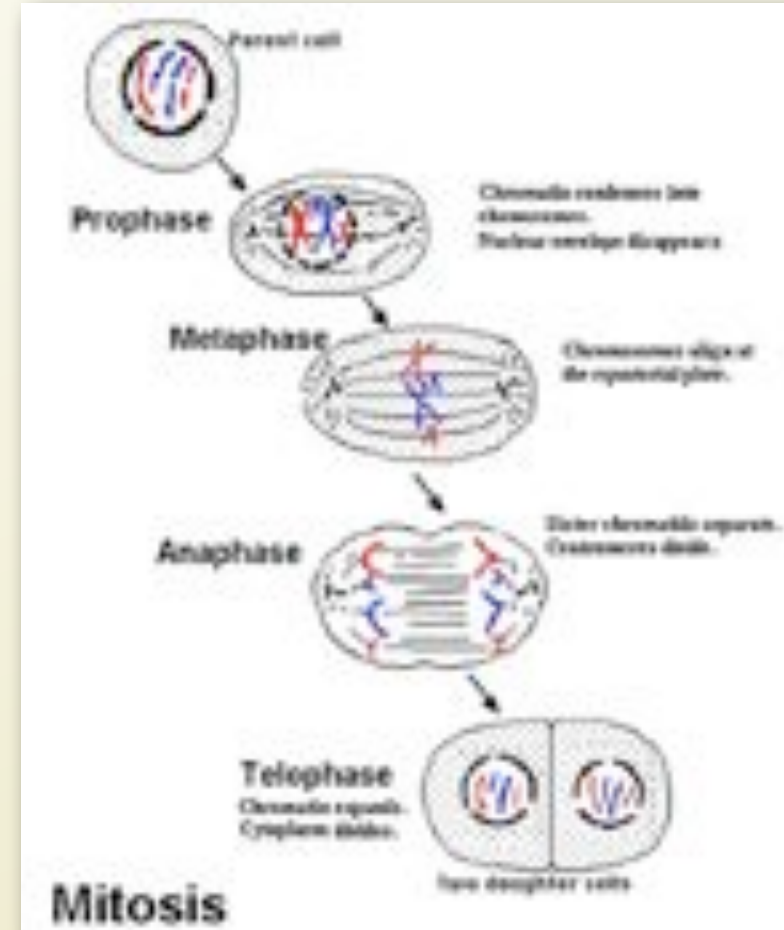
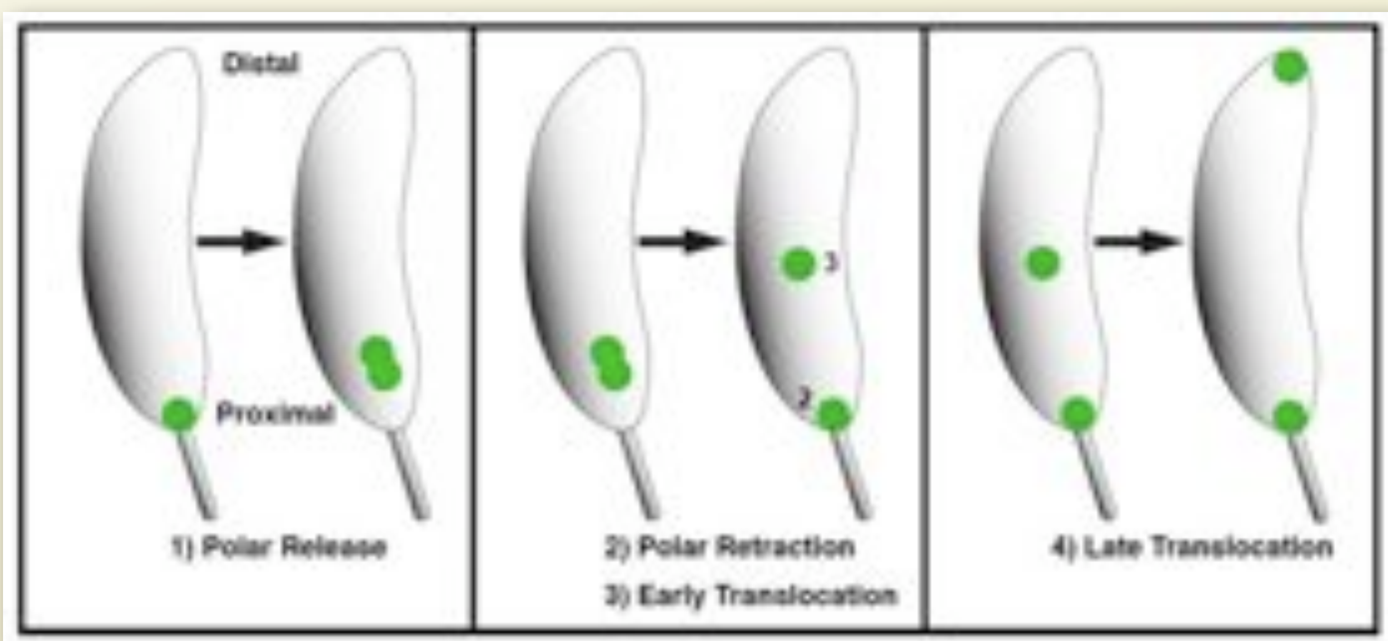
Lee & Liu *Biophys. J.* **95**, 4529–4539 (2008)
Lee & Liu *Biophys. J.* **97**, 1295–1304 (2009)

ASYMMETRIC CELL DIVISION

- Most cells divide symmetrically



Caulobacter crescentus and *Vibrio cholerae* divide asymmetrically



- What is the mechanism for chromosomal motility?

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Disassembly driven motility

- How does the chromosome move across the cell during chromosomal segregation in certain asymmetric bacteria?

Caulobacter crescentus

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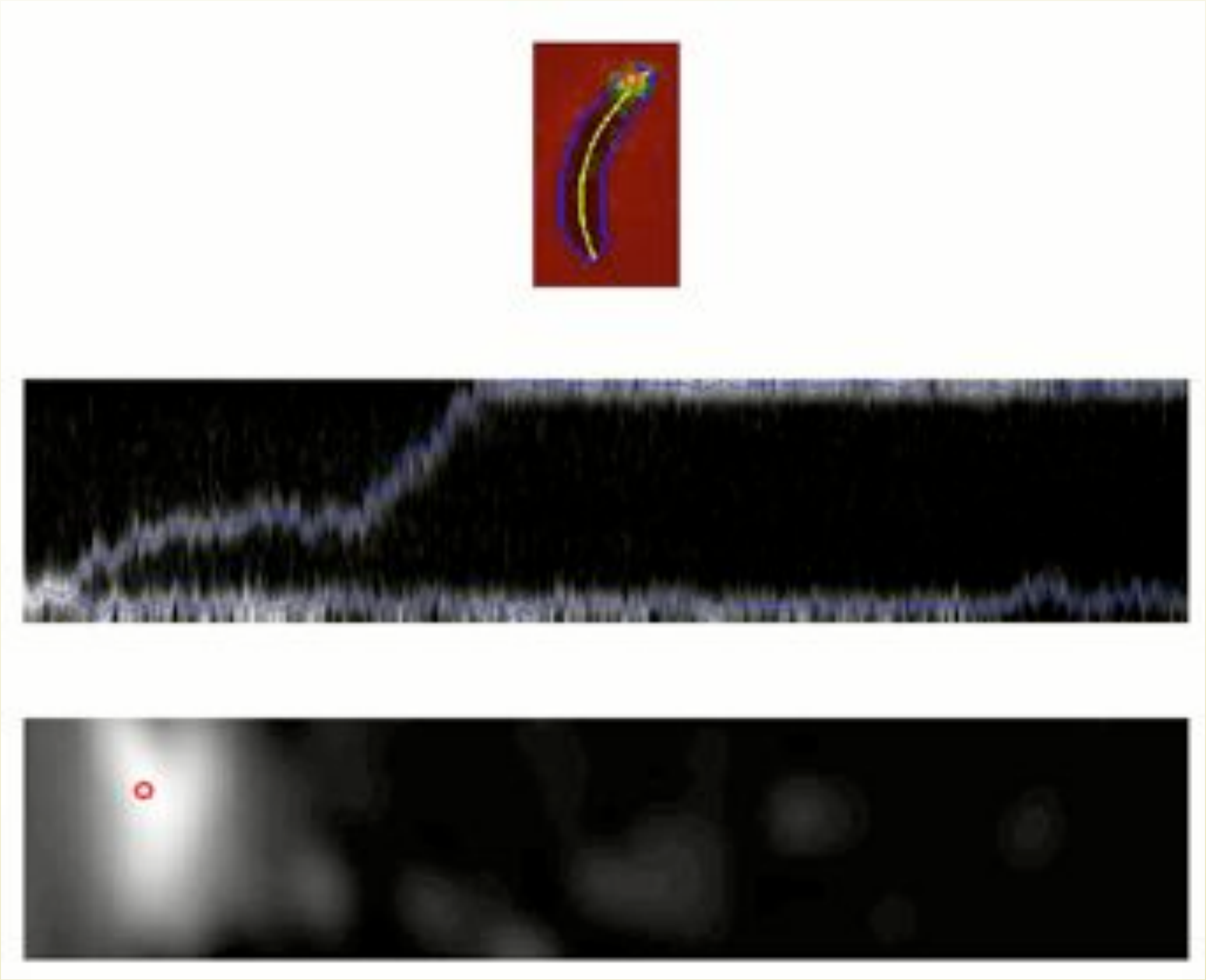
Courtesy of C. W. Shebelut, J. M. Guberman and Z. Gitai

Disassembly driven motility

- How does the chromosome move across the cell during chromosomal segregation in certain asymmetric bacteria?



Caulobacter crescentus

Courtesy of C. W. Shebelut, J. M. Guberman and Z. Gitai

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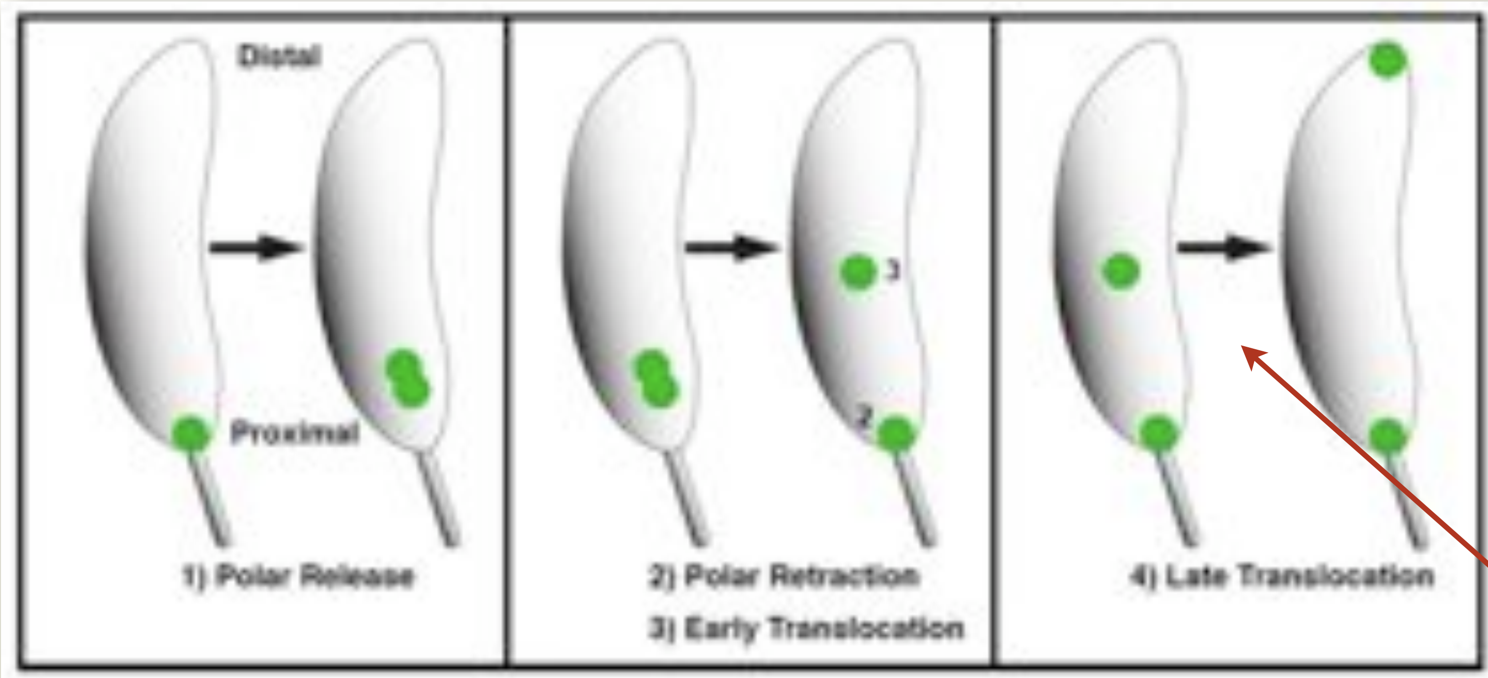
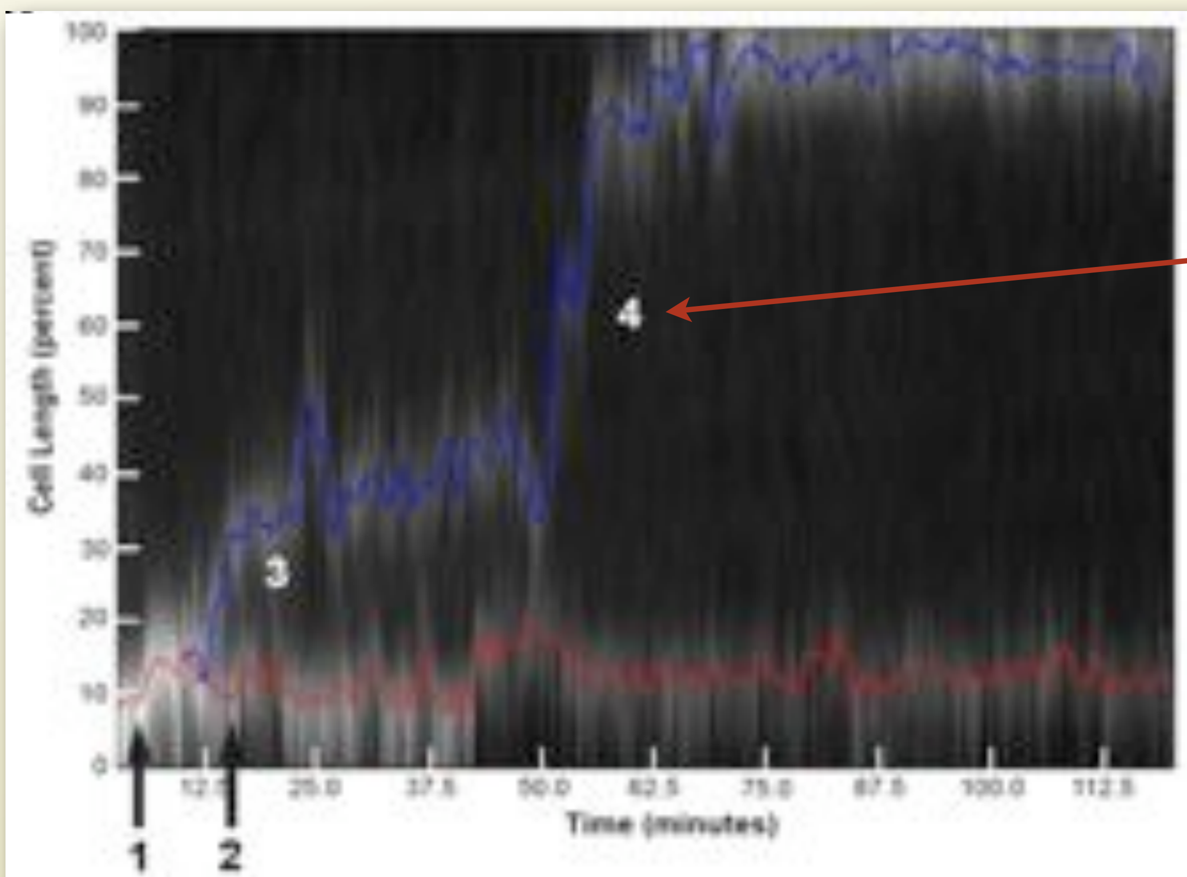
Chromosomal segregation in C. crescentus and V. cholerae

Caulobacter crescentus





We are interested in the 4th stage

How does the chromosome (ori) scoot across the cell?

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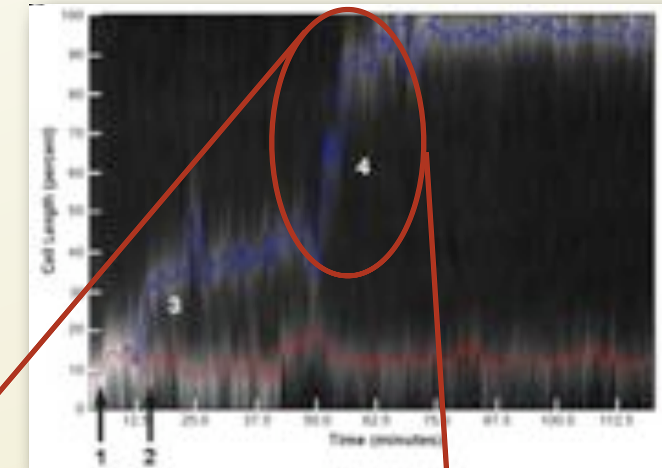
Courtesy of C. W. Shebelut, J. M. Guberman and Z. Gitai

VIBRIO CHOLERAE

Courtesy of Popular Logistics



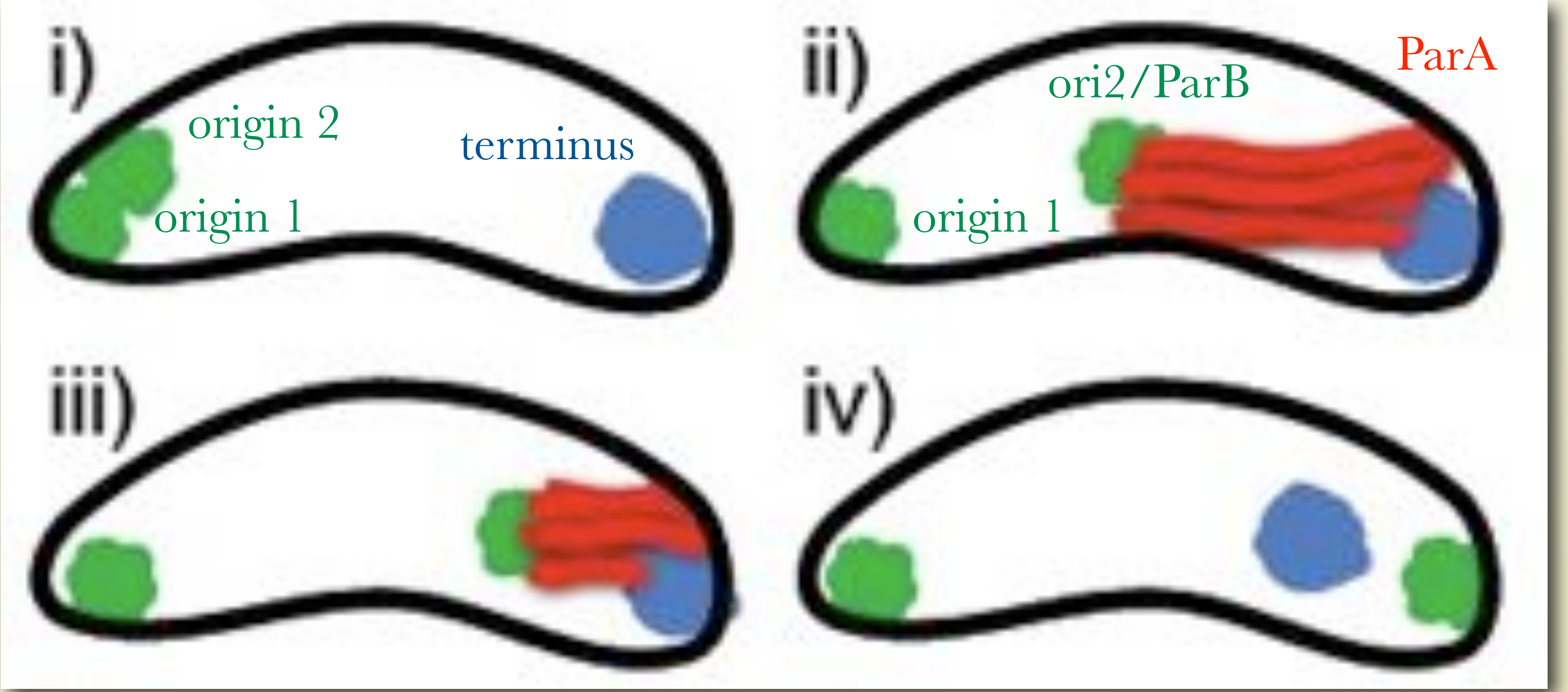
FOGEL & WALDOR *Genes & Dev.* **20**, 3269–3282 (2006)

A closer look at the process

Fogel & Waldor *Genes & Dev.* **20**, 3269–3282 (2006)

Replication

ParB on origin attaches to ParA



ParA disassembles and origin moves

Origin and terminus switch places

- Origin is decorated with ParB which binds to and hydrolyses ParA
- ParA filament structure depolymerises and drags ParB along

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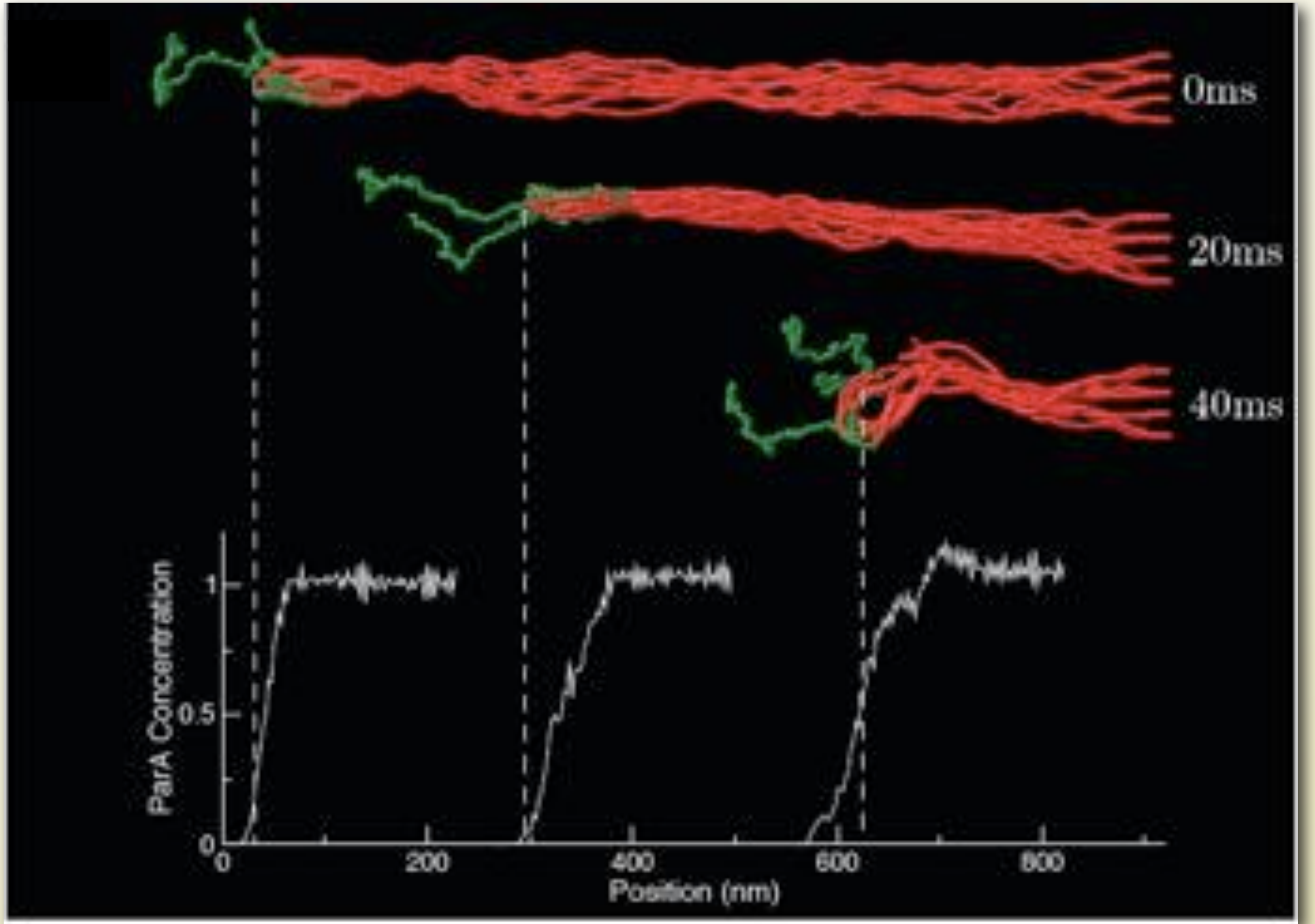
CONCENTRATION GRADIENT DRIVES MOTION



- System uses depolymerisation to create a steady-state concentration gradient to move up

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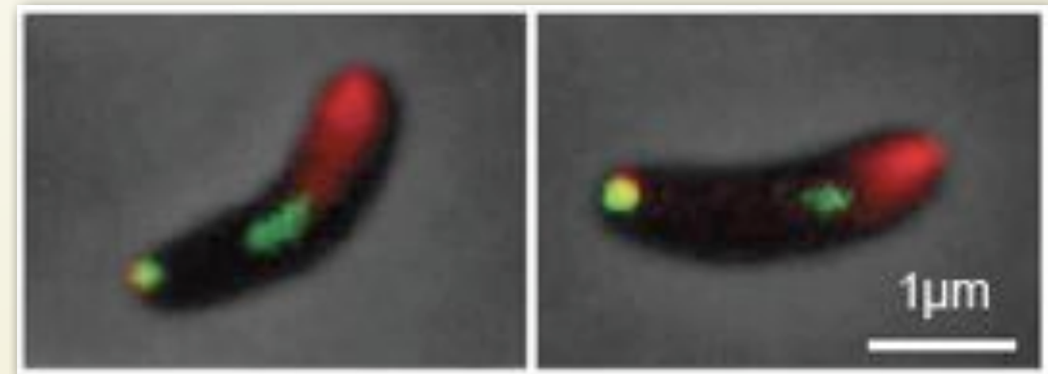
BANIGAN ET AL *submitted* (2011)

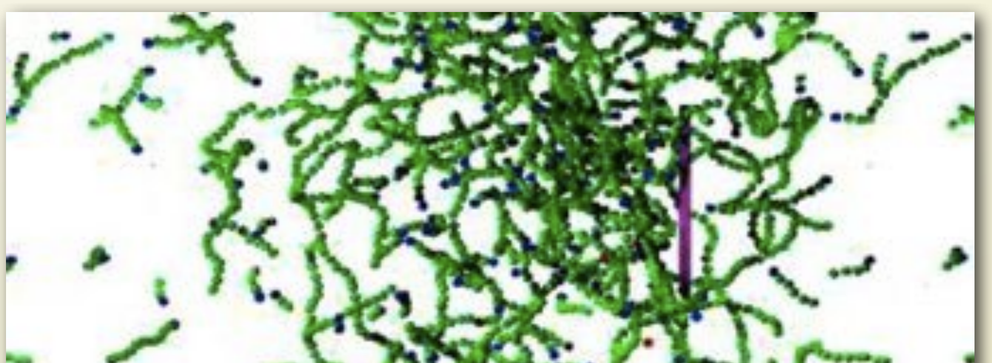
BIOLOGICAL MOTILITY

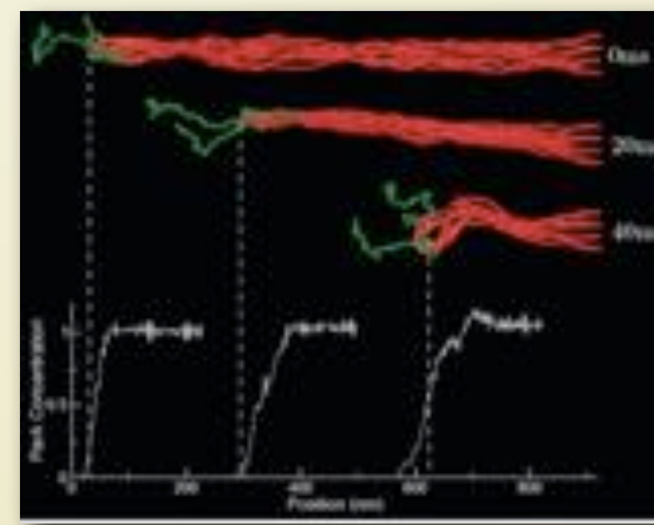
- Two examples of motion involving the assembly or disassembly of filaments

Courtesy of J. Theriot



FOGEL & WALDOR *Genes & Dev.* **20**, 3269–3282 (2006)

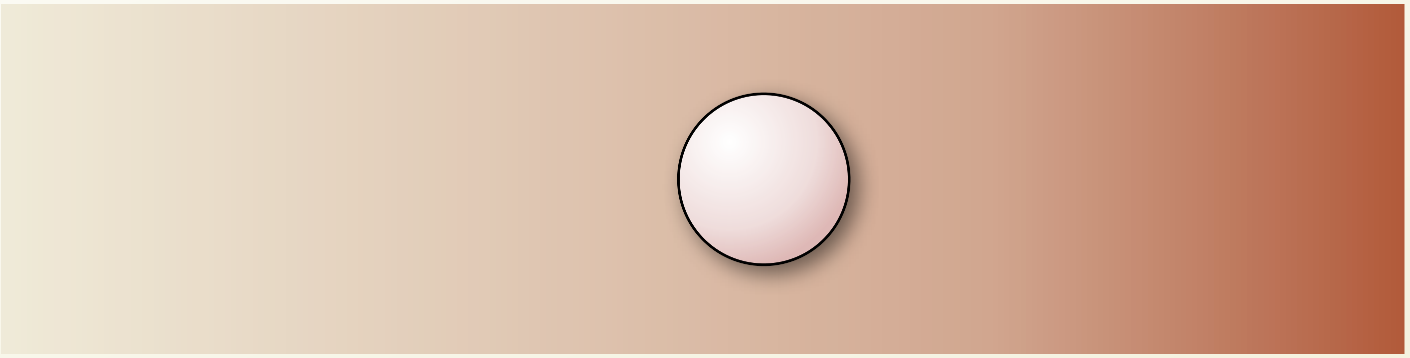




- Simulations replicate interaction with filaments - suggests motion in a filament concentration gradient

but without fluid flow

Particle motion in a concentration gradient



Particle interacts with the concentration field and moves

Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)

Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)

Particle motion in a concentration gradient

Particle interacts with the concentration field and moves down the gradient if it is repelled

Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)

Particle motion in a concentration gradient

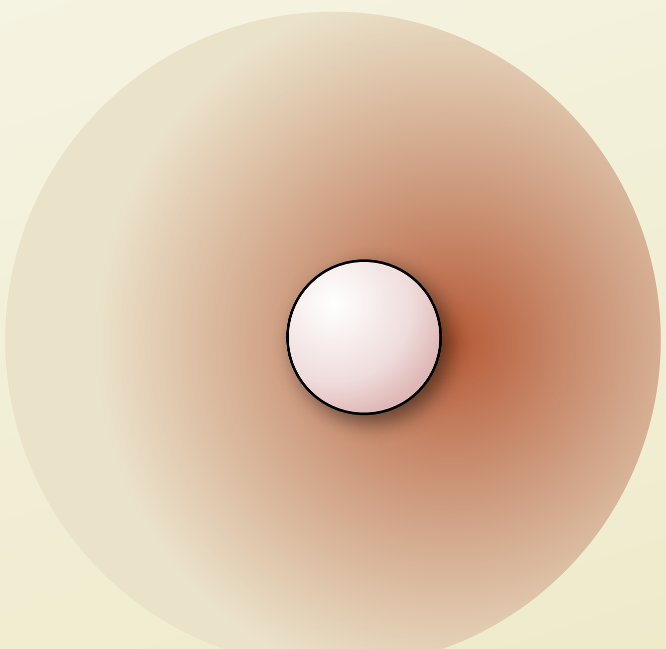
Particle interacts with the concentration field and moves up the gradient if it is attracted

ANDERSON *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
GOLESTANIAN, LIVERPOOL & AJDARI *Phys. Rev. Lett.* **94**, 220801 (2005)

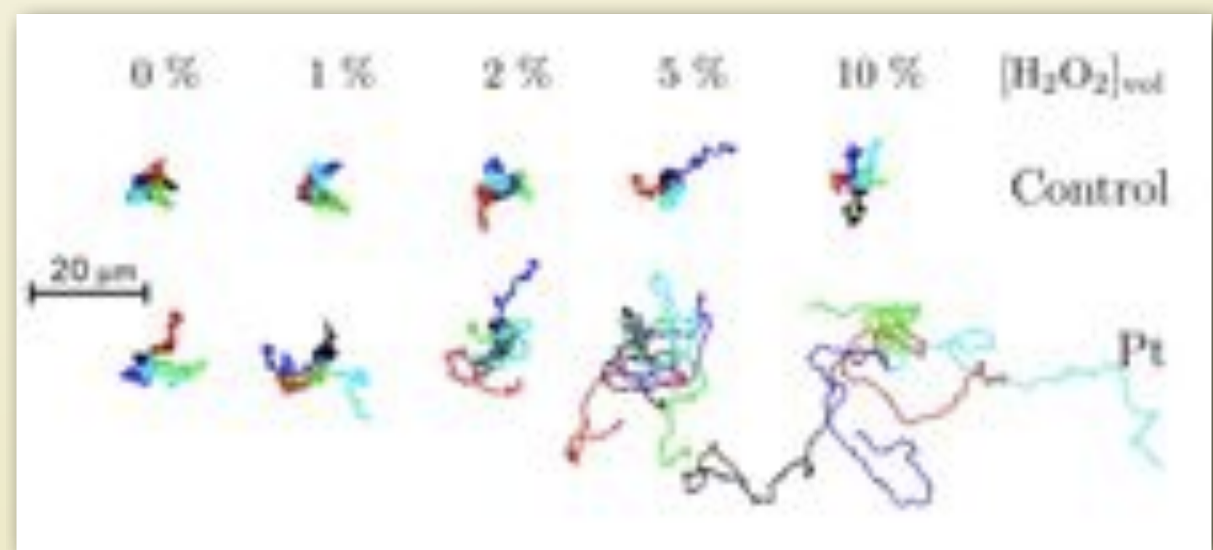
Particle motion in a concentration gradient

Particle interacts with the concentration field and moves up the gradient if it is attracted

In self-diffusiophoresis the particle itself generates the concentration gradient



Example: catalytic decomposition of H_2O_2

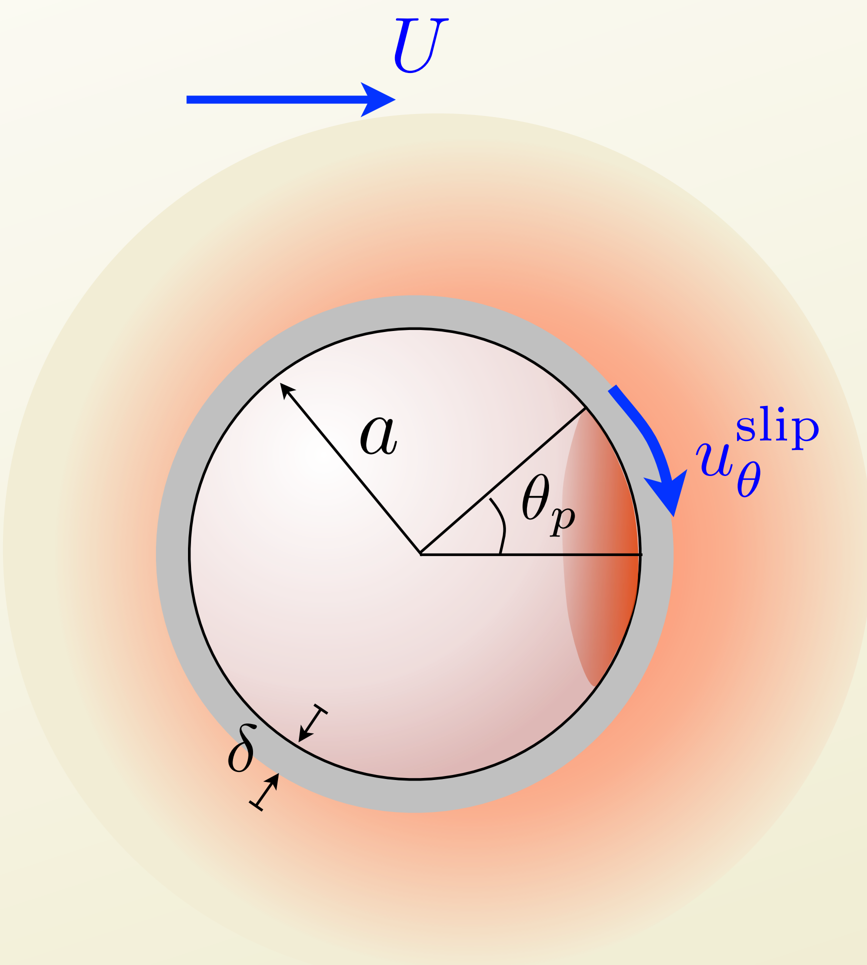


Howse et al *Phys. Rev. Lett.* **99**, 048102 (2007)

Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)

Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)

A PARED DOWN MODEL



Rest frame of the particle

- Spherical particle
- Axisymmetry, steady state
- Single solute species
- Purely radial potential with compact support
- Zero Reynolds number

Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)
Golestanian, Liverpool & Ajdari *New J. Phys.* **9**, 126 (2007)

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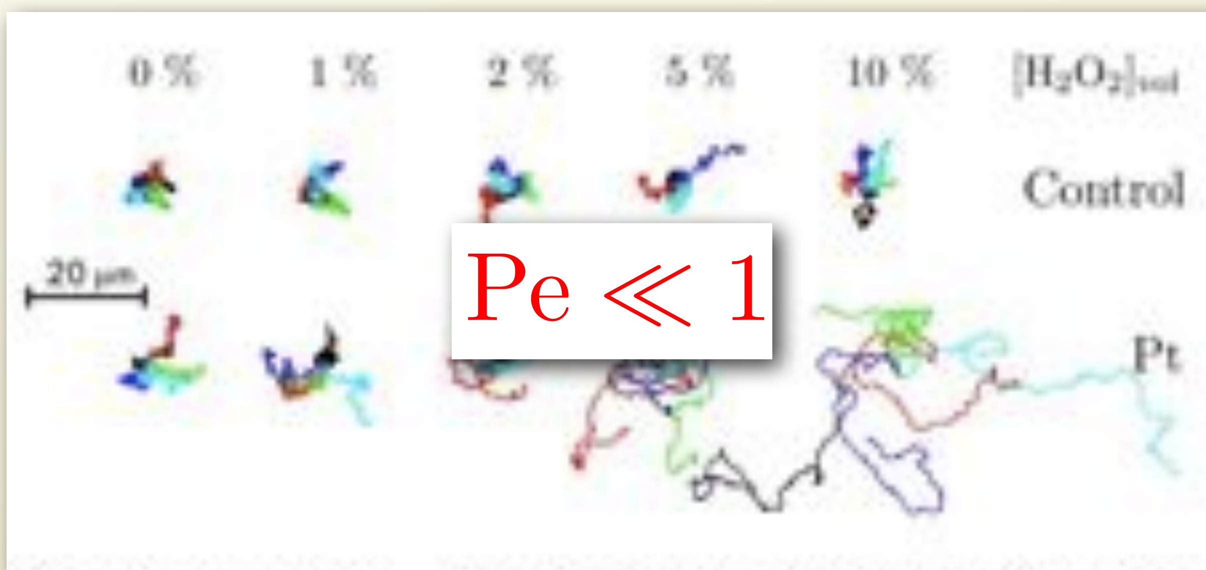
PARTICLE MOTION IN A CONCENTRATION GRADIENT

Motion involves a balance between diffusion and advection

$$\mathrm{Pe} = \frac{\text{advection}}{\text{diffusion}} = \frac{U a}{D}$$

Diffusion dominated

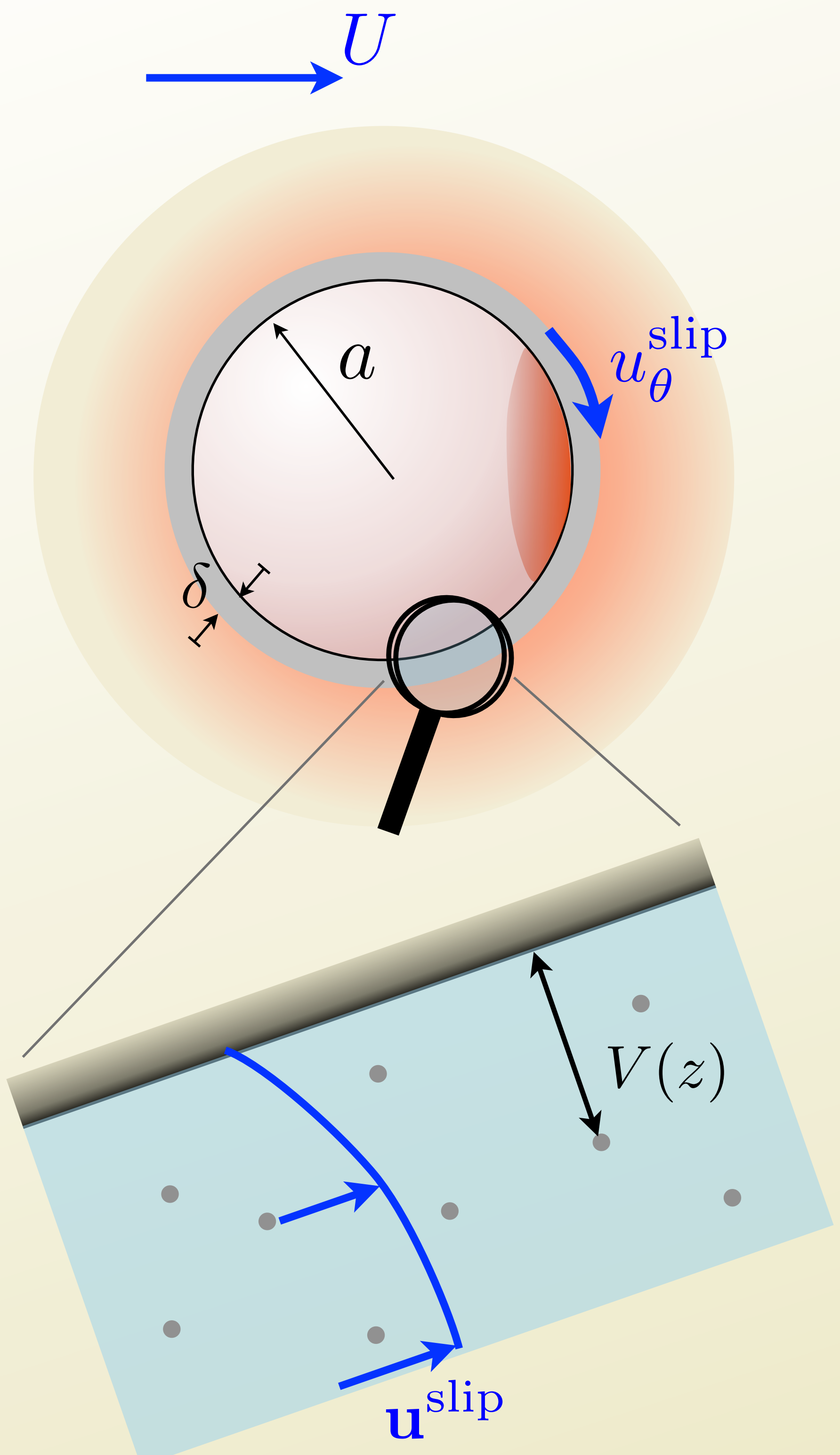


HOWSE ET AL *Phys. Rev. Lett.* **99**, 048102 (2007)

Advection dominated

Courtesy of Julie Theriot
http://cmgm.stanford.edu/theriot/movies.htm

Boundary layer analysis



The interaction occurs close to the surface in a boundary layer of thickness δ

Solve Stokes in the upper half space

$$\mathbf{0} = -\nabla p + \mu \nabla^2 \mathbf{u} - c\,\nabla V \qquad 0 = \nabla \cdot \mathbf{u}$$

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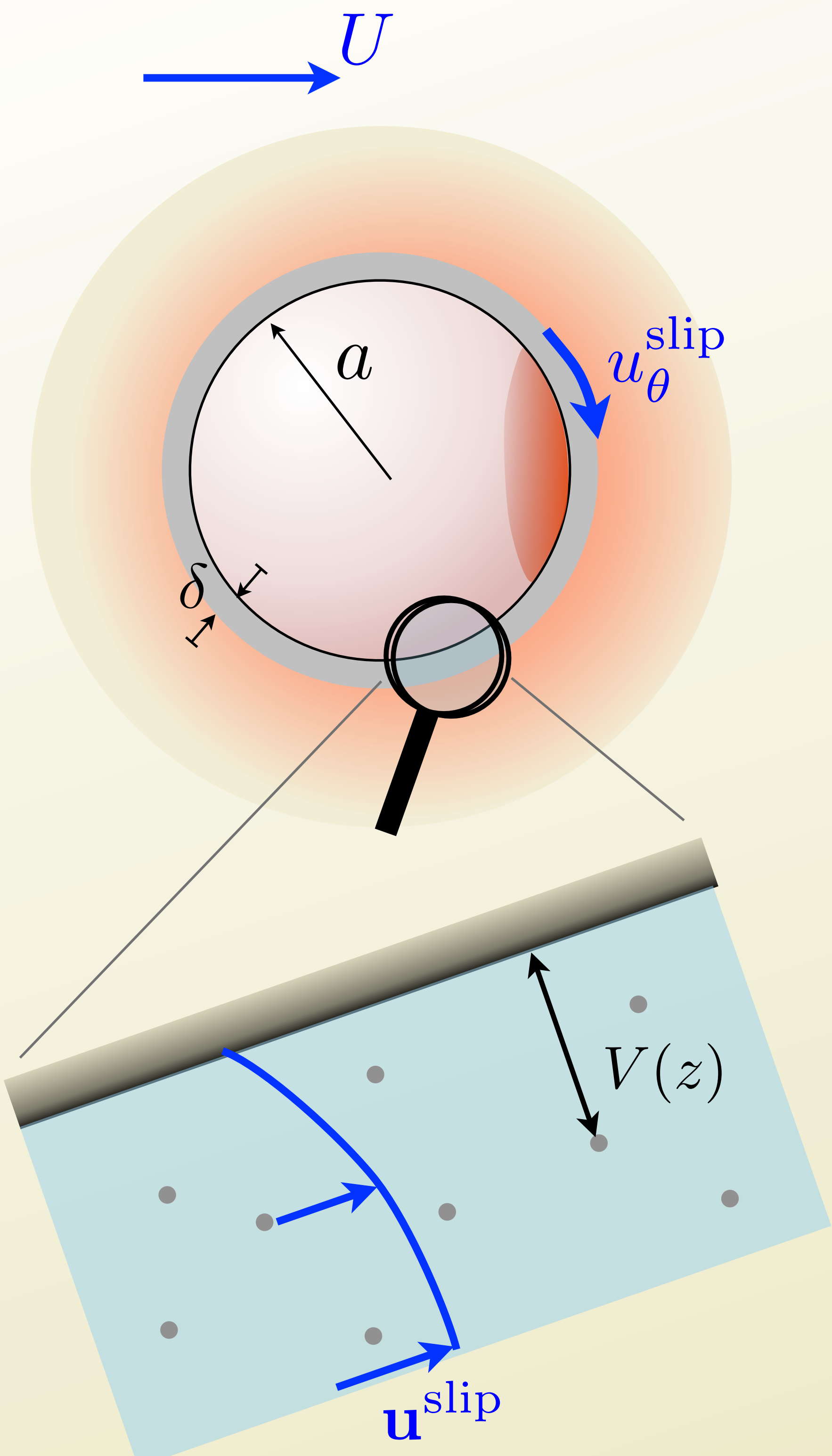
Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
Jülicher, Prost *Eur. Phys. J. E* **29**, 27–36 (2009)

Boundary layer analysis



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Tangential slip velocity

$$\boxed{\mathbf{u}^{\text{slip}} = m^D \nabla_{\|} c\Big|_{z=1}}$$

diffusiophoretic mobility (Derjaguin)

$$m^D = \frac{k_B T}{\mu}\delta^2 \int_0^1 \mathrm{d}z\, z\left[1 - \mathrm{e}^{-V/k_B T}\right]$$

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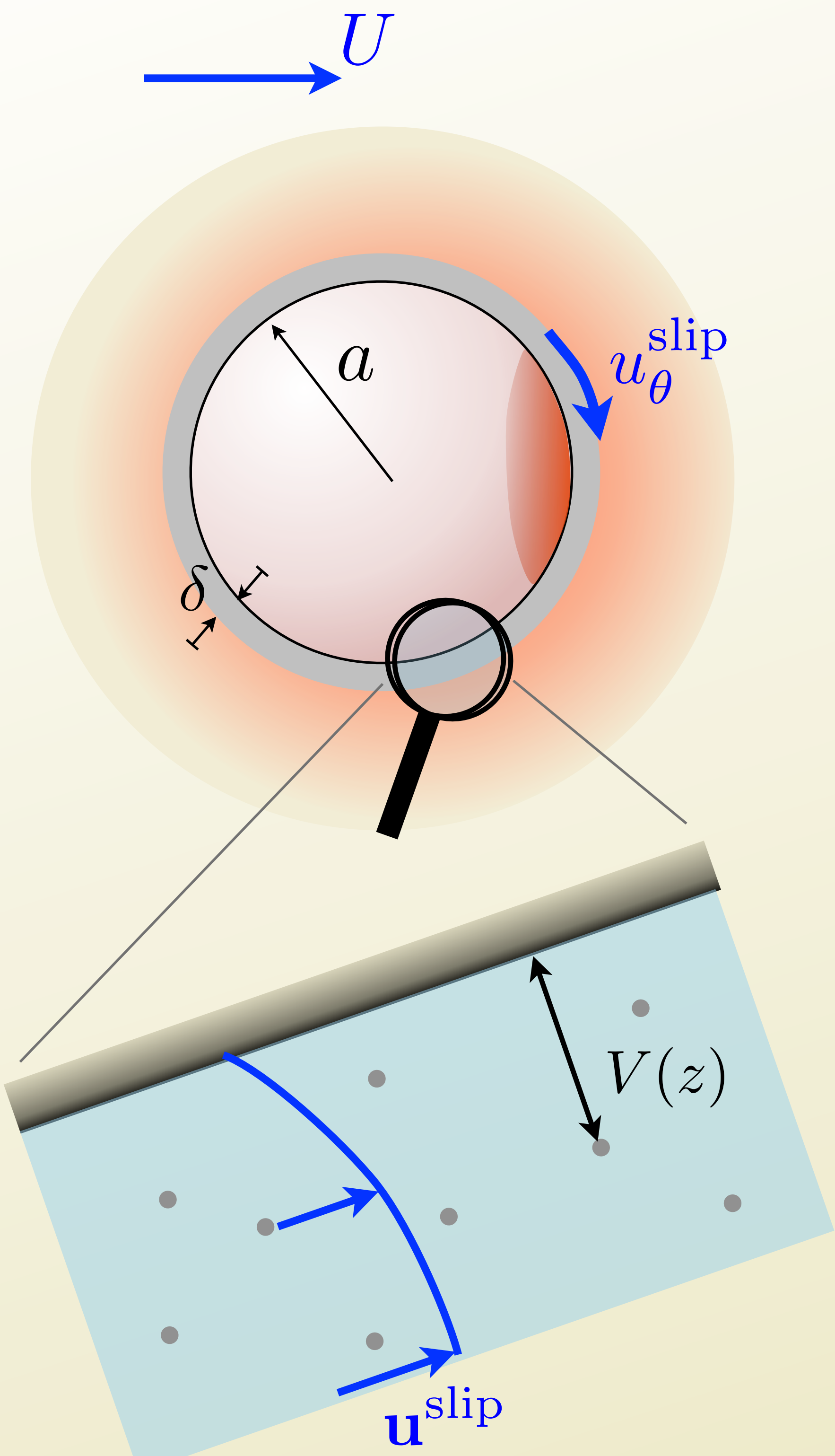
Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
Jülicher, Prost *Eur. Phys. J. E* **29**, 27–36 (2009)

Squirmers



Slip velocity provides an inner boundary condition for the exterior flow

General solution provided by Lighthill's squirmer model

$$u_r = U\left[1 - \left(\tfrac{a}{r}\right)^3\right]\cos(\theta) + \sum_{l=2}^{\infty} B_l\left[\left(\tfrac{a}{r}\right)^l - \left(\tfrac{a}{r}\right)^{l+2}\right]P_l\big(\cos(\theta)\big)$$

$$u_\theta = -U\sin(\theta) + \sum_{l=2}^{\infty} B_l\left[\tfrac{l-2}{2}\left(\tfrac{a}{r}\right)^l - \tfrac{l}{2}\left(\tfrac{a}{r}\right)^{l+2}\right]V_l\big(\cos(\theta)\big)$$

Matching the boundary condition gives the speed

$$\boxed{U = \frac{2m^D}{3a}c_1}$$

first Legendre coefficient

$$c\big|_{r=a+\delta} = \sum_{l=0}^{\infty} c_l P_l\big(\cos(\theta)\big)$$

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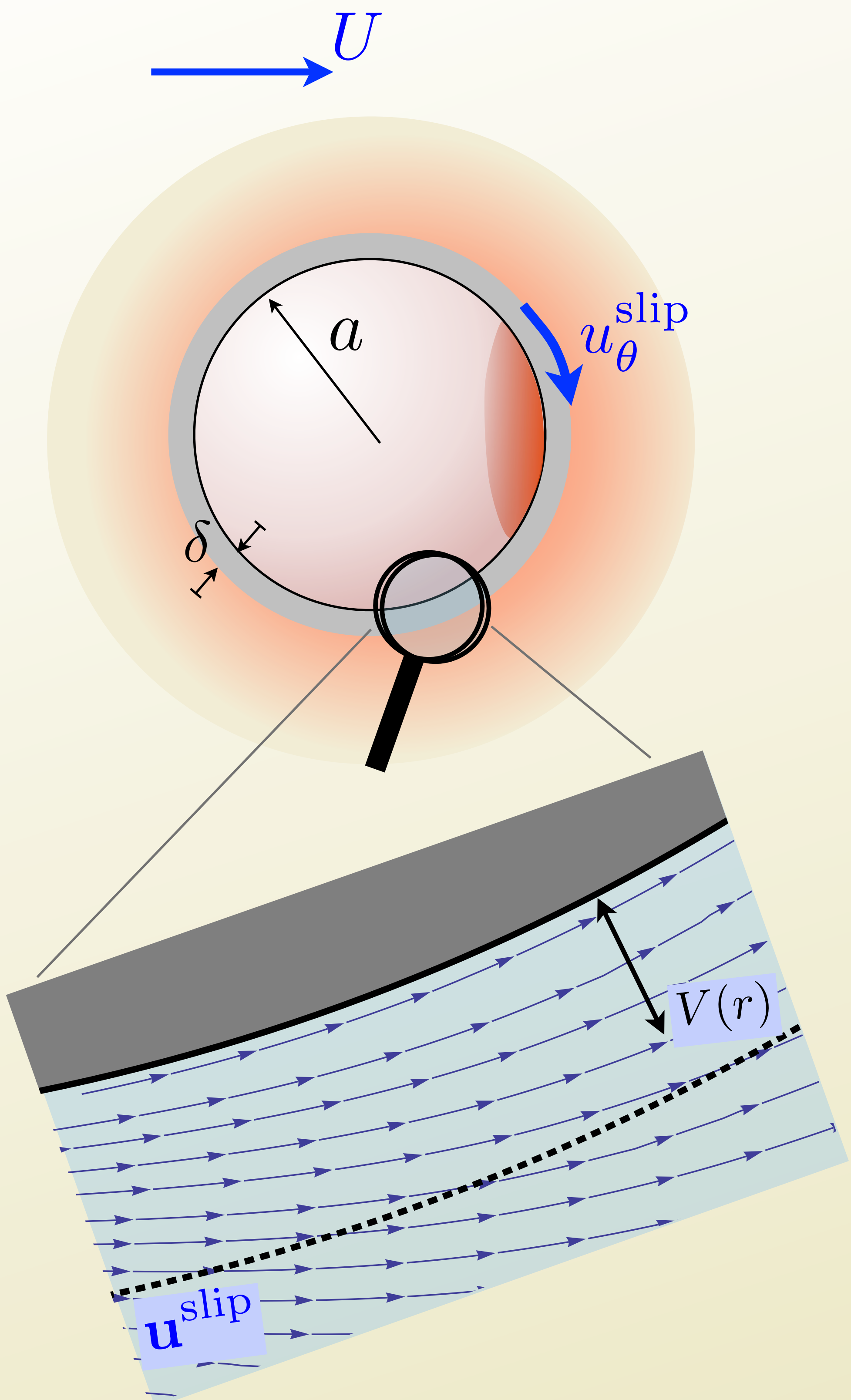
Blake *J. Fluid Mech* **46**, 199–208 (1971)
Lighthill *Commun. Pure Appl. Math.* **5**, 109–118 (1952)

Solute profile



Outside the boundary layer the solute is conserved

$$\partial_t c + \mathbf{u} \cdot \nabla c - D\nabla^2 c = 0$$

Rate of transport across the boundary layer equals rate of production

$$\int_{r=a+\delta} \left(u_r^{\text{slip}} c - D\partial_r c\right) = \int_{r=a} \alpha$$

Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)
Golestanian, Liverpool & Ajdari *New J. Phys.* **9**, 126 (2007)

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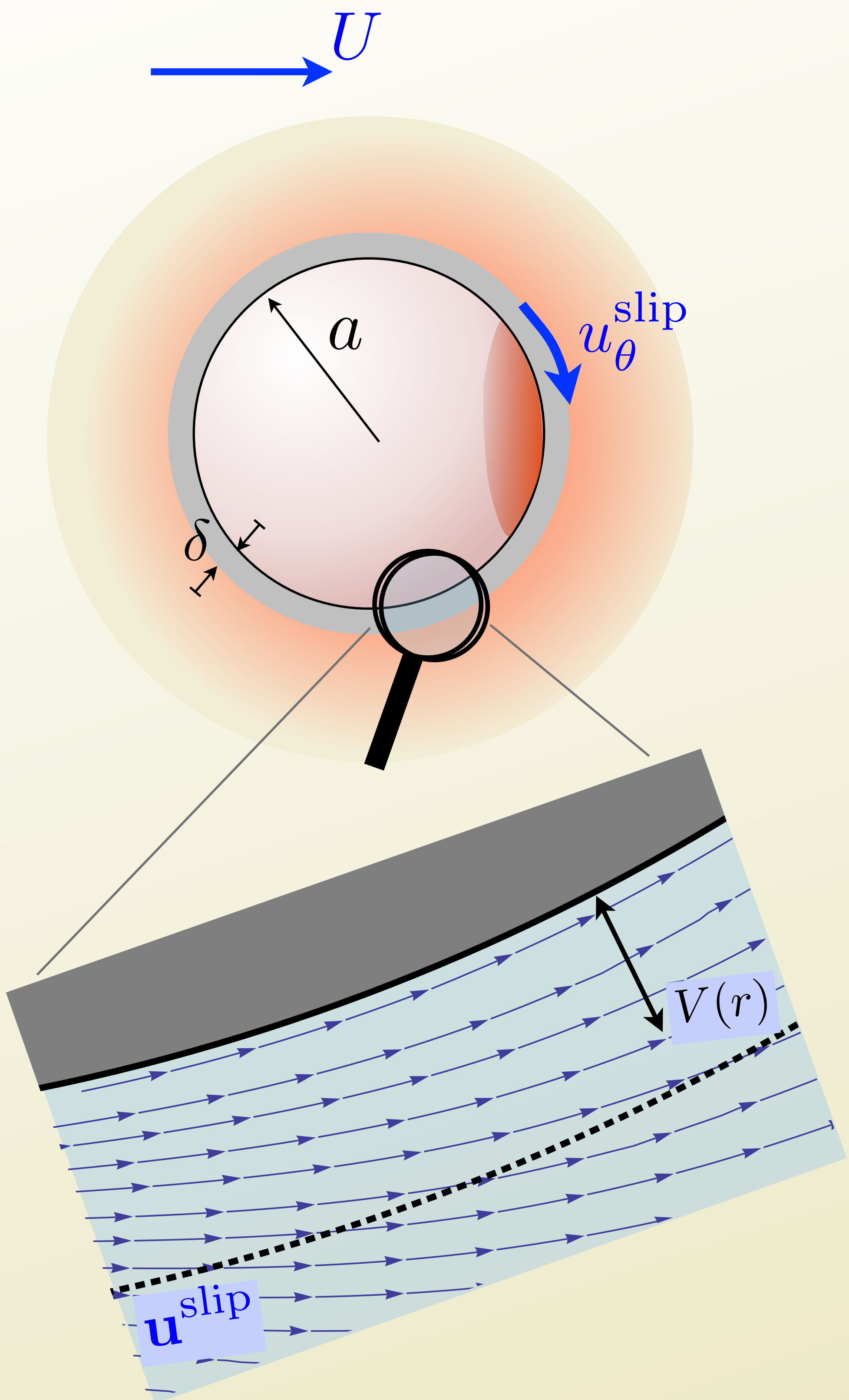

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$$\cancel{\partial_t c + \mathbf{u}\cdot\nabla c} - D\nabla^2 c = 0 \qquad \boxed{\mathrm{Pe} \ll 1}$$

Rate of transport across the boundary layer equals rate of production

$$\int_{r=a+\delta} \left(\cancel{u_r^{\text{slip}}} c - D\partial_r c\right) = \int_{r=a} \alpha$$

solve pointwise

$$c(r,\theta) = \sum_{l=0}^{\infty} \frac{a+\delta}{(l+1)D}\,\alpha_l \left(\frac{a+\delta}{r}\right)^{l+1} P_l(\cos(\theta))$$

Anderson *Ann. Rev. Fluid Mech.* **21**, 61–99 (1989)
Golestanian, Liverpool & Ajdari *Phys. Rev. Lett.* **94**, 220801 (2005)
Golestanian, Liverpool & Ajdari *New J. Phys.* **9**, 126 (2007)

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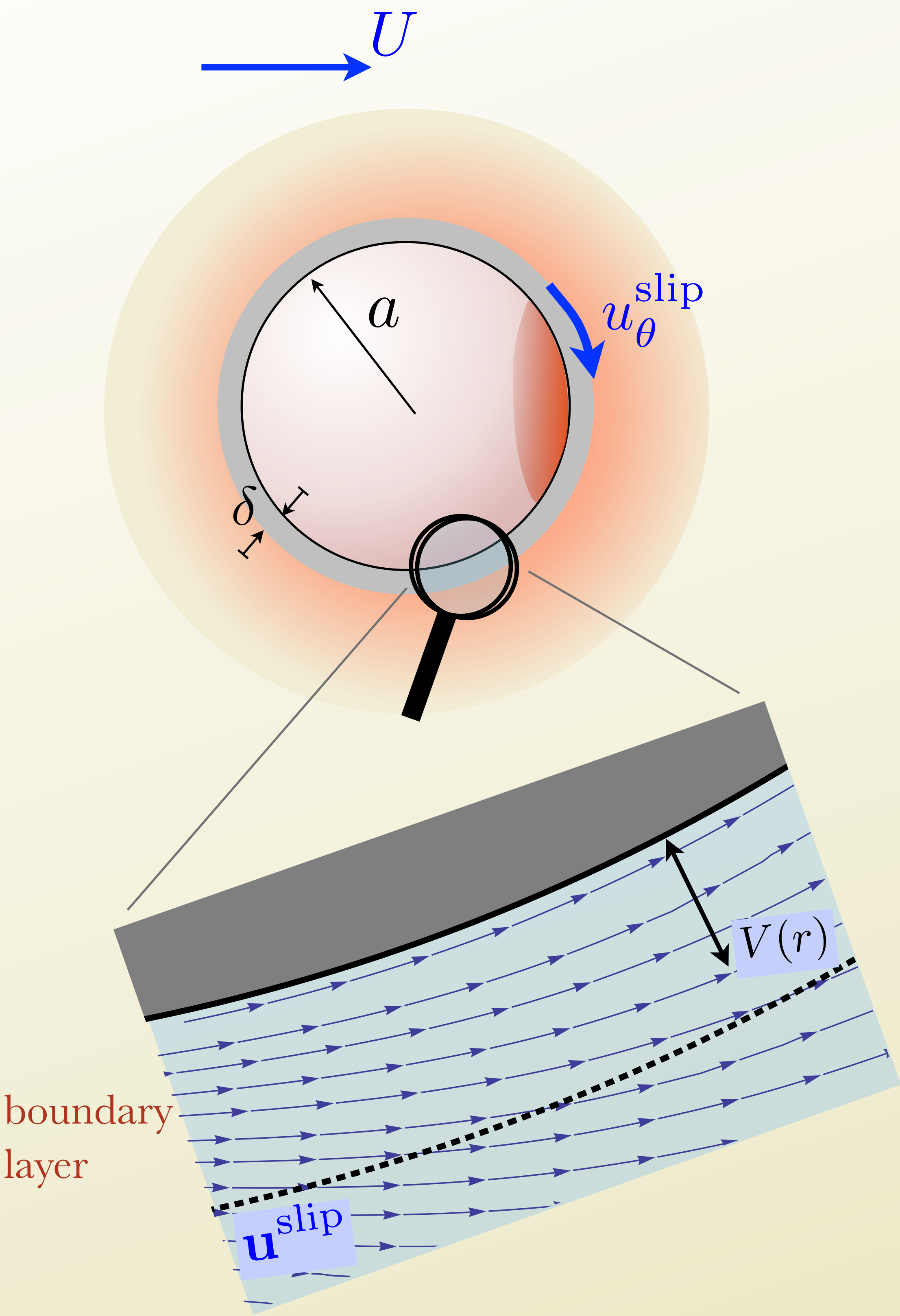
DIGRESSION



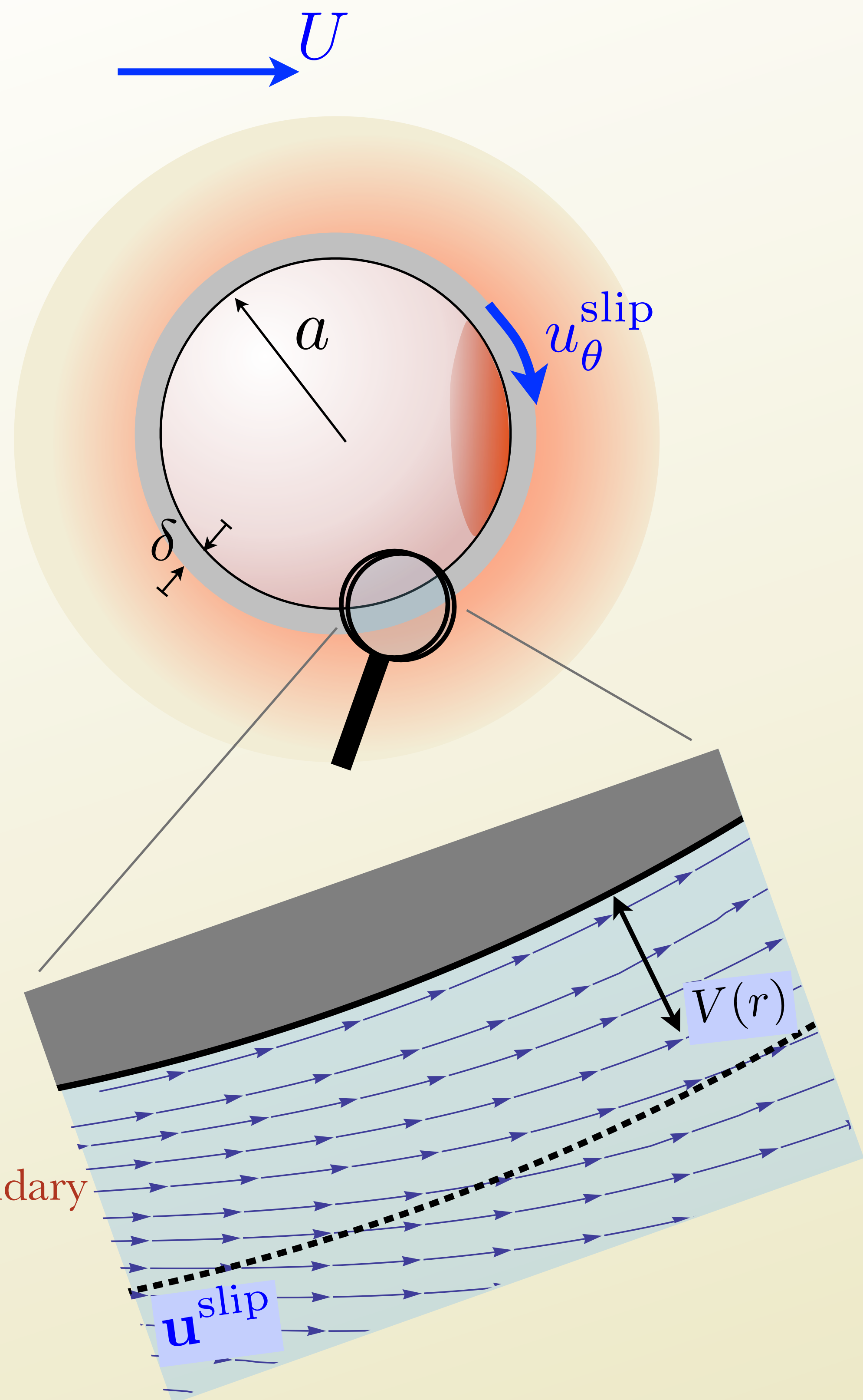
Boundary layer analysis neglects

- fluid continuity; radial slip

$$\frac{1}{r^2}\partial_r\left(r^2 u_r\right) + \frac{1}{r\sin(\theta)}\partial_\theta\left(\sin(\theta)u_\theta\right) = 0$$

not constant

DIGRESSION



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$$\frac{1}{r^2}\partial_r\left(r^2 u_r\right) + \frac{1}{r\sin(\theta)}\partial_\theta\left(\sin(\theta)u_\theta\right) = 0$$

not constant

Scaling $\quad u_r^{\text{slip}} \sim \frac{\delta}{a}u_\theta^{\text{slip}}$

generically small

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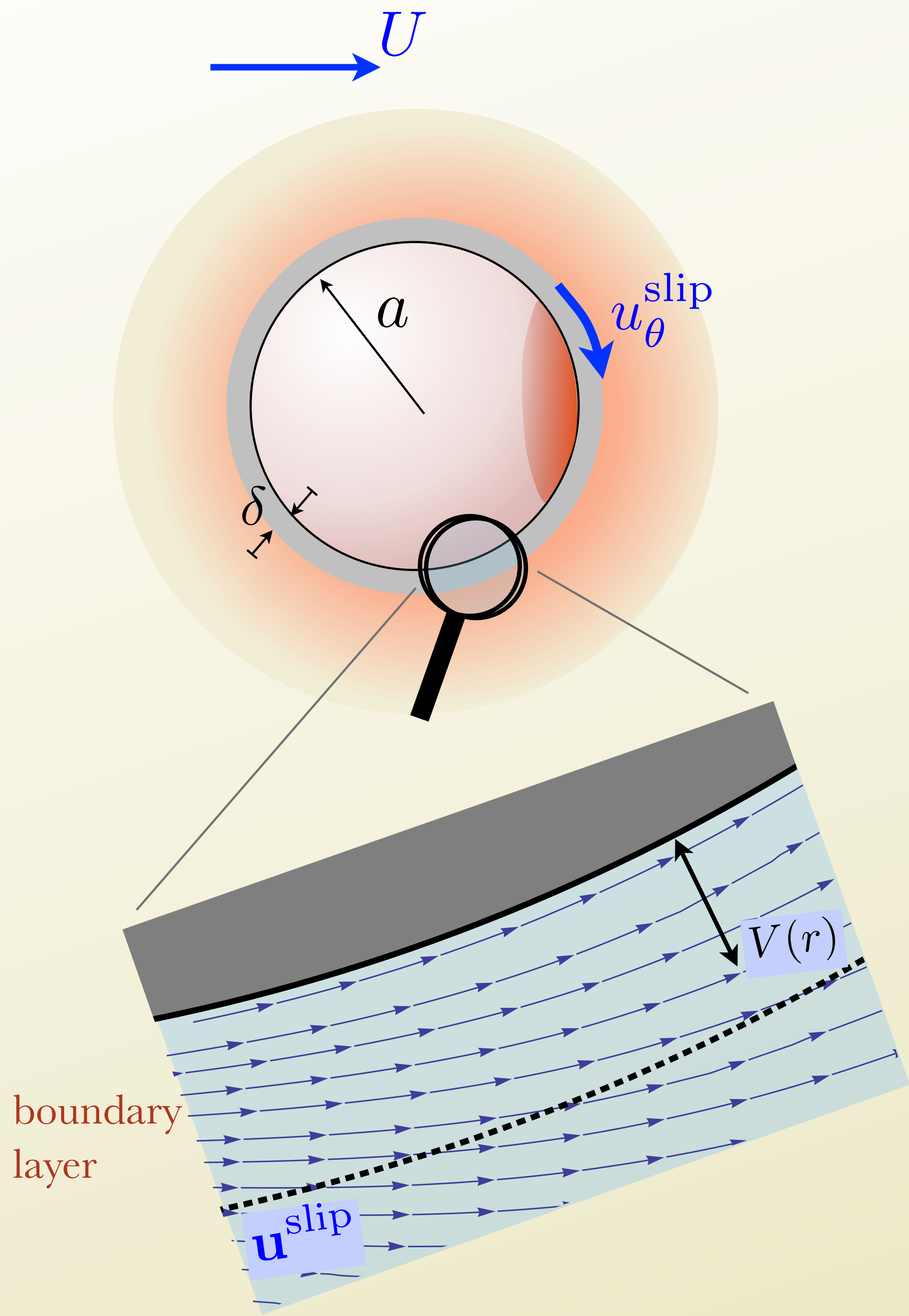
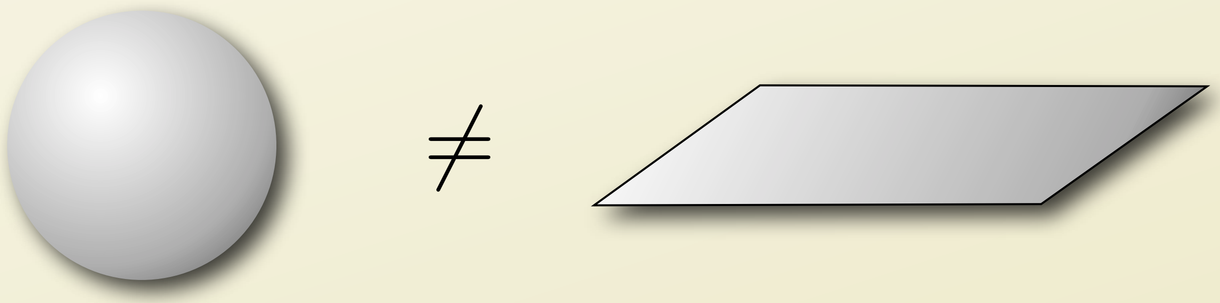
Digression



Boundary layer analysis neglects

- fluid continuity; radial slip
- topology of the sphere



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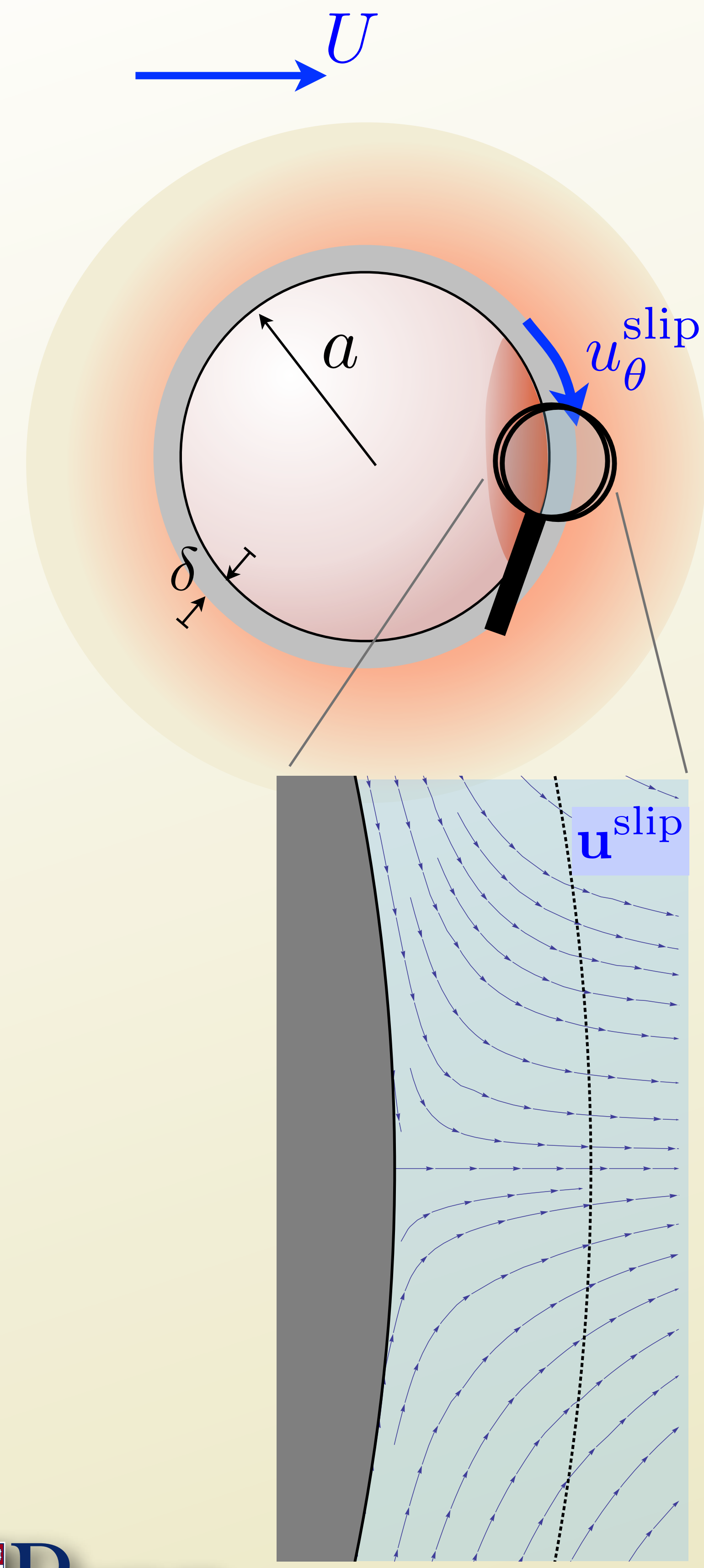

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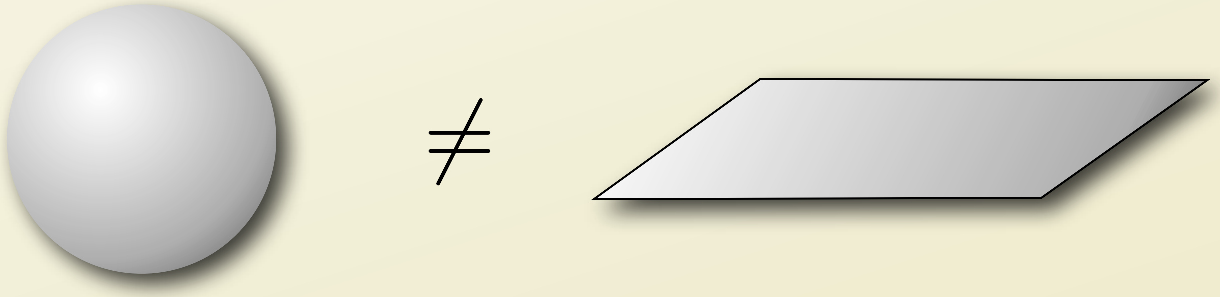

Neglecting the radial slip is not a good approximation near $\theta = 0, \pi$

consequence of topology rather than axisymmetry (Poincaré-Hopf)

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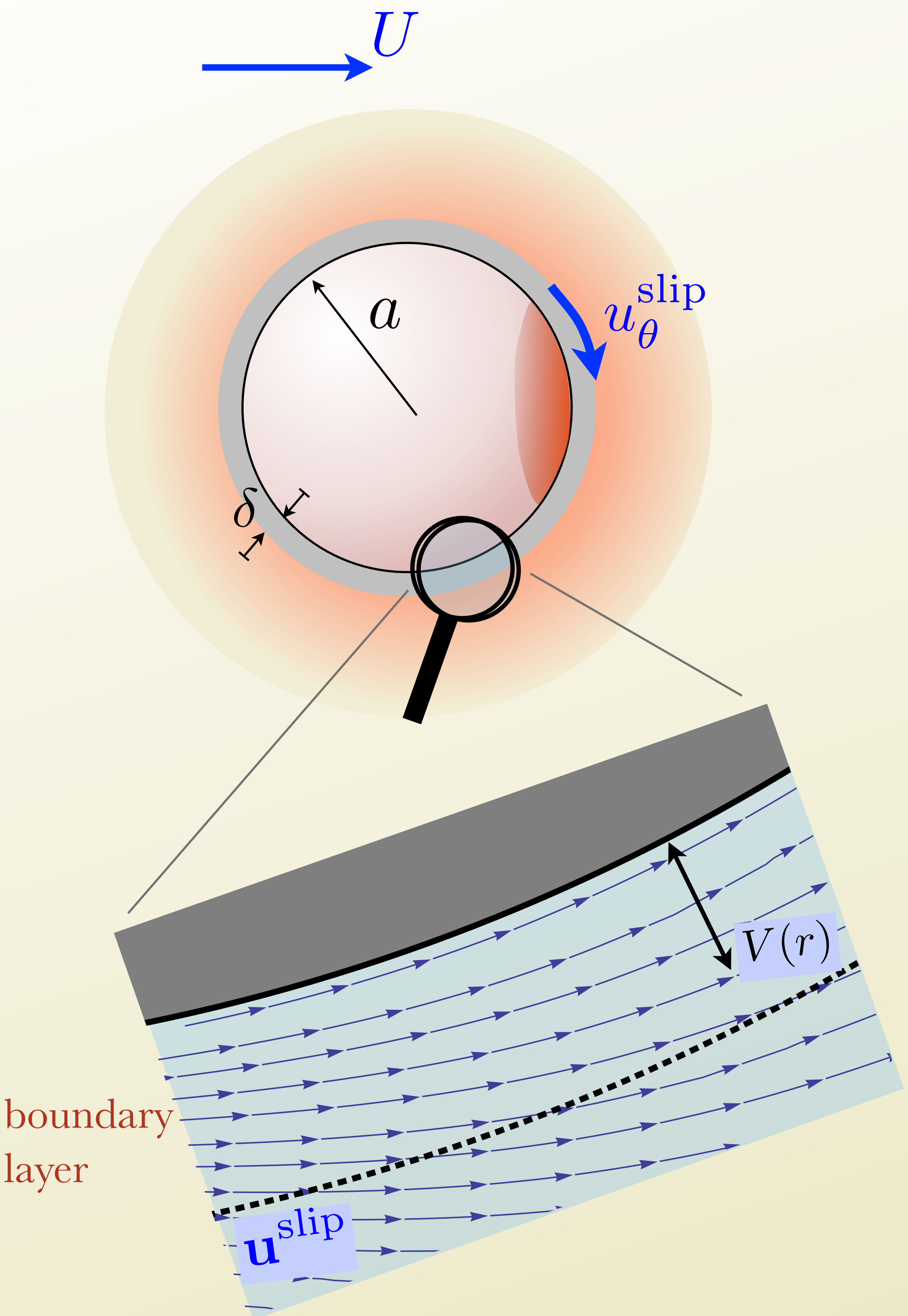


Boundary layer analysis neglects

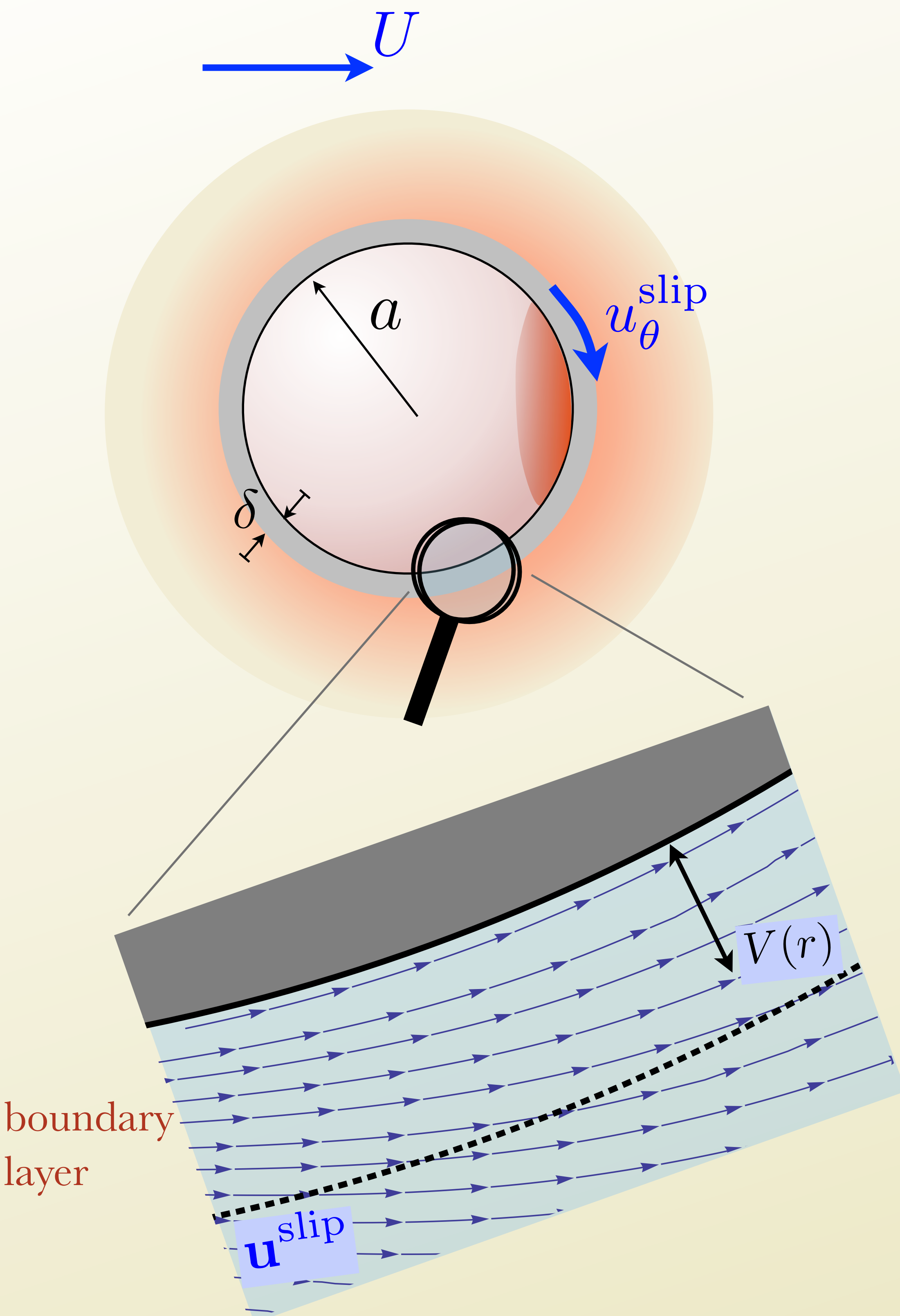
- fluid continuity; radial slip $u_r^{\text{slip}} \sim \frac{\delta}{a} u_\theta^{\text{slip}}$
- topology of the sphere Poincaré-Hopf

Such a simple problem can be solved exactly

$$\mathbf{0} = -\nabla p + \mu \nabla^2 \mathbf{u} - \mathbf{e}_r\, c(r,\theta)\, \partial_r V \qquad 0 = \nabla \cdot \mathbf{u}$$

- Spherical particle
- Axisymmetry, steady state
- Single solute species
- Purely radial potential
- Zero Reynolds number
- Zero Péclet number

Digression



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- topology of the sphere $\quad$ Poincaré-Hopf

Such a simple problem can be solved exactly

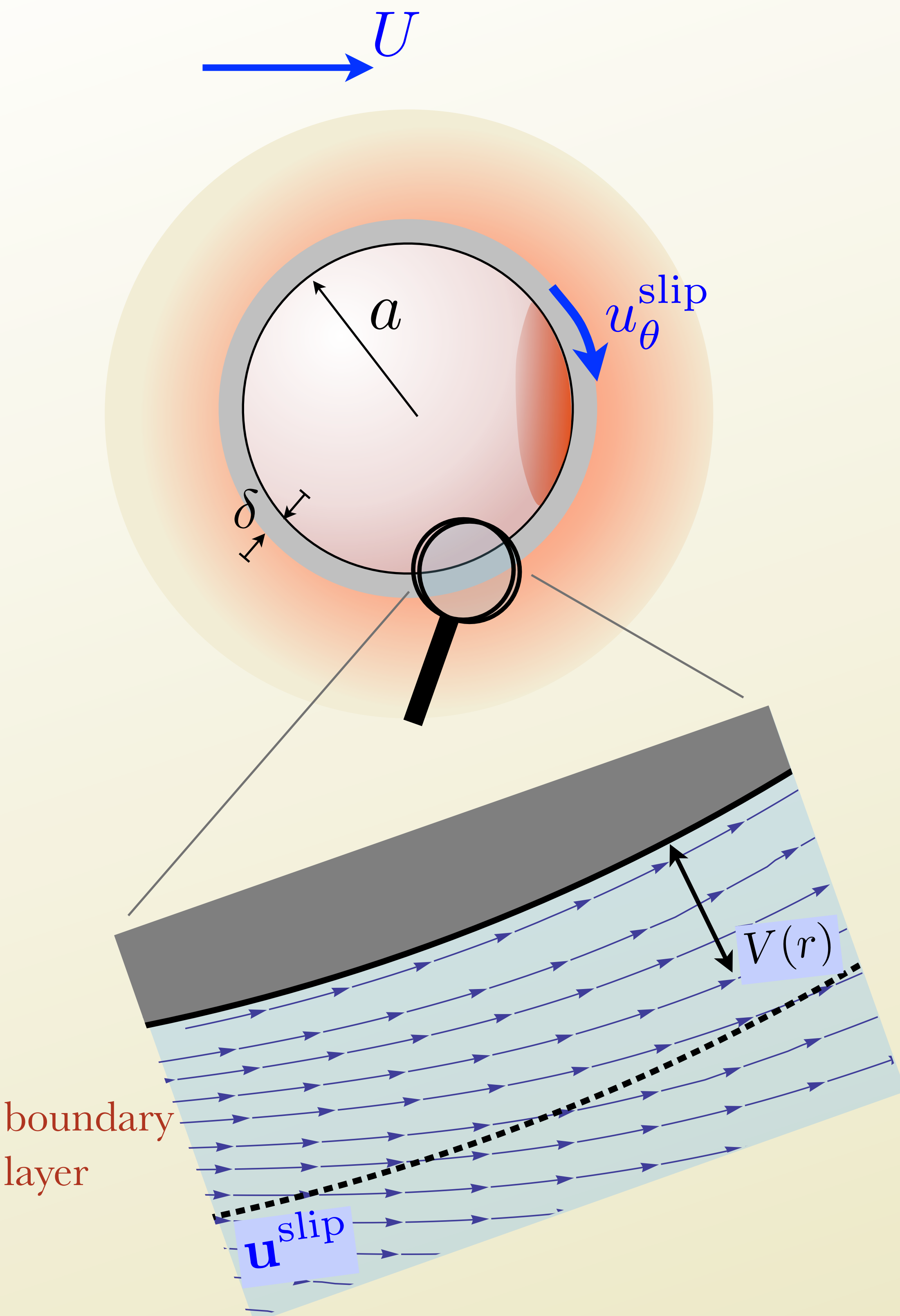
$$\mathbf{0} = -\nabla p + \mu \nabla^2 \mathbf{u} - \mathbf{e}_r\, c(r,\theta)\, \partial_r V \qquad 0 = \nabla \cdot \mathbf{u}$$

E.g., the speed is

$$U = \frac{2}{3a} \frac{k_B T}{\mu} c_1 \delta^2 \int_0^1 \mathrm{d}z\, z \Big(1 - \underbrace{\frac{\delta}{6a} \frac{3z + 2\delta z^2/a}{(1+\delta z/a)^2}}_{\text{correction}}\Big) \left[1 - \mathrm{e}^{-V/k_B T}\right]$$

Alexander & Idema *in preparation* (2011)

Digression



Boundary layer analysis neglects

- fluid continuity; radial slip $u_r^{\text{slip}} \sim \frac{\delta}{a} u_\theta^{\text{slip}}$
- topology of the sphere Poincaré-Hopf

Such a simple problem can be solved exactly

$$\mathbf{0} = -\nabla p + \mu \nabla^2 \mathbf{u} - \mathbf{e}_r \, c(r,\theta)\, \partial_r V \qquad 0 = \nabla \cdot \mathbf{u}$$

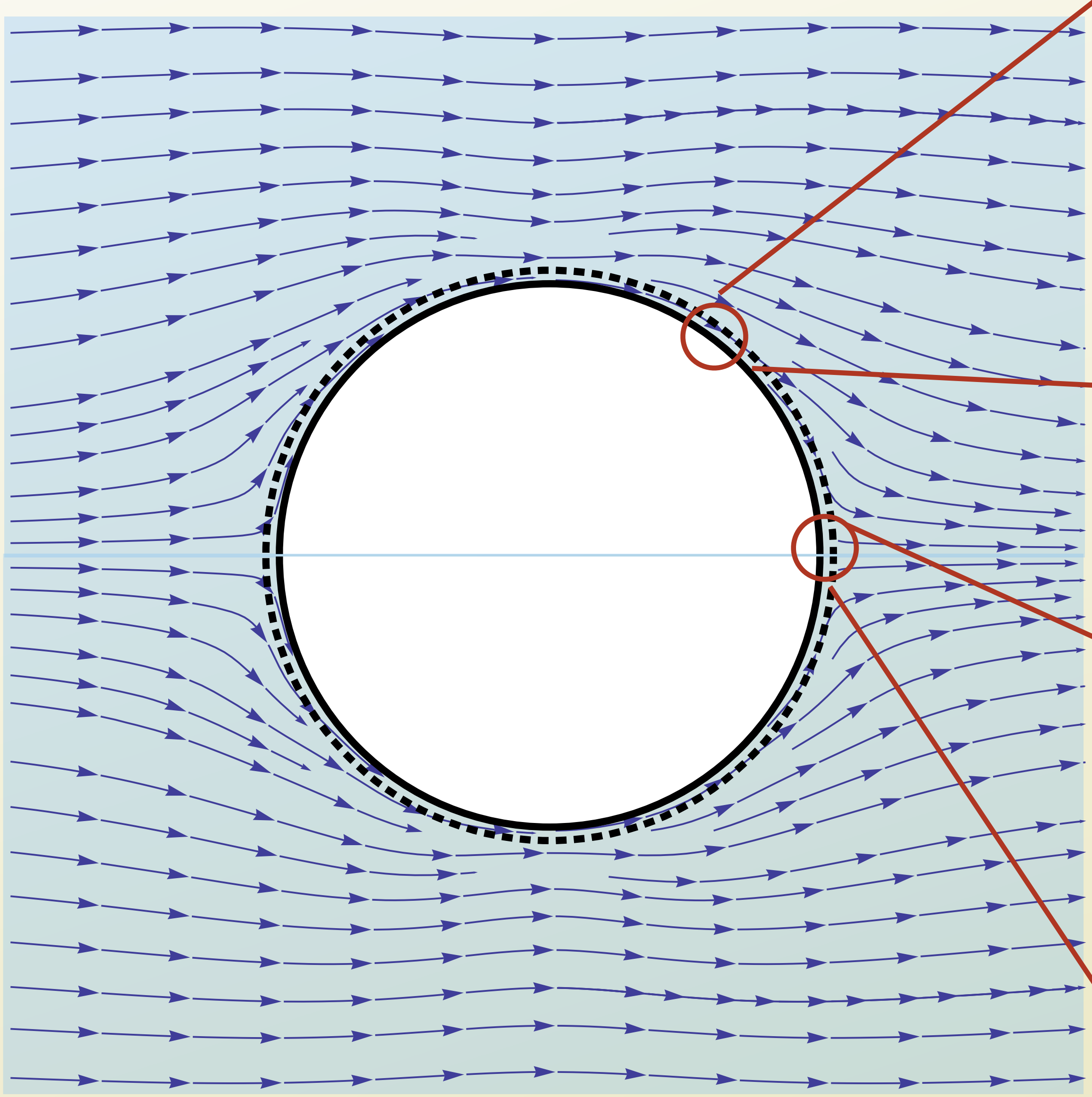
E.g., the speed is

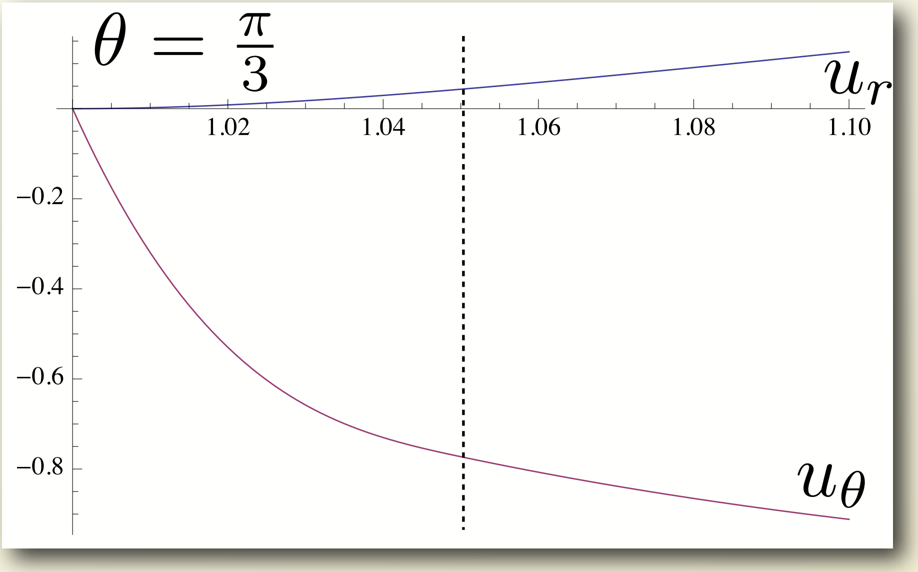
$$U = \frac{2}{3a} \frac{k_B T}{\mu} c_1 \delta^2 \int_0^1 \mathrm{d}z\, z \left(1 - \frac{\delta}{6a} \frac{3z + 2\delta z^2/a}{(1+\delta z/a)^2}\right) \left[1 - \mathrm{e}^{-V/k_B T}\right]$$

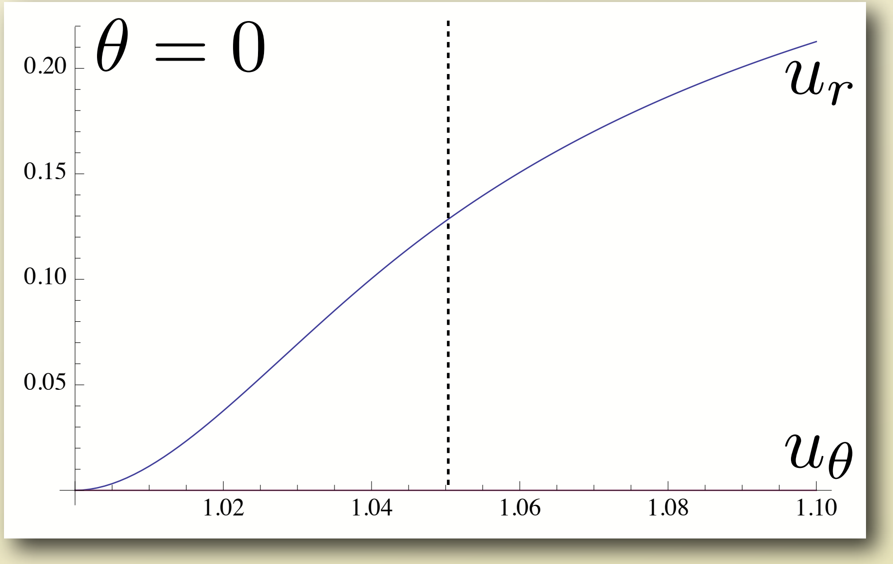
scaling $\quad U \sim \dfrac{k_B T \alpha_1}{\mu D} \delta^2 \left(1 + \frac{\delta}{a}\right)$

Sabass, Seifert *Phys. Rev. Lett.* **105**, 218103 (2010)
Alexander & Idema *in preparation* (2011)

Boundary layer flow

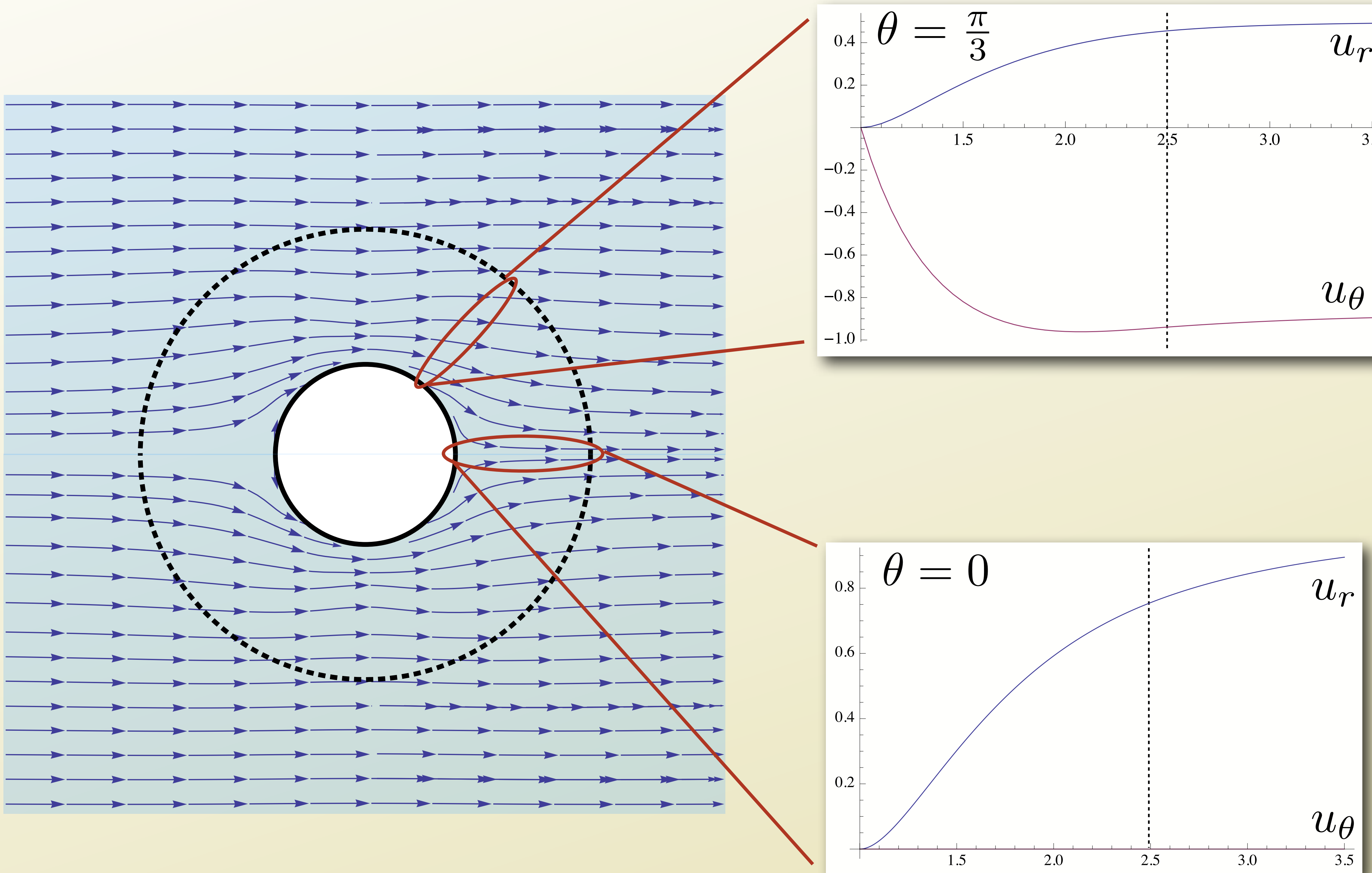






Alexander & Idema *in preparation* (2011)

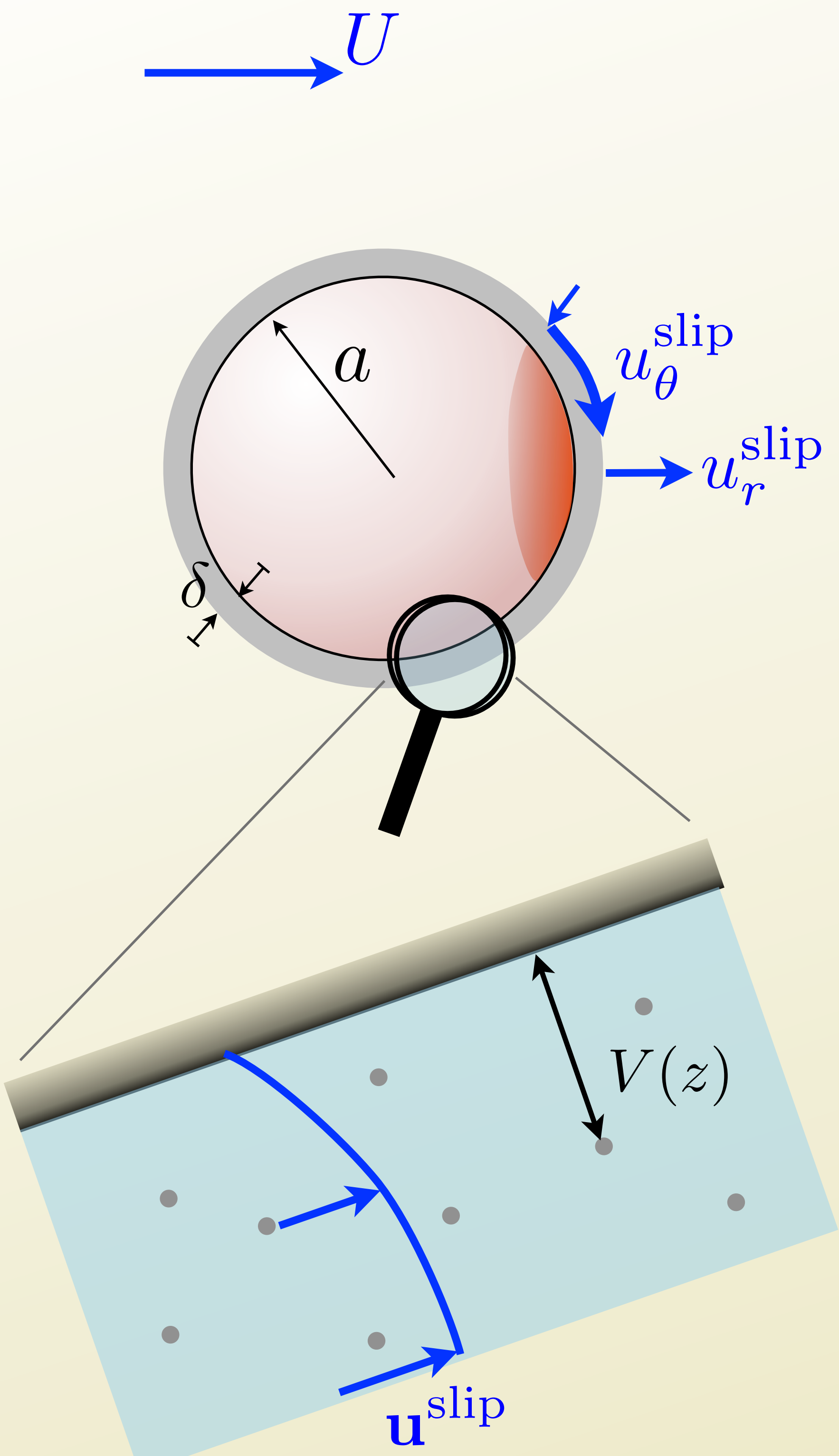
Boundary layer flow



Penn

Alexander & Idema *in preparation* (2011)

WHAT CHANGES IF THE PÉCLET NUMBER IS LARGE?

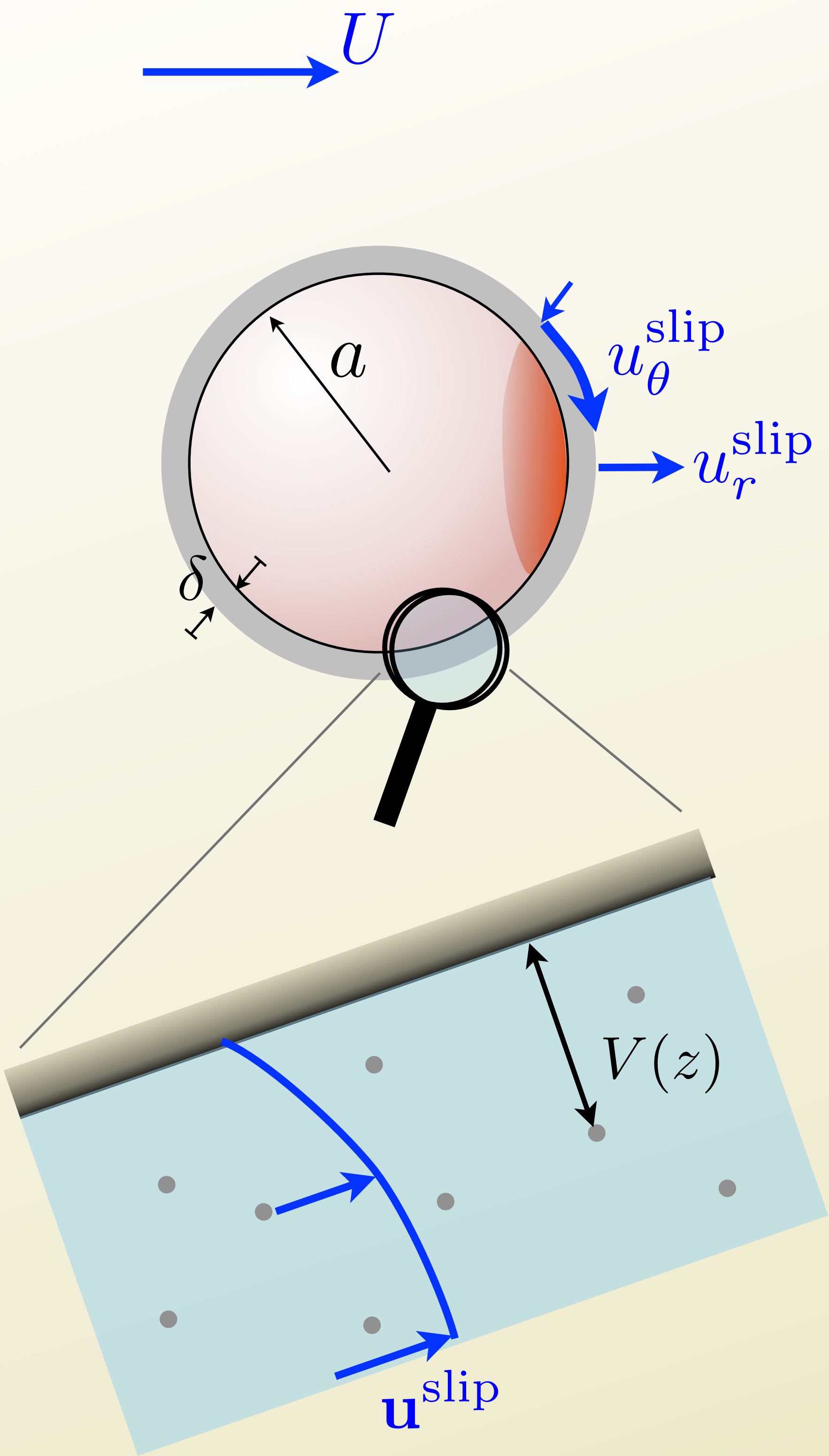


Basic mechanism remains unchanged

- concentration gradients drive tangential flow in a thin boundary layer

postulate $\qquad \mathbf{u}^{\mathrm{slip}} = m^A \nabla_{\|} c$

What changes if the Péclet number is large?



Basic mechanism remains unchanged

- concentration gradients drive tangential flow in a thin boundary layer

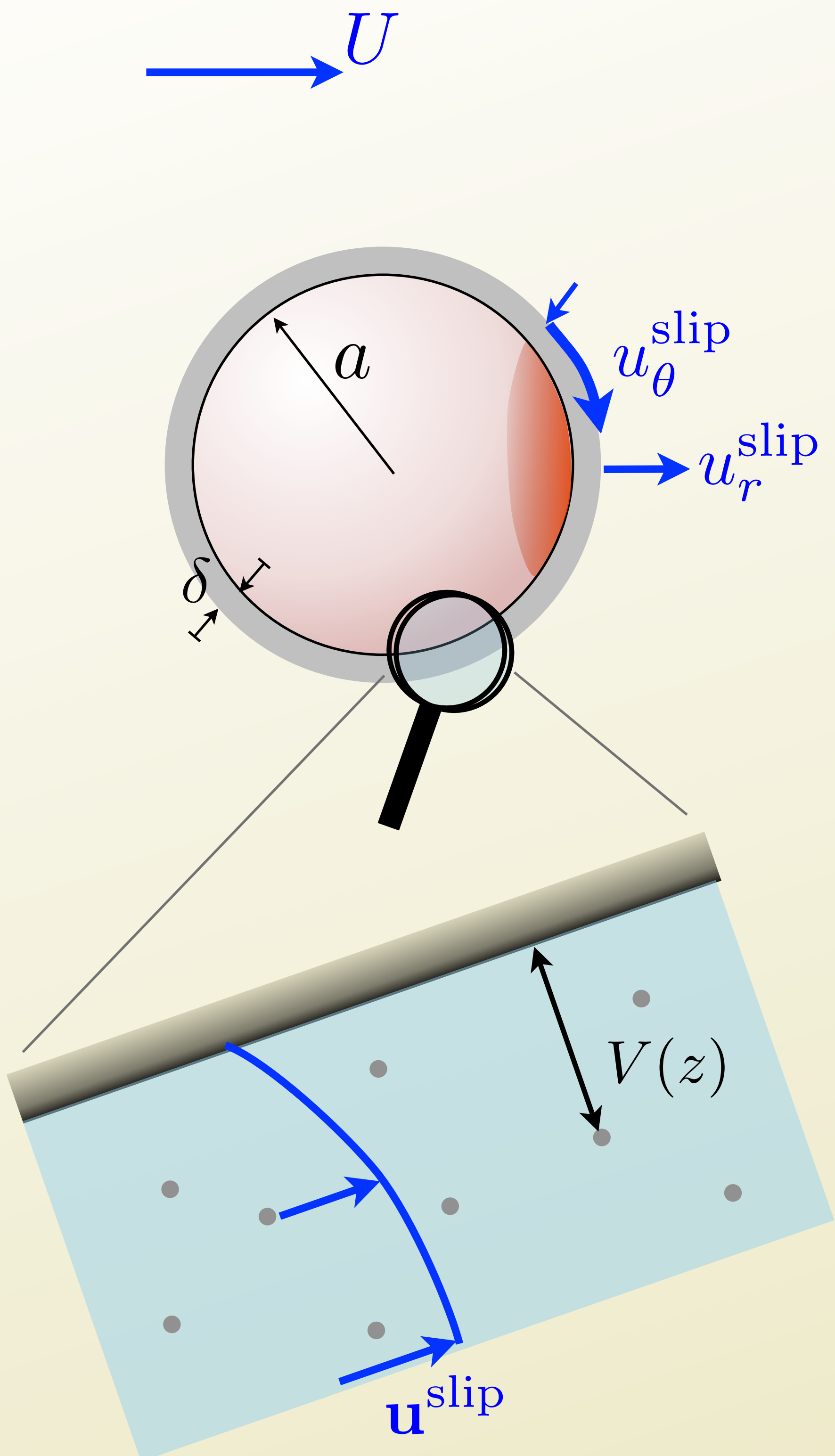
postulate $\quad \mathbf{u}^{\mathrm{slip}} = m^A \nabla_{\|} c$

But ...

- if the solute does not diffuse then it will only be found where it is produced
- tangential slip only generated within the active patch
- radial influx at the boundary and outflux from the interior

Penn

Solute transport



Outside the boundary layer the solute is conserved

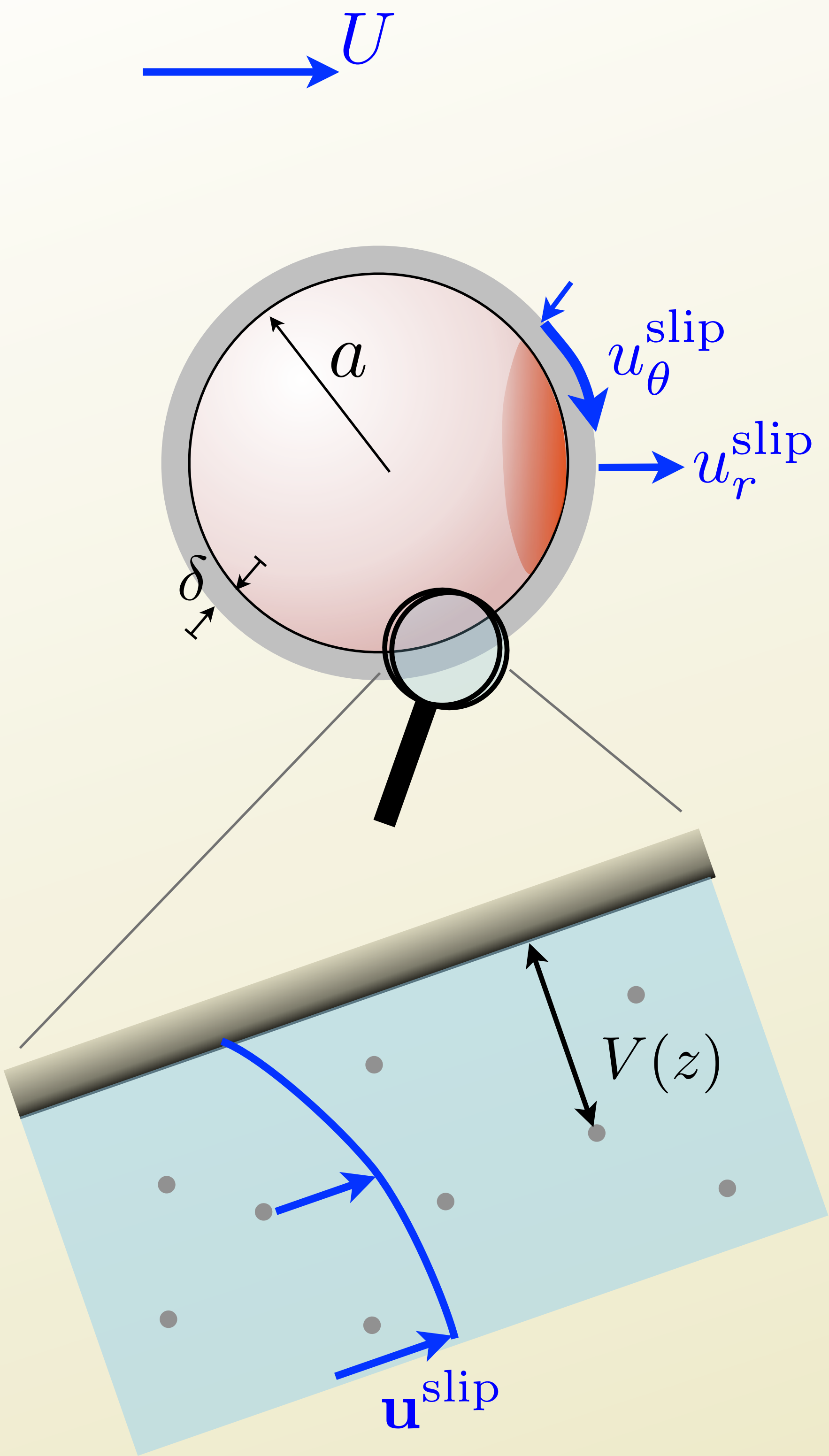
$$\partial_t c + \mathbf{u} \cdot \nabla c - D\nabla^2 c = 0$$

Rate of transport across the boundary layer equals rate of production

$$\int_{r=a+\delta} \left(u_r^{\text{slip}} c - D\partial_r c \right) = \int_{r=a} \alpha$$

Penn

Solute transport



Outside the boundary layer the solute is conserved

$$\partial_t c + \mathbf{u}\cdot\nabla c - \cancel{D\nabla^2 c} = 0 \qquad \boxed{\mathrm{Pe} \gg 1}$$

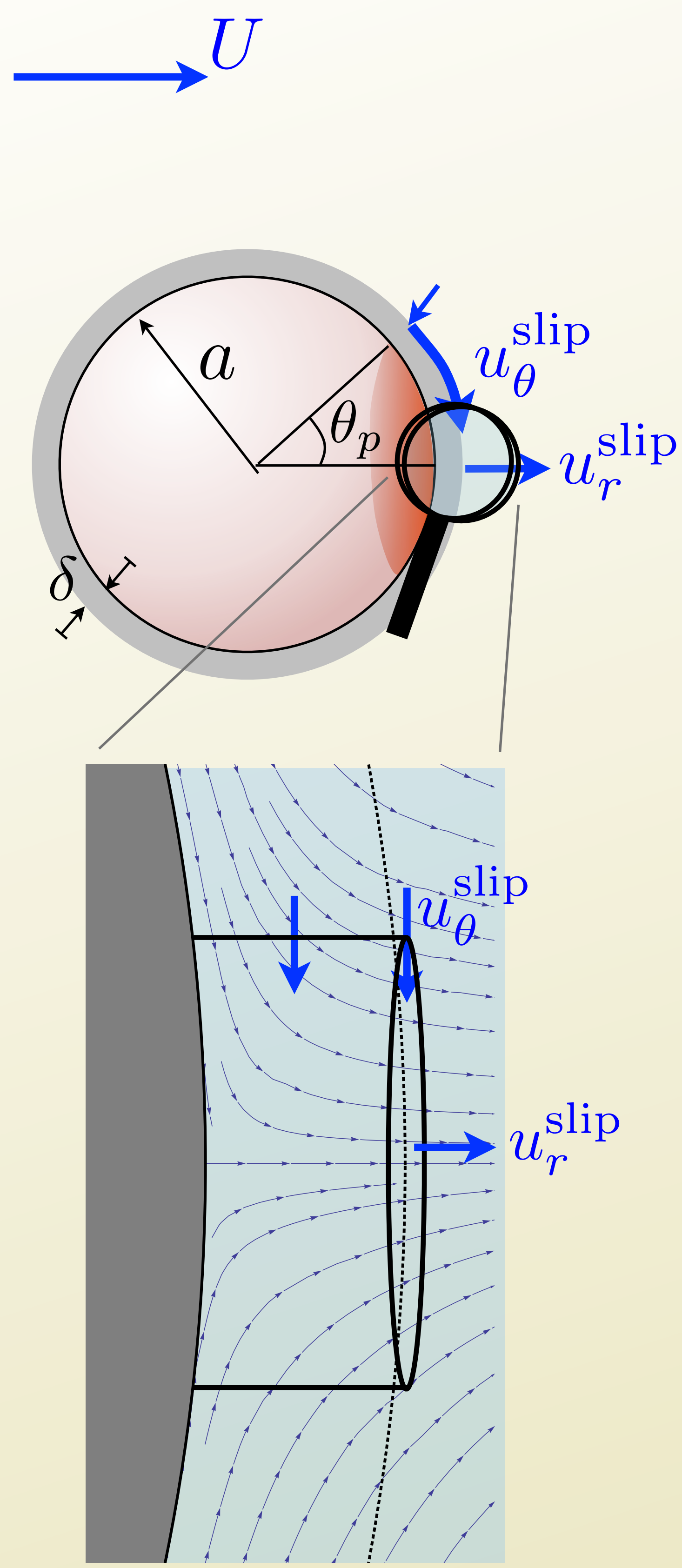
Rate of transport across the boundary layer equals rate of production

$$\int_{r=a+\delta} \left(u_r^{\mathrm{slip}} c - \cancel{D\partial_r c}\right) = \int_{r=a} \alpha$$

radial slip is important

Penn

RADIAL SLIP



Radial outflux

$$2\pi(a+\delta)^2 \int_0^{\theta_q} \mathrm{d}\theta\, \sin(\theta)\, u_r^{\text{slip}}$$

Tangential influx

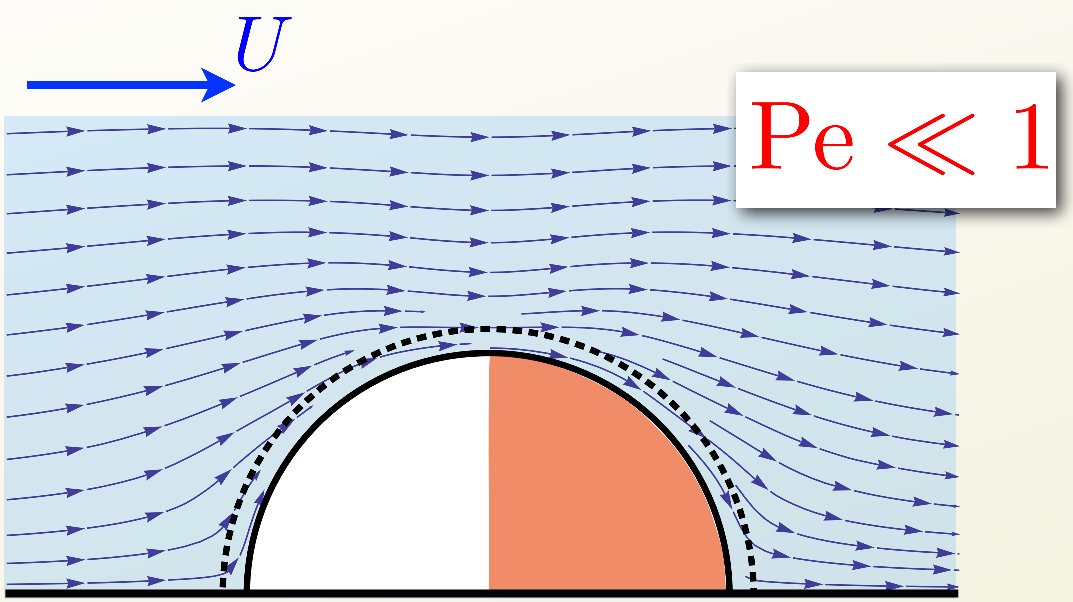
$$-2\pi \int_a^{a+\delta} \mathrm{d}r\, r \sin(\theta)\, u_\theta(r,\theta)$$

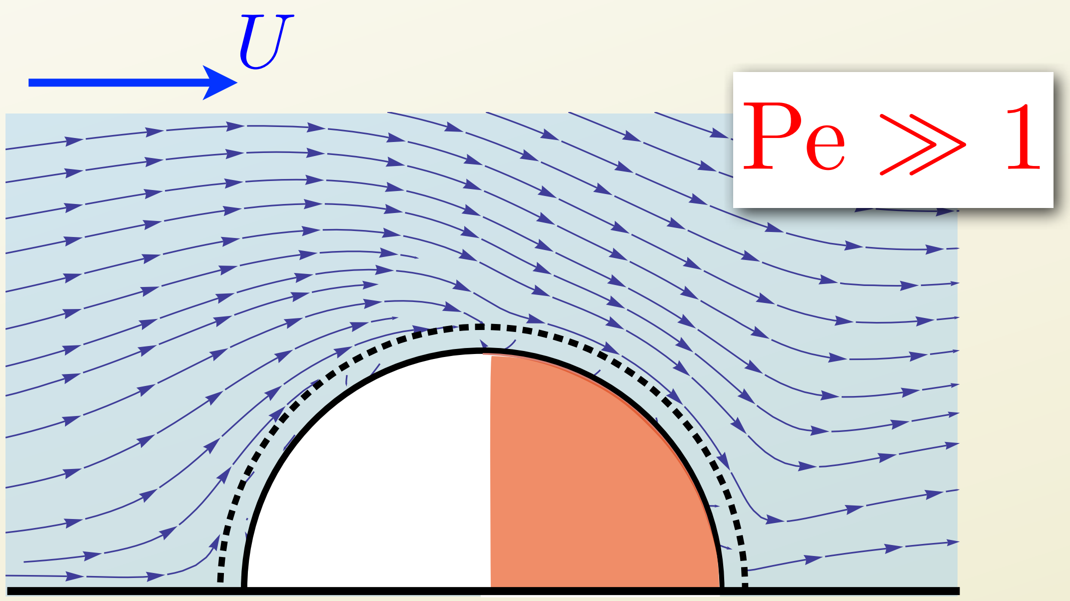
$$\approx -2\pi a \delta \beta\, \sin(\theta)\, u_\theta^{\text{slip}}$$

$\mathcal{O}(1)$

conserved away from $\theta = 0$

So, what's different?





$$U \sim \frac{m^D \bar{\alpha}}{D} \sin^2(\theta_p)$$

$$U \sim \left(\frac{m^A \bar{\alpha}}{\delta \ln\left(\frac{a}{\delta}\right)} \right)^{1/2} \sin^3(\tfrac{1}{2}\theta_p)$$

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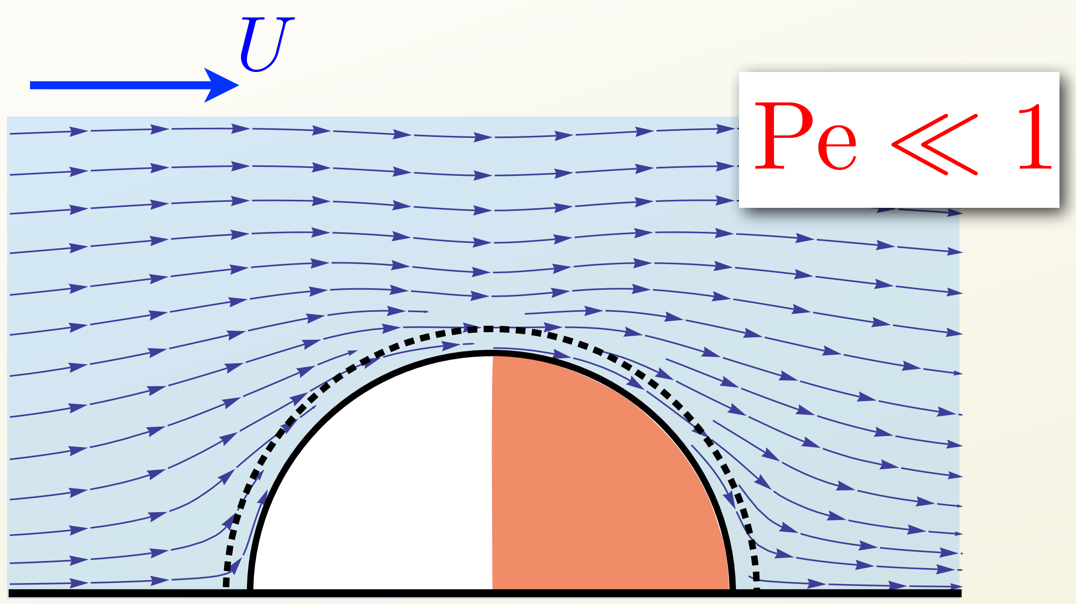

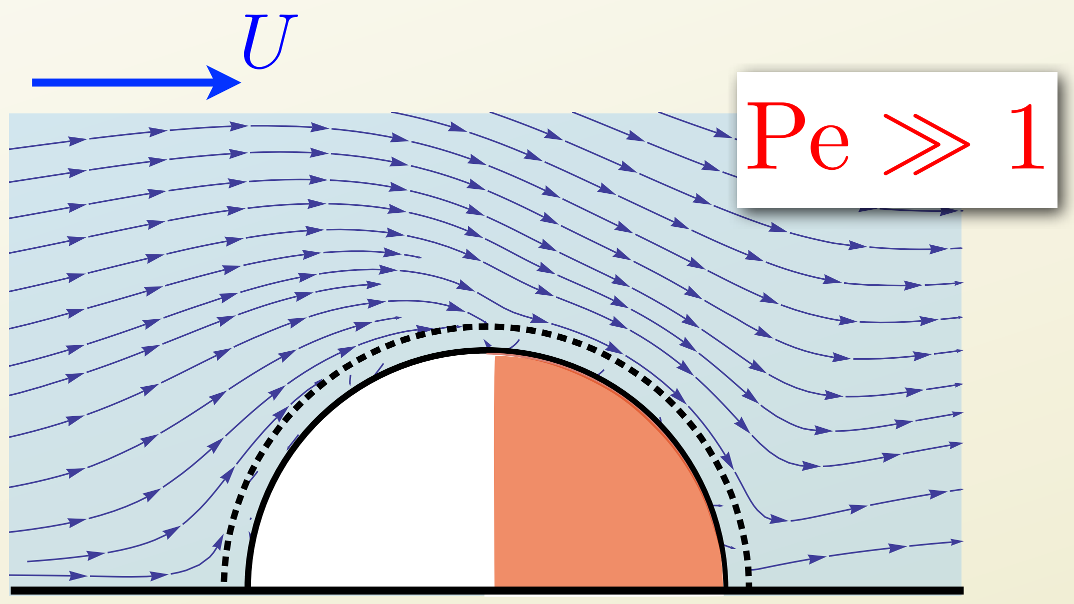

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Penn

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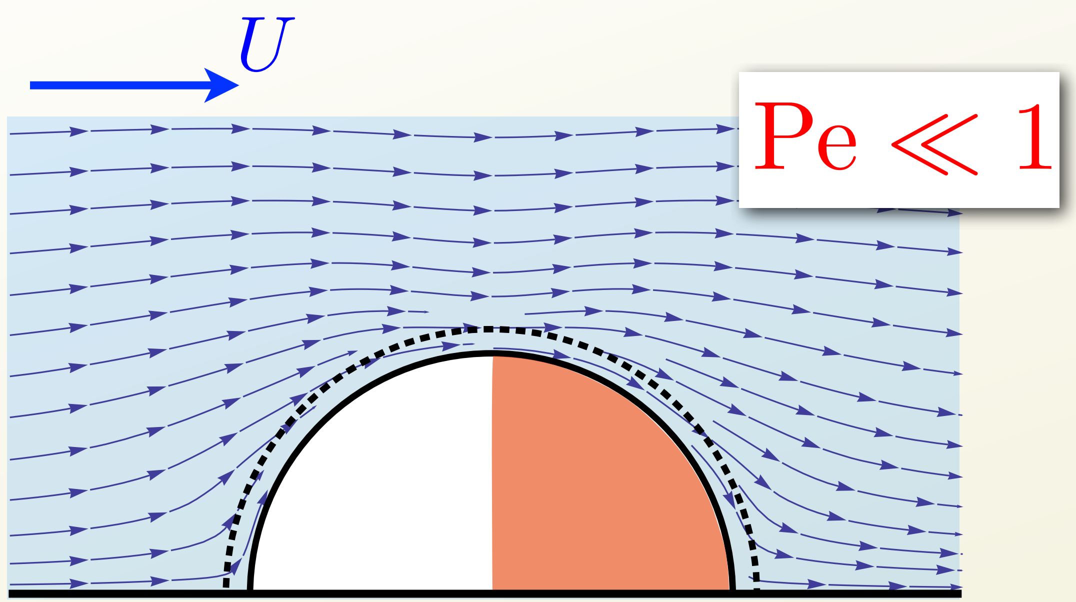

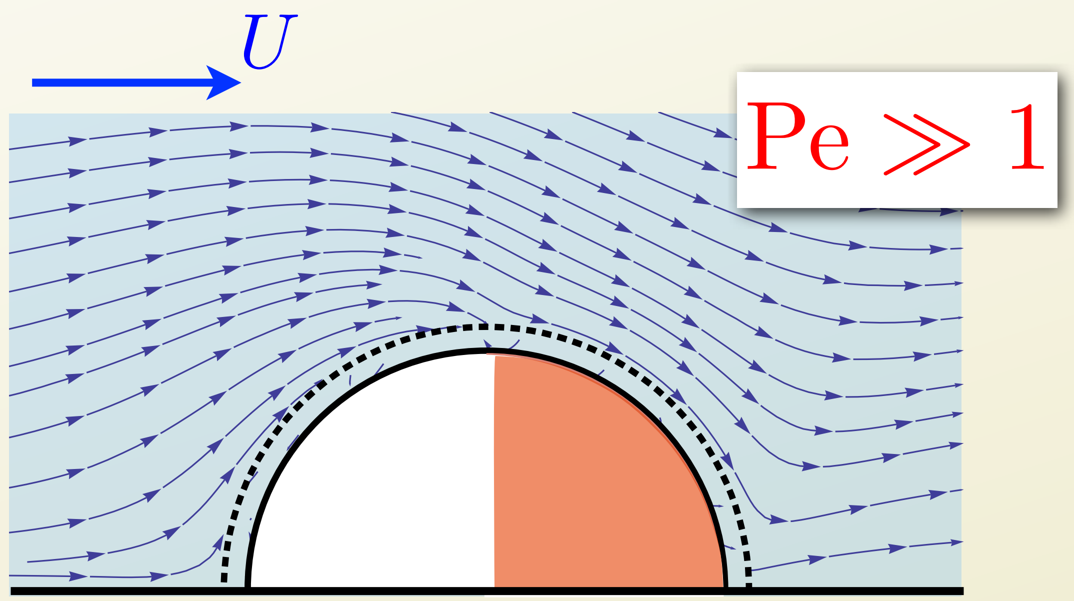

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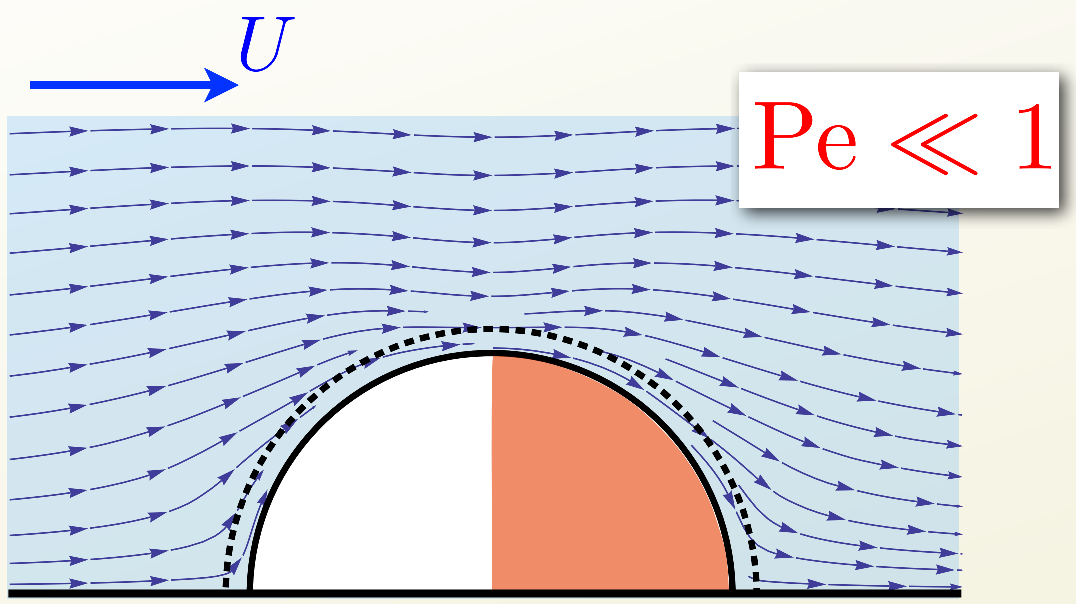
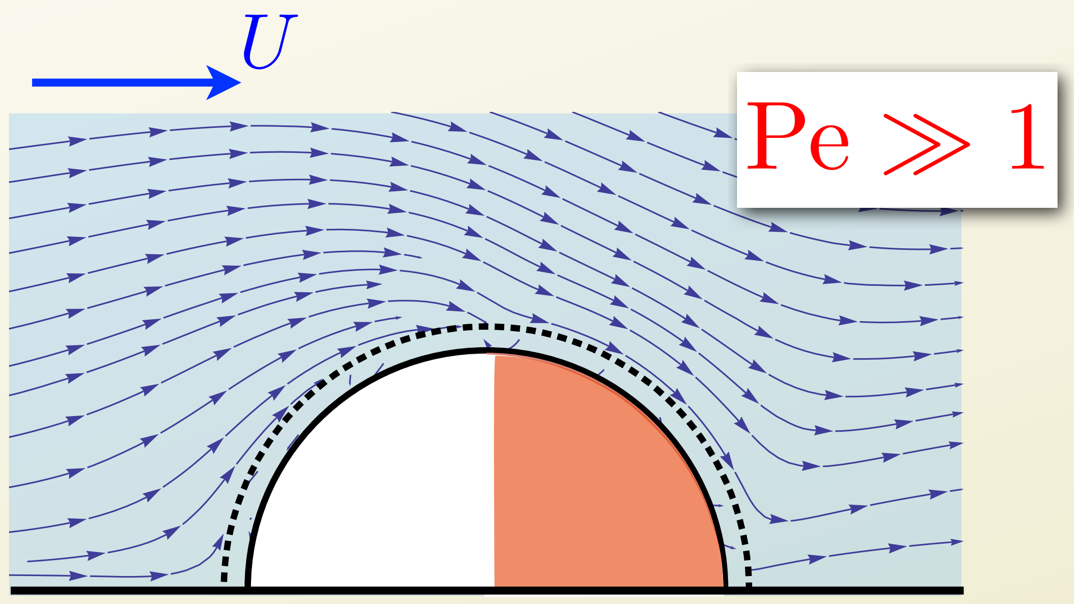
Transport balance $$\int_{r=a+\delta} \left(u_r^{\mathrm{slip}} c - D \partial_r c \right) = \int_{r=a} \alpha$$

$\propto |u_\theta^{\mathrm{slip}}|$

So, what's different?





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$\propto |\sin(\theta) u_\theta^{\mathrm{slip}}|$

by fluid continuity

$\propto |u_\theta^{\mathrm{slip}}|$

by postulate

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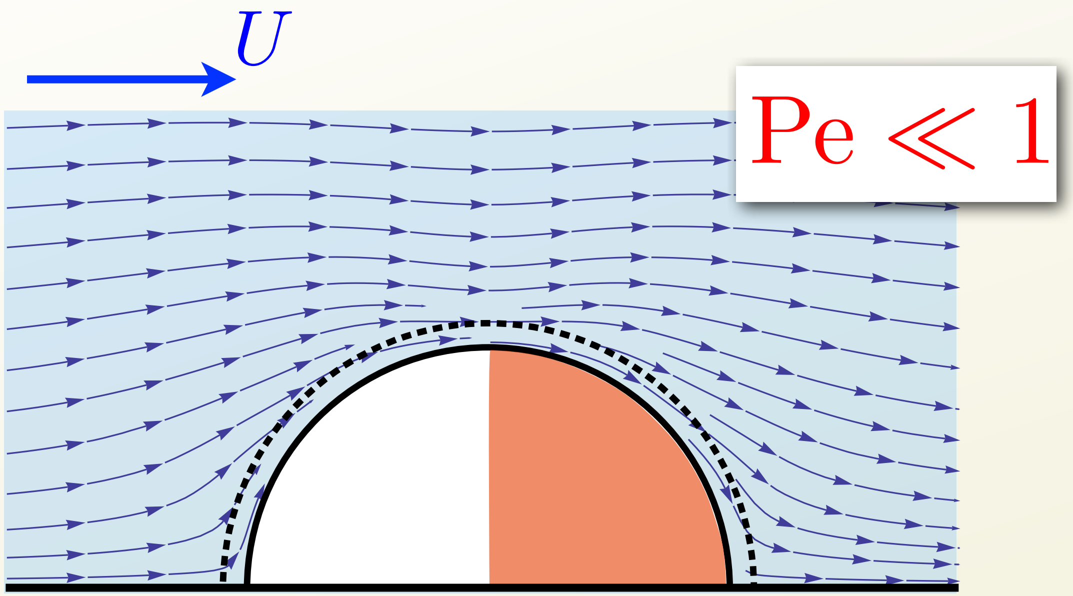
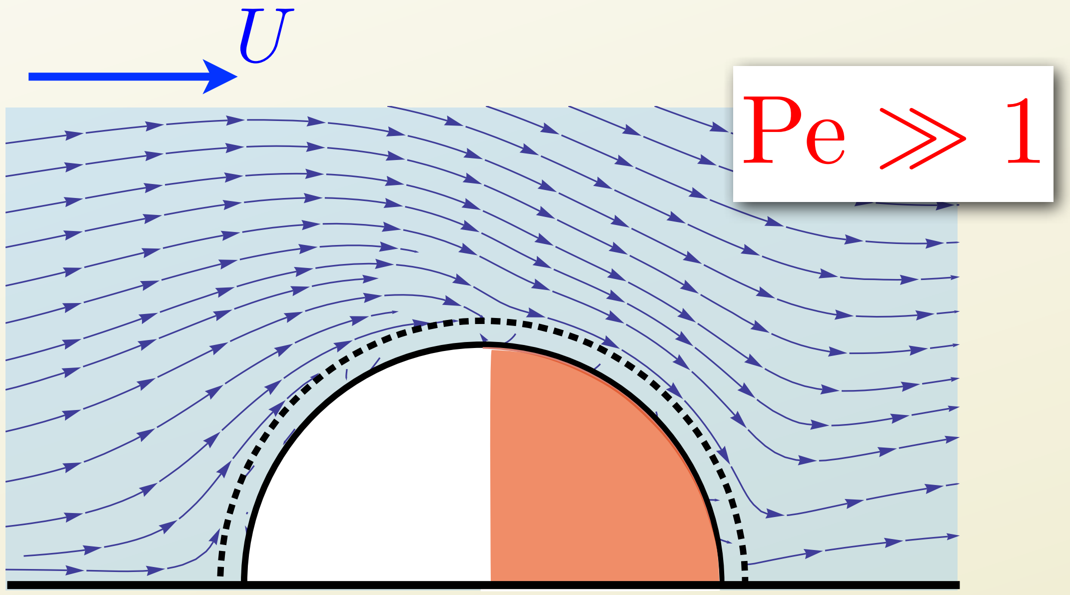
So, what's different?





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Penn

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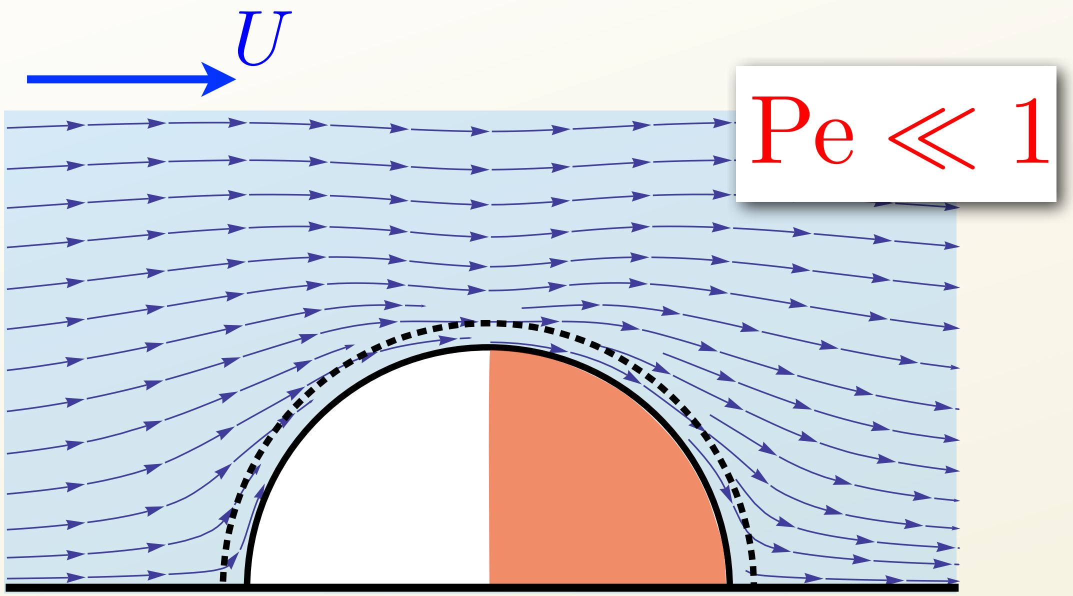

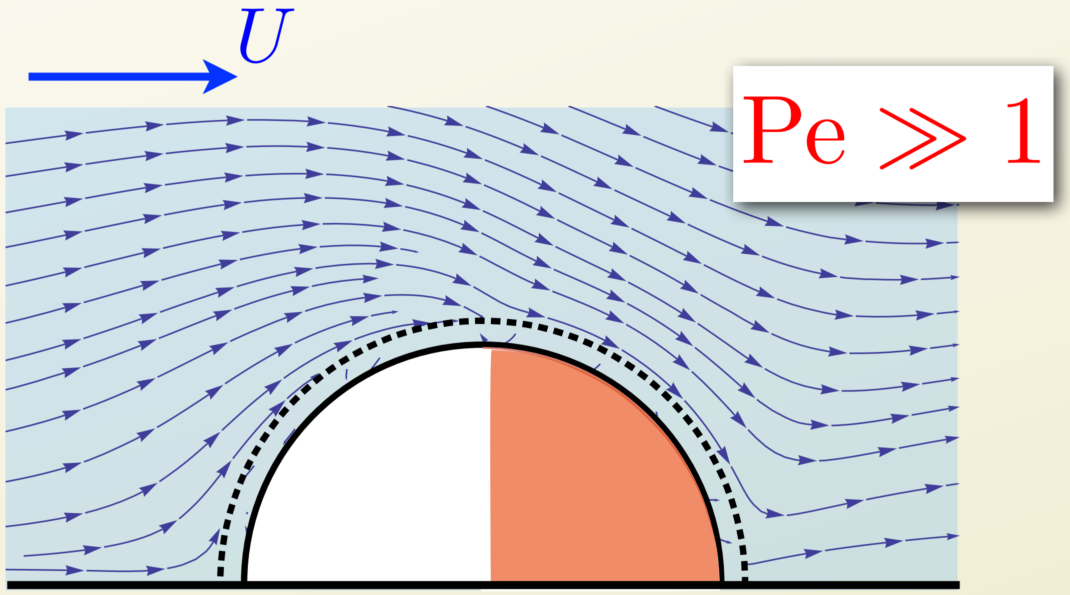

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$$U \sim \alpha_1 \sim \int_{\cos(\theta_p)}^{1} \mathrm{d}s\, s\bar{\alpha} \sim \sin^2(\theta_p)$$

first Legendre
coefficient of activity

$$U \sim \int_0^{\theta_p} \mathrm{d}\theta\, \sin^2(\theta) u_\theta^{\mathrm{slip}} \sim \sin^2\left(\tfrac{1}{2}\theta_p\right)$$

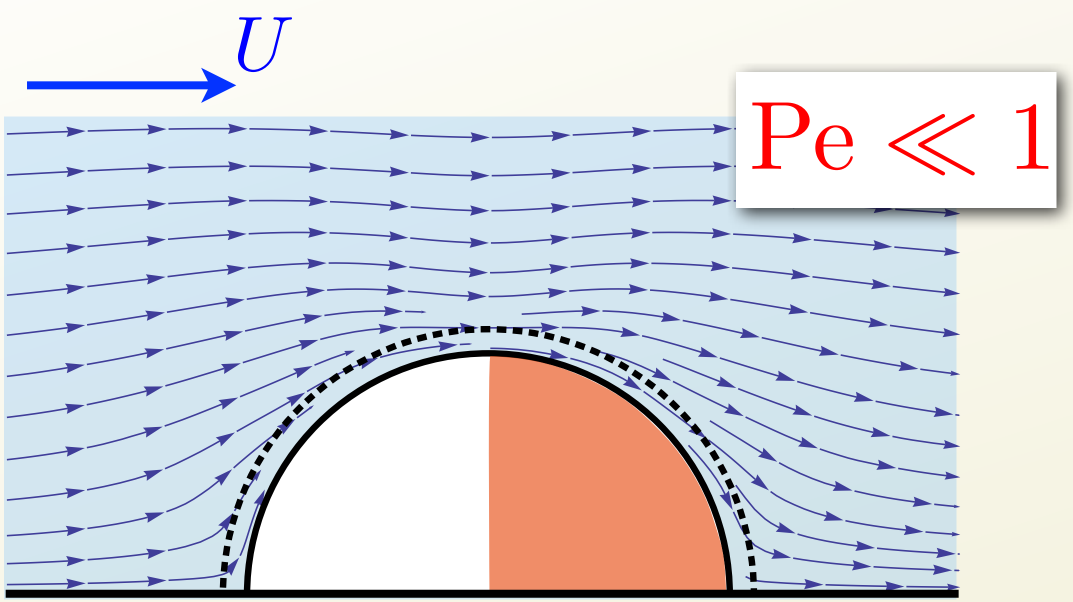
by, e.g., Stone & Samuel

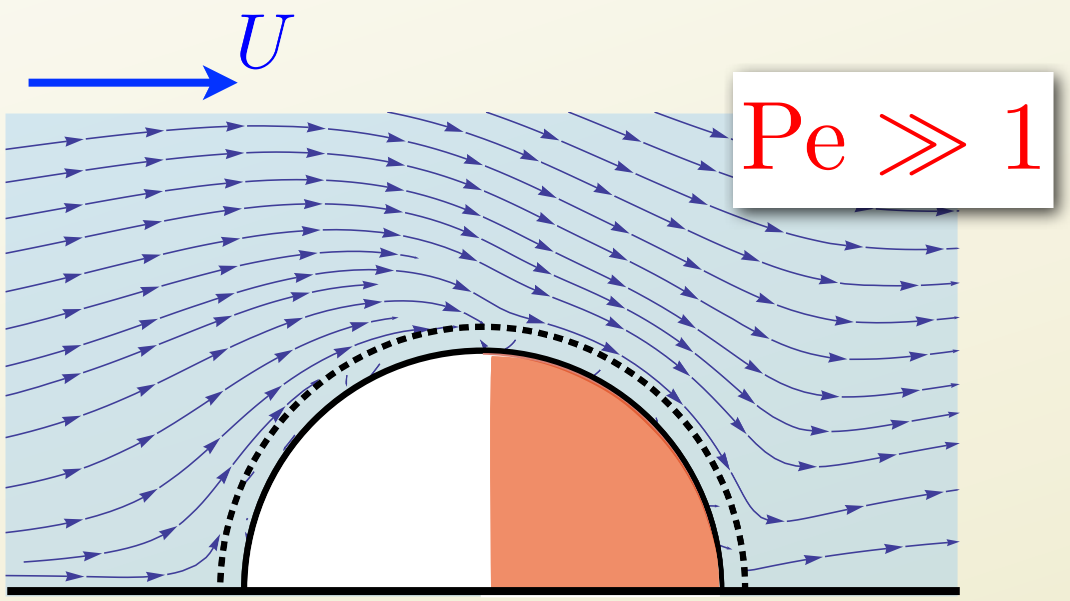
tangential flux
is conserved

Penn

Stone & Samuel *Phys. Rev. Lett.* **77**, 4102–4104 (1996)

So, what's different?





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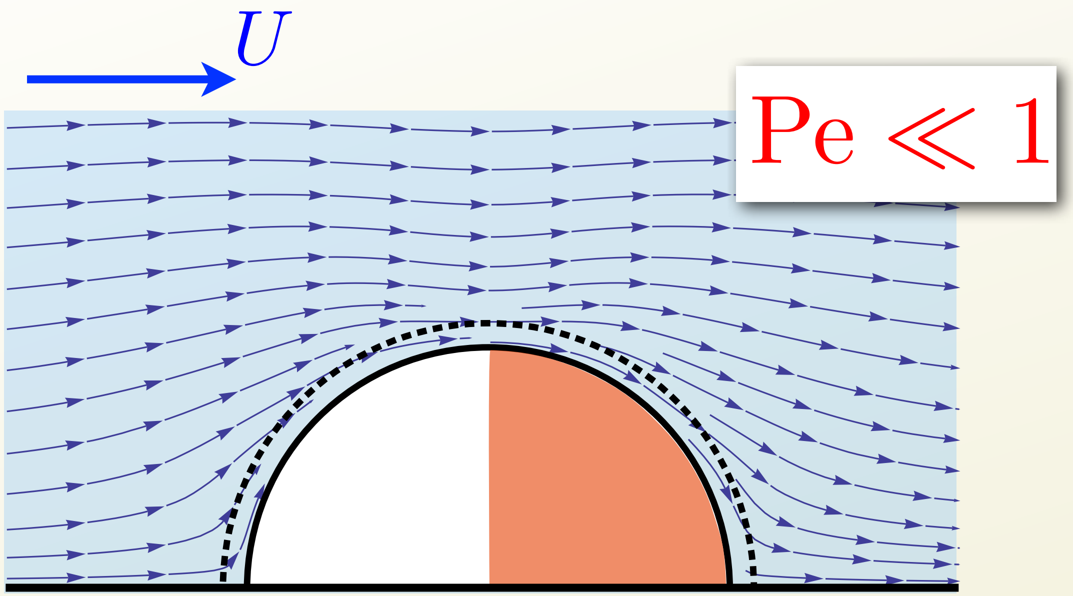
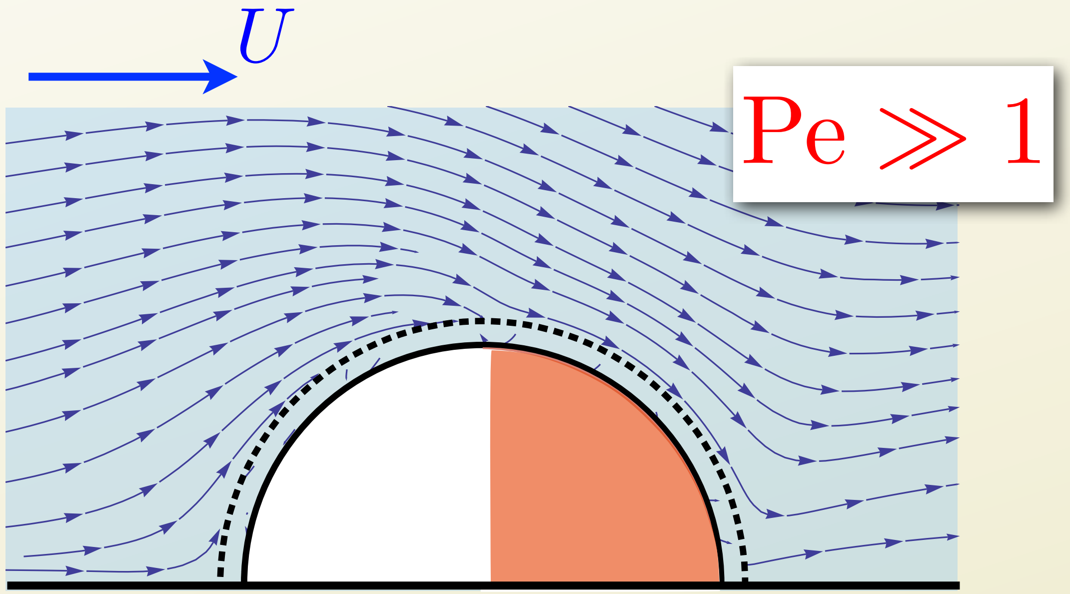
does not vanish for
total coverage $\theta_p \to \pi$

Penn

So, what's different?





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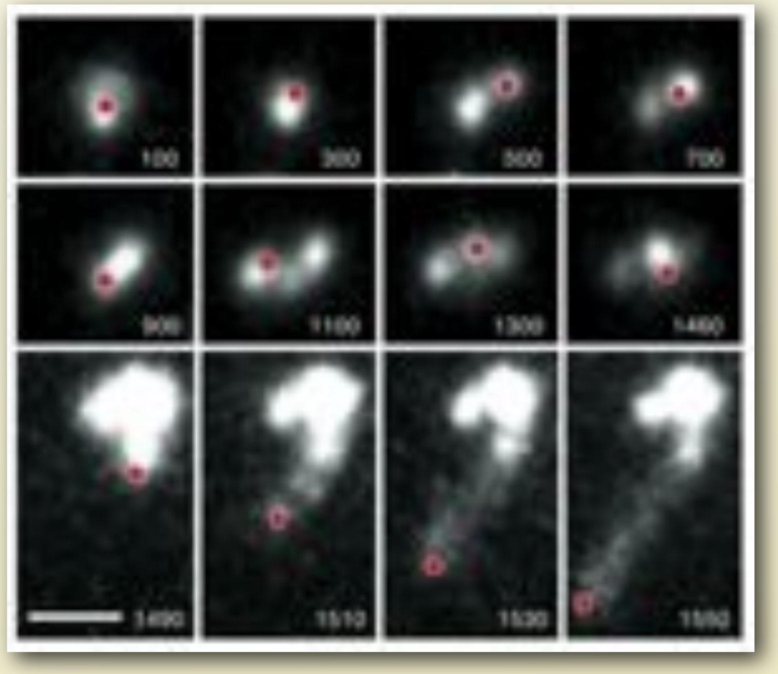

Cameron et al *Proc. Natl. Acad. Sci. USA* **96**, 4908–4913 (1999)

- is state of total coverage unstable?
- if so, what is the critical Péclet number for the instability?

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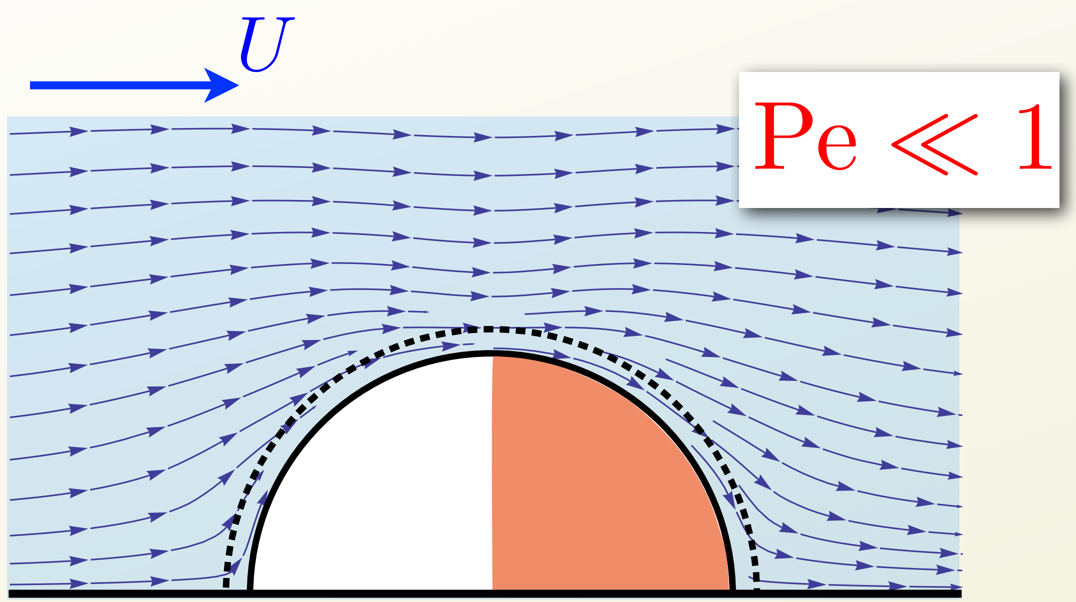
Different scenarios



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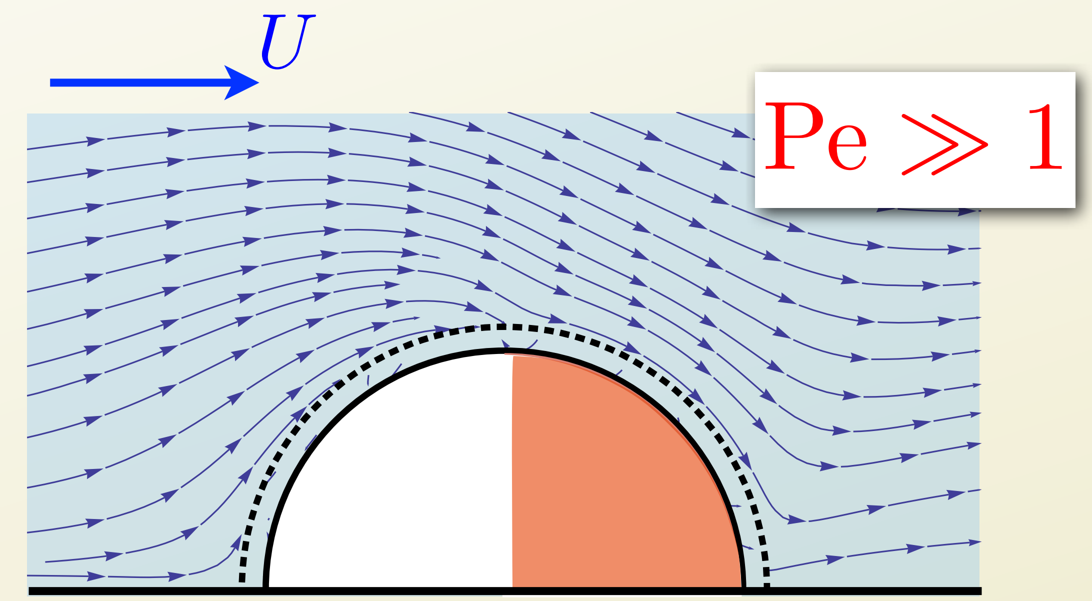

$$U \sim \left(\frac{m^A \bar{\alpha}}{\delta \ln\left(\frac{a}{\delta}\right)} \right)^{1/2} \sin^3\left(\tfrac{1}{2}\theta_p\right)$$

Four scenarios

repulsive producer $V > 0,\ \alpha > 0$	attractive producer $V < 0,\ \alpha > 0$
repulsive consumer $V > 0,\ \alpha < 0$	attractive consumer $V < 0,\ \alpha < 0$

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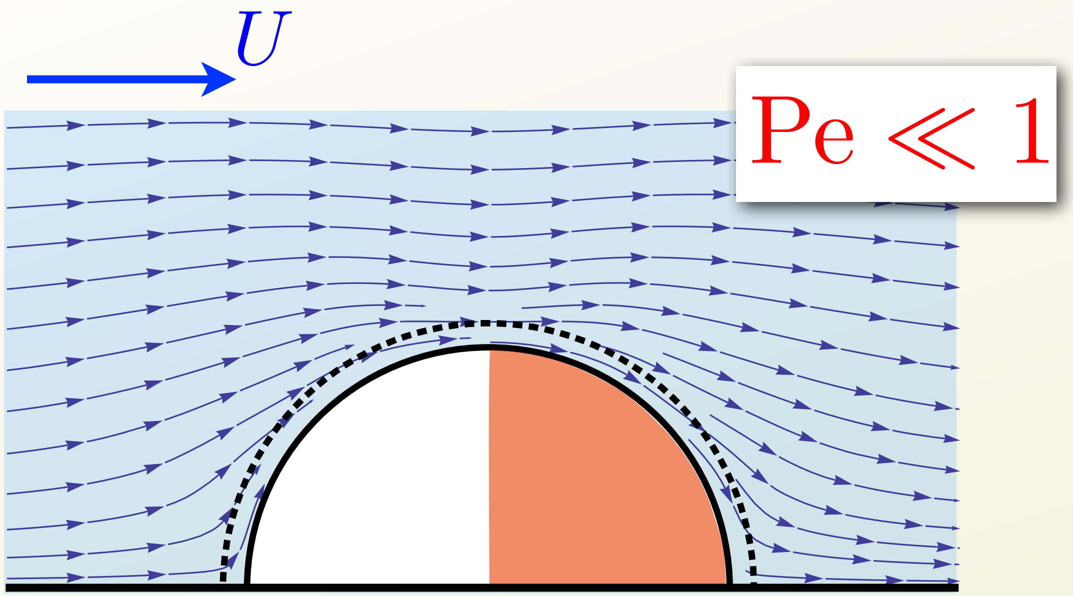
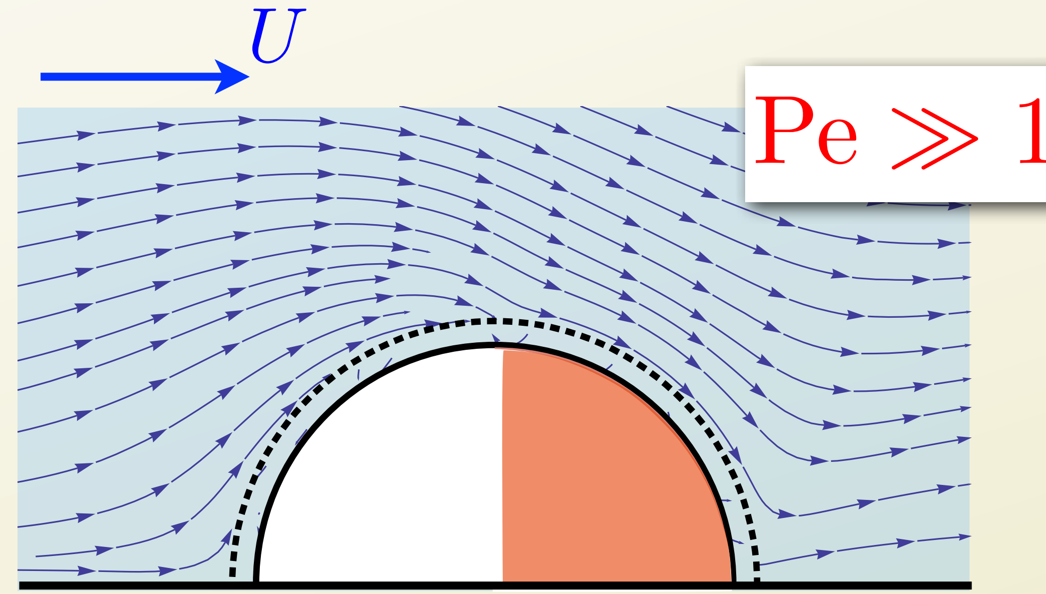
Different scenarios





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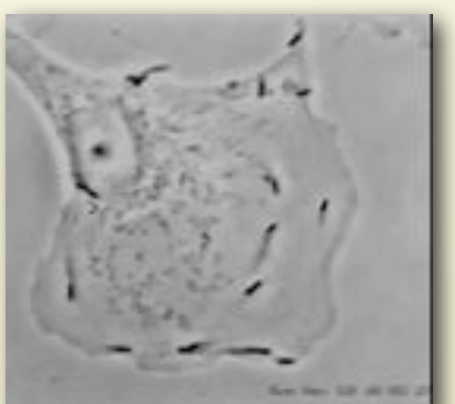

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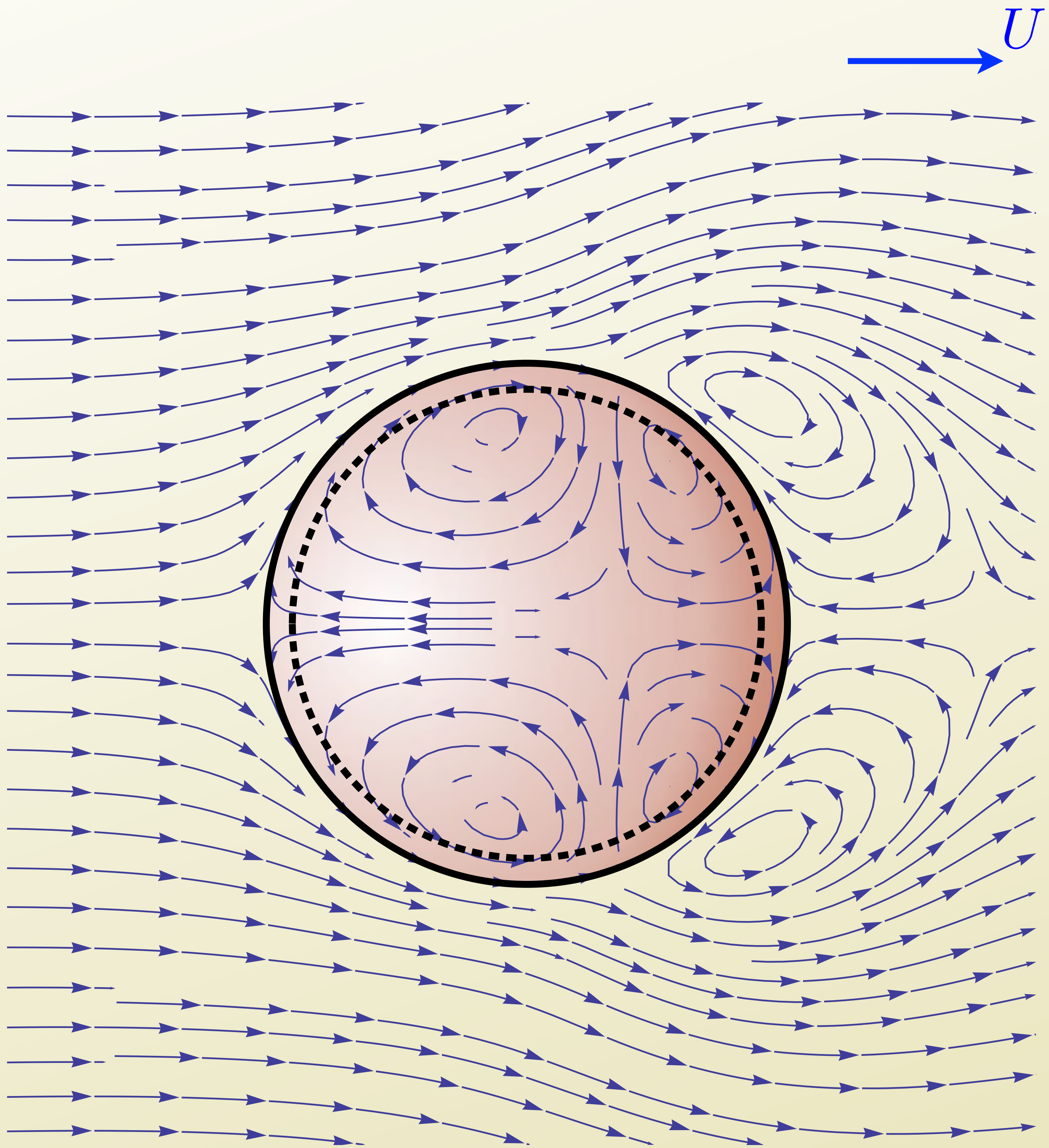
Fluid droplets



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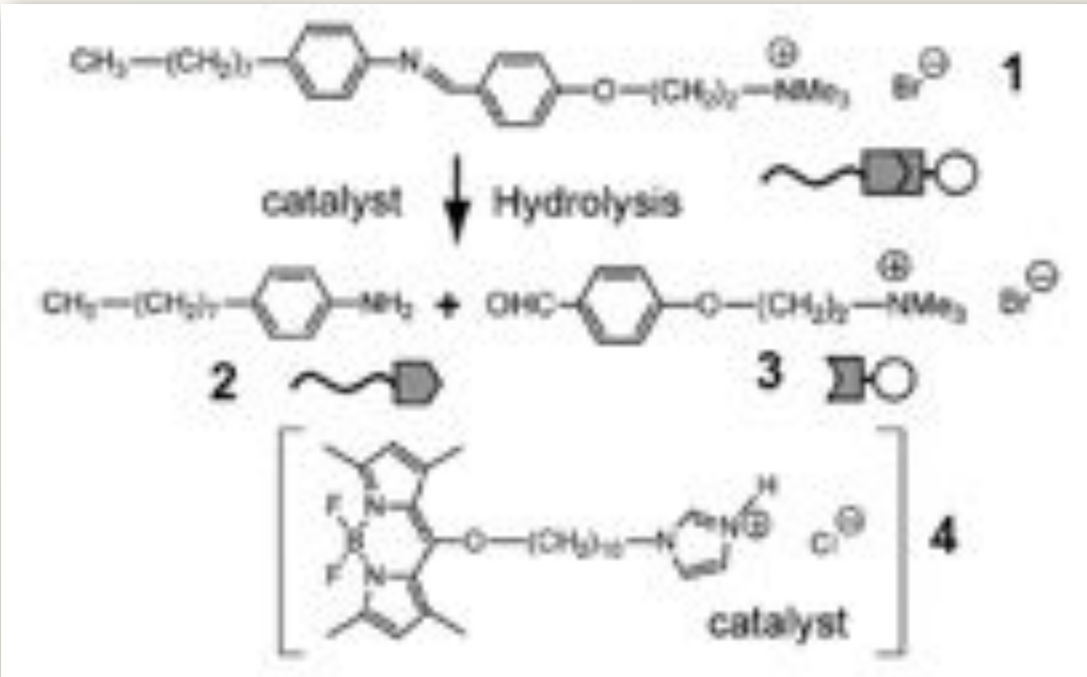
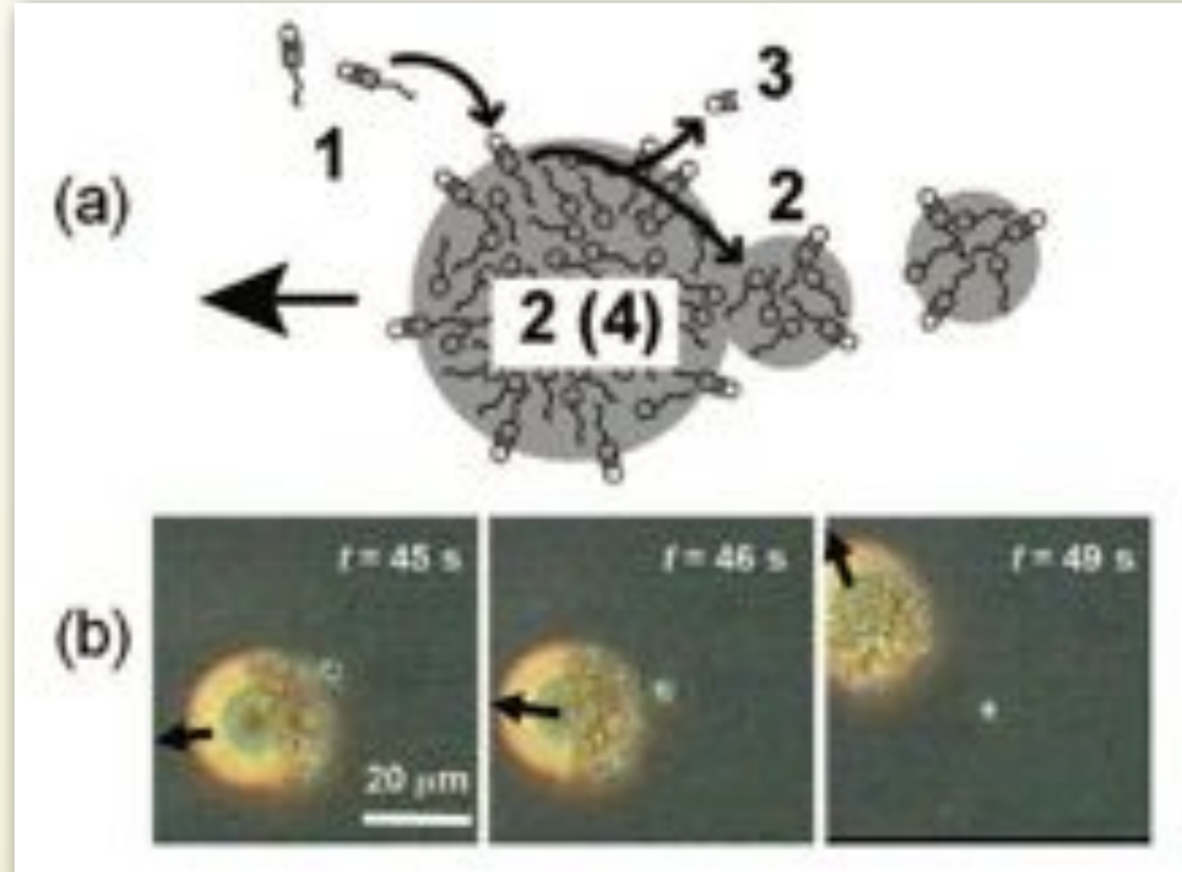
FLUID DROPLETS





- Droplets containing a catalyst dispersed in a bulk fuel
- Fuel hydrolysed at the surface
- Waste product accumulates on the surface and is released at the rear
- Self-maintained surface tension gradients drive Marangoni flows

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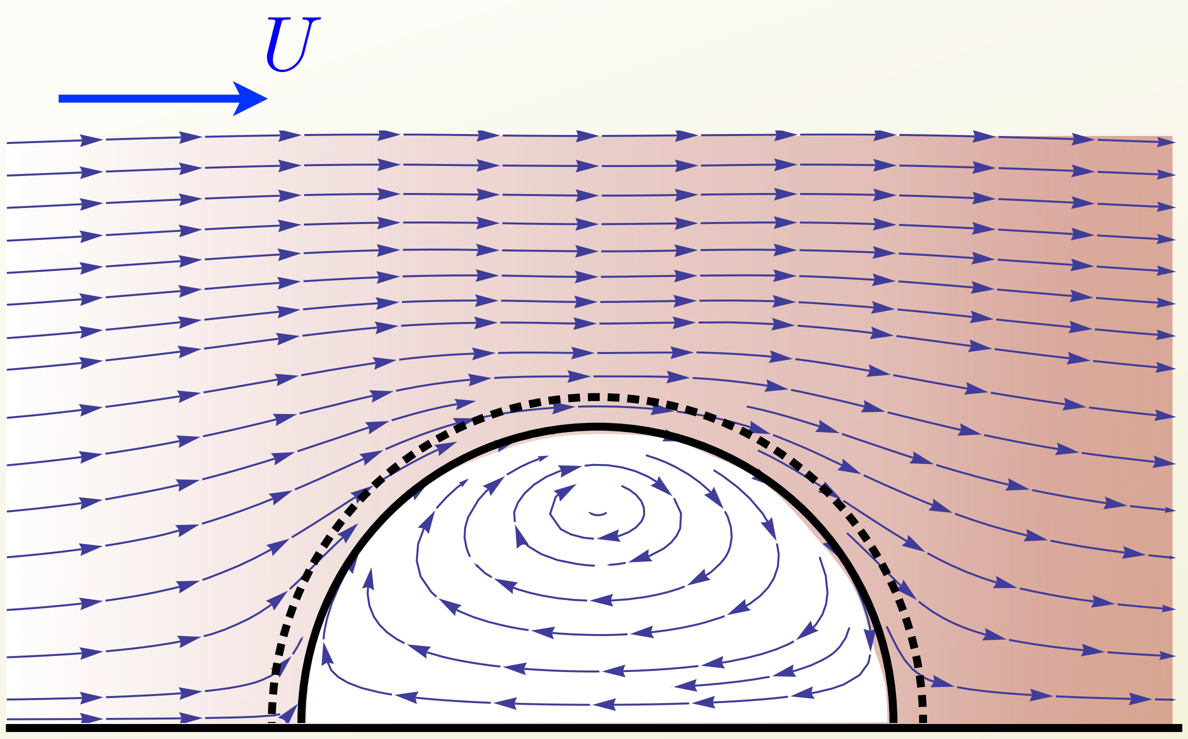
TOYOTA ET AL *J. Am. Chem. Soc.* **131**, 5012–5013 (2009)

FLUID DROPLETS

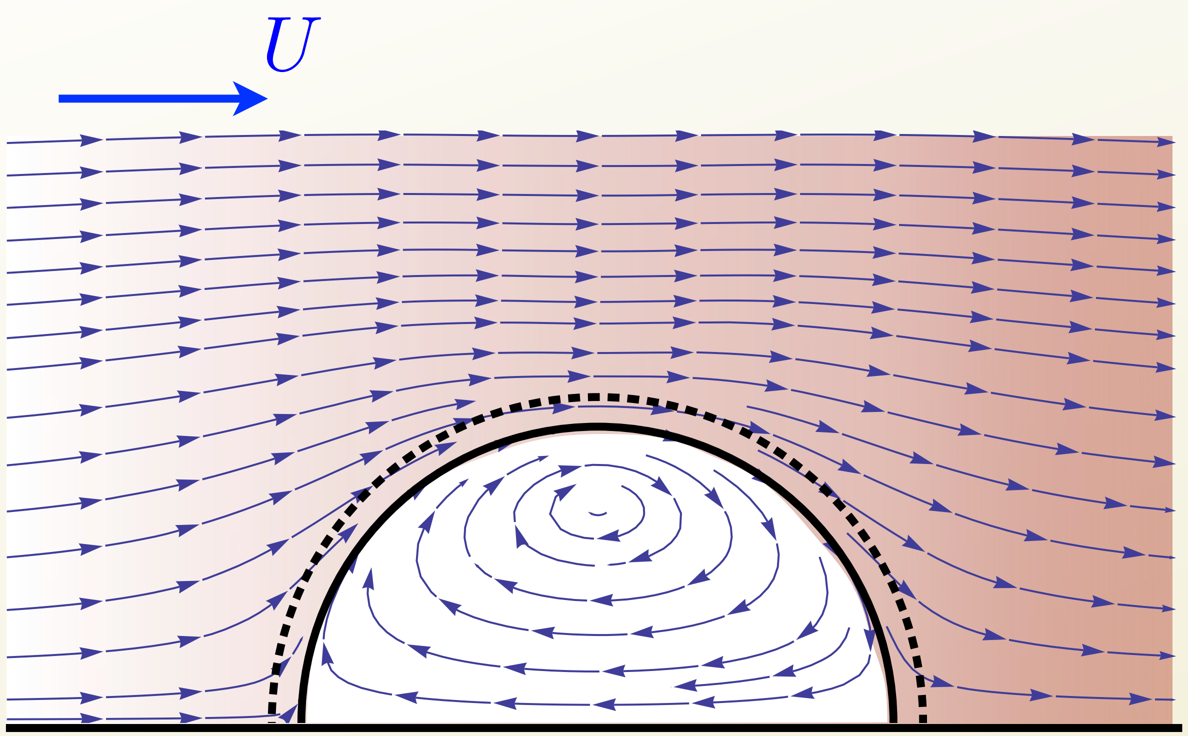


Rest frame of the particle

- Spherical particle
- Axisymmetry, steady state
- Single solute species
- Purely radial potential with compact support
- Zero Reynolds number
- Zero Péclet number

LEVICH, KRYLOV *Ann. Rev. Fluid Mech.* **1**, 293–316 (1969)
LEVICH, KUZNETSOV *Dokl. Akad. SSSR* **146**, 145–147 (1962)
REDNIKOV, RYAZANTSEV, VERLARDE *Phys. Fluids* **6**, 452–468 (1994)

Fluid droplets



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Stress balance at the interface

$$(\sigma_{ab} - \tilde{\sigma}_{ab})\, n_b - \gamma(2H)\, n_a + \nabla_a \gamma = 0$$

external fluid stress

internal fluid stress

surface normal

surface tension

mean curvature

Marangoni stress

Levich, Krylov *Ann. Rev. Fluid Mech.* **1**, 293–316 (1969)
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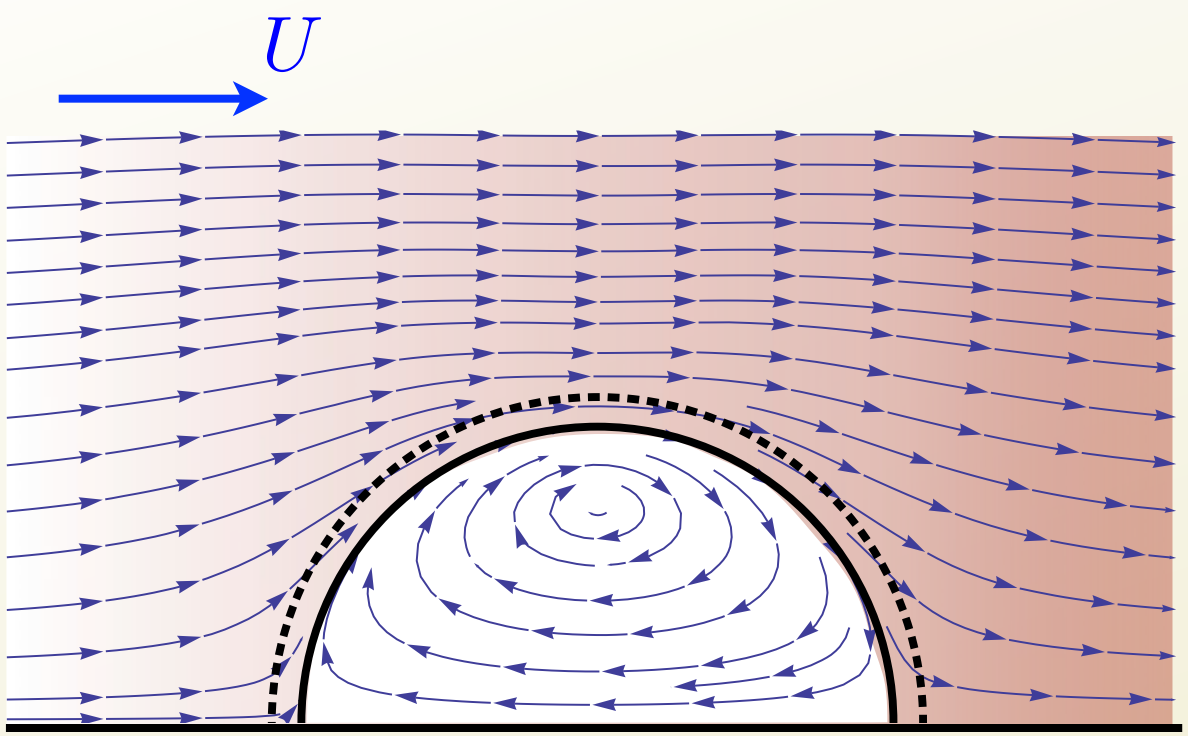

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Solve as before; e.g., the speed is

$$U = \frac{k_B T}{3\mu} \frac{\mu + \tilde{\mu}}{\mu + \frac{3}{2}\tilde{\mu}} \frac{c_1 \delta^2}{a} \int_0^1 \mathrm{d}z \left(\left(2 + \frac{\tilde{\mu}}{\mu + \tilde{\mu}} \right) z + \frac{\mu}{\mu + \tilde{\mu}} \frac{a}{\delta} - \frac{\delta}{2a} \frac{\tilde{\mu}}{\mu + \tilde{\mu}} \frac{3z^2 + 2\delta z^3/a}{(1 + \delta z/a)^2} \right) \left[1 - \mathrm{e}^{-V/k_B T} \right]$$

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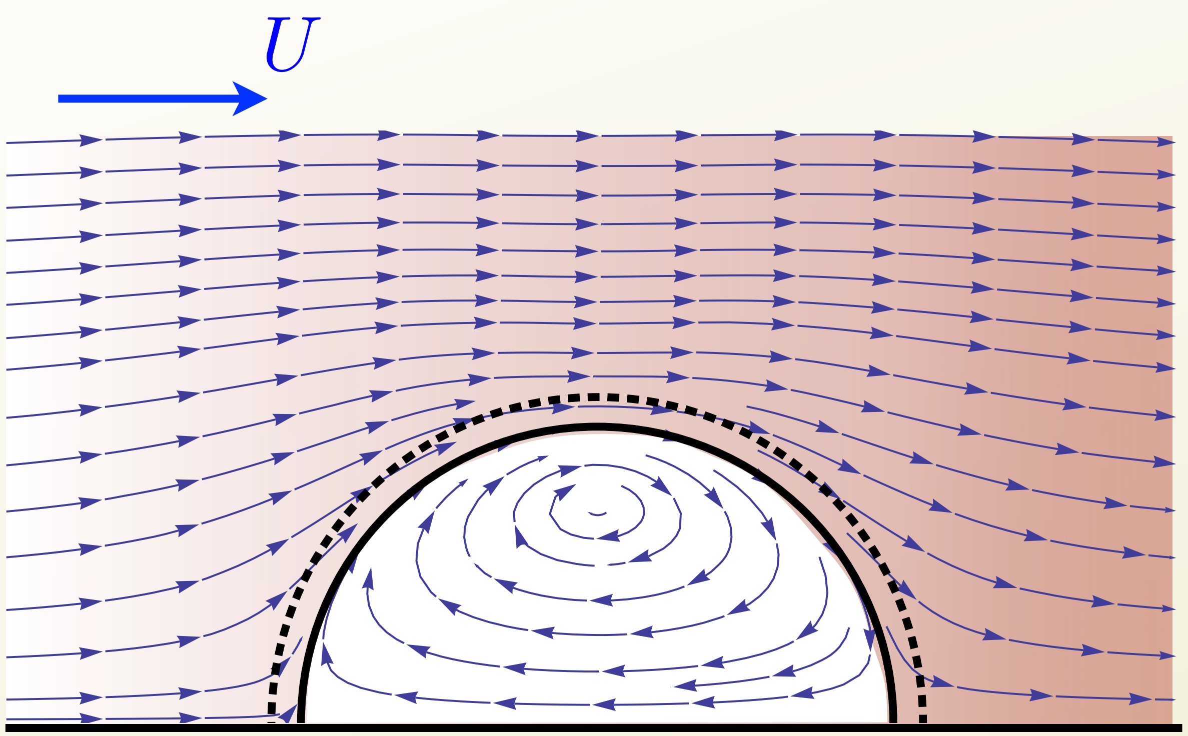
FLUID DROPLETS



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limit $\frac{\mu}{\tilde{\mu}} \longrightarrow 0$
(solid particle)

$$\boxed{U \sim \frac{k_B T \alpha_1}{\mu D} \delta^2}$$

independent of particle size

$$\boxed{U \sim \frac{k_B T \alpha_1}{\mu D} a\delta}$$

proportional to particle radius

limit $\frac{\tilde{\mu}}{\mu} \longrightarrow 0$
(gas bubble)

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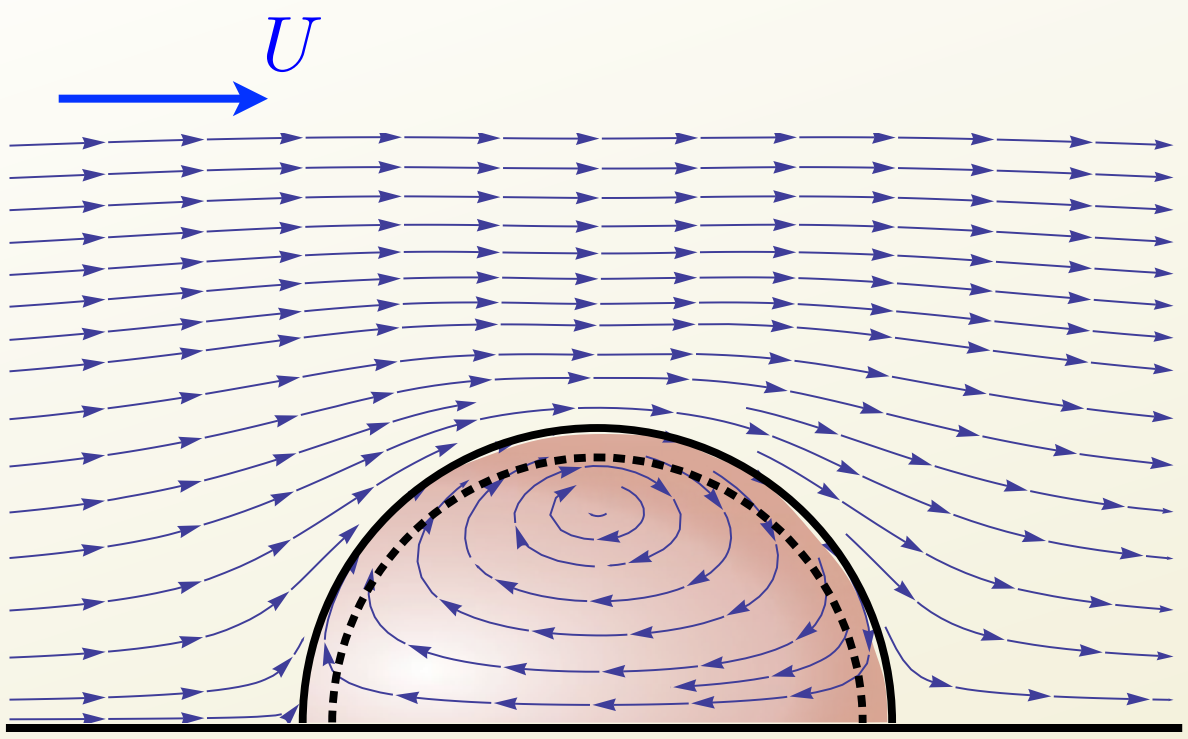
Activity on the Inside



Rest frame of the particle

- No need for a favourable environment -- take everything you need with you!
- “Clean” system; everything is internal
- Only interaction between droplets is hydrodynamic

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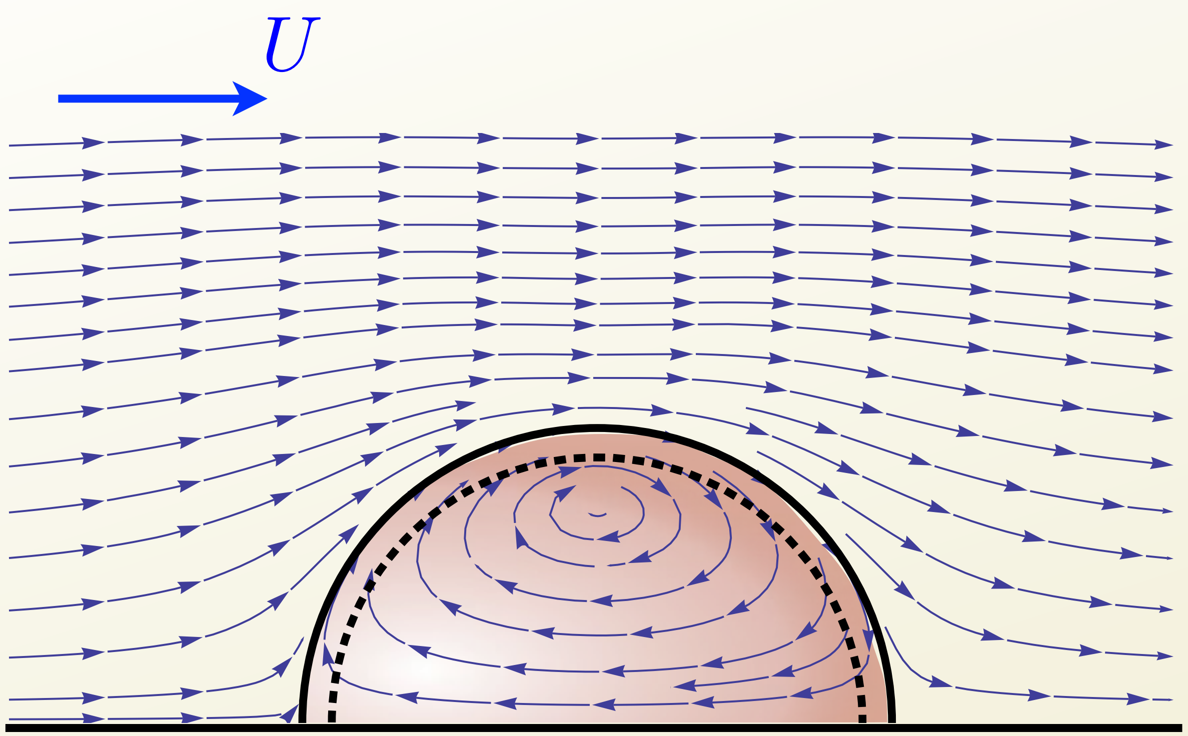
ACTIVITY ON THE INSIDE



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- "Clean" system; everything is internal
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Solve as before; e.g., the speed is

$$U = \frac{k_B T}{3\mu} \frac{\mu}{\mu + \frac{3}{2}\tilde{\mu}} \tilde{c}_1 \tilde{\delta} \int_0^1 \mathrm{d}z \left(1 - 5\frac{\tilde{\delta}}{a} z + 6\left(\frac{\tilde{\delta}}{a}\right)^2 z^2 - 2\left(\frac{\tilde{\delta}}{a}\right)^3 z^3\right) \left[1 - \mathrm{e}^{-\tilde{V}/k_B T}\right]$$

Scaling $\quad \boxed{U \sim \frac{k_B T \alpha_1}{\mu D} \frac{\mu}{\mu + \frac{3}{2}\tilde{\mu}} a\tilde{\delta}}$

same as the
"gas bubble"

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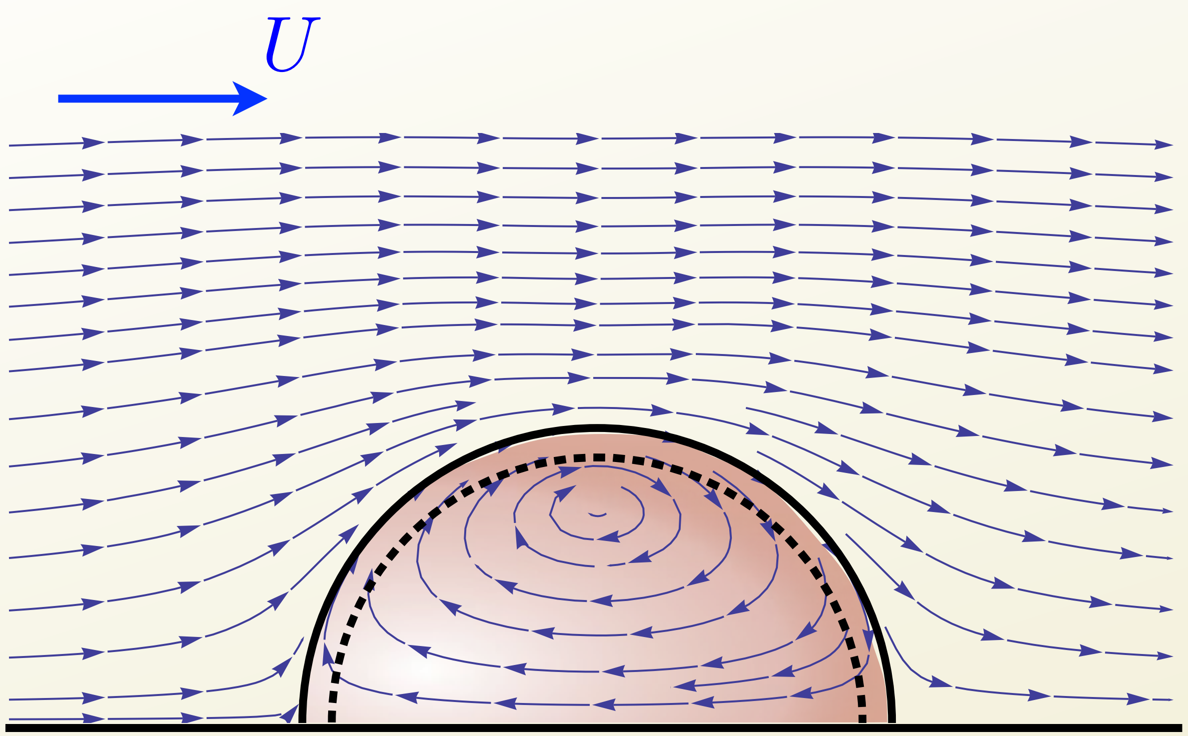
SURFACE TENSION GRADIENTS



Rest frame of the particle

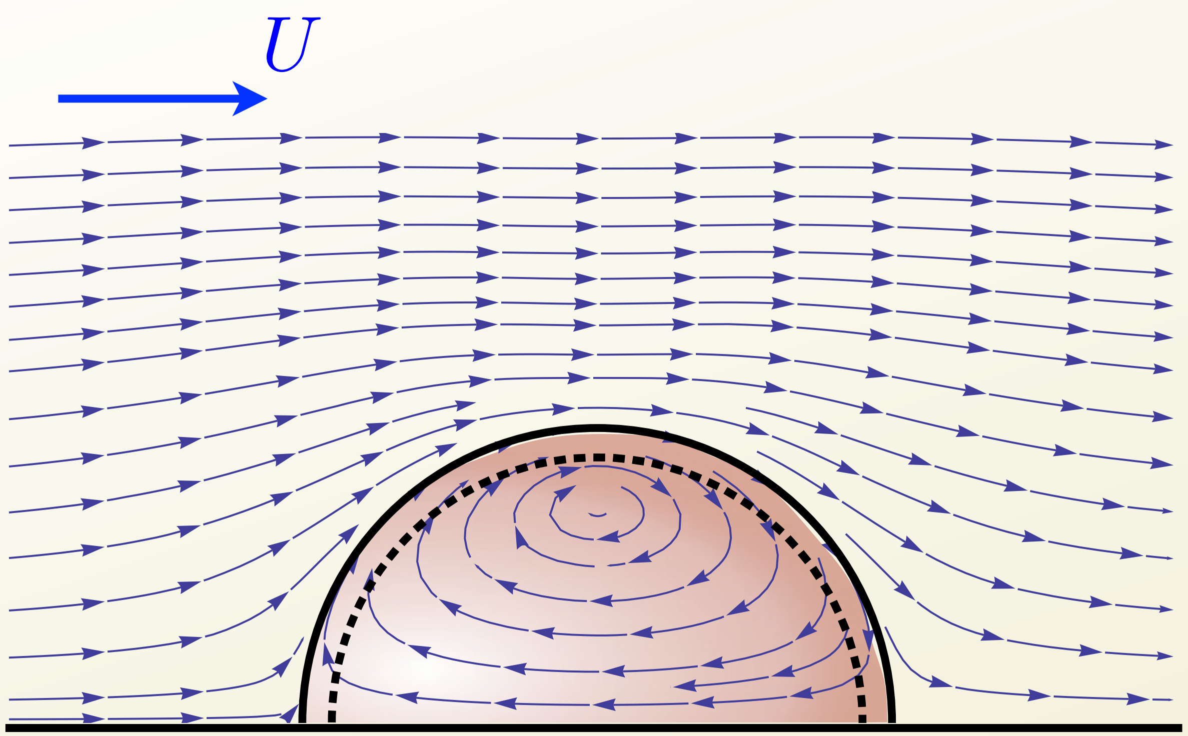
- Activity due to a surface active catalyst
- Surface adsorbed species lower the surface tension
- Chemical reaction near surface produces local heating; lowers surface tension
- Non-uniform surface tension drives Marangoni flows

Stress balance at the interface

$$(\sigma_{ab} - \tilde{\sigma}_{ab})\, n_b - \gamma(2H)\, n_a + \nabla_a \gamma = 0$$

Pearson *J. Fluid Mech.* **4**, 489–500 (1958)
Sternling, Scriven *AIChE J.* **5**, 514–523 (1959)
Levich, Krylov *Ann. Rev. Fluid Mech.* **1**, 293–316 (1969)

Surface tension gradients



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Additional contribution to the speed $$\frac{\mu+\tilde{\mu}}{3\mu\left(\mu+\frac{3}{2}\tilde{\mu}\right)}\gamma_1$$

typically surface tension is **lowered** so that γ_1 is **negative** Marangoni flows then **oppose** self-diffusiophoresis

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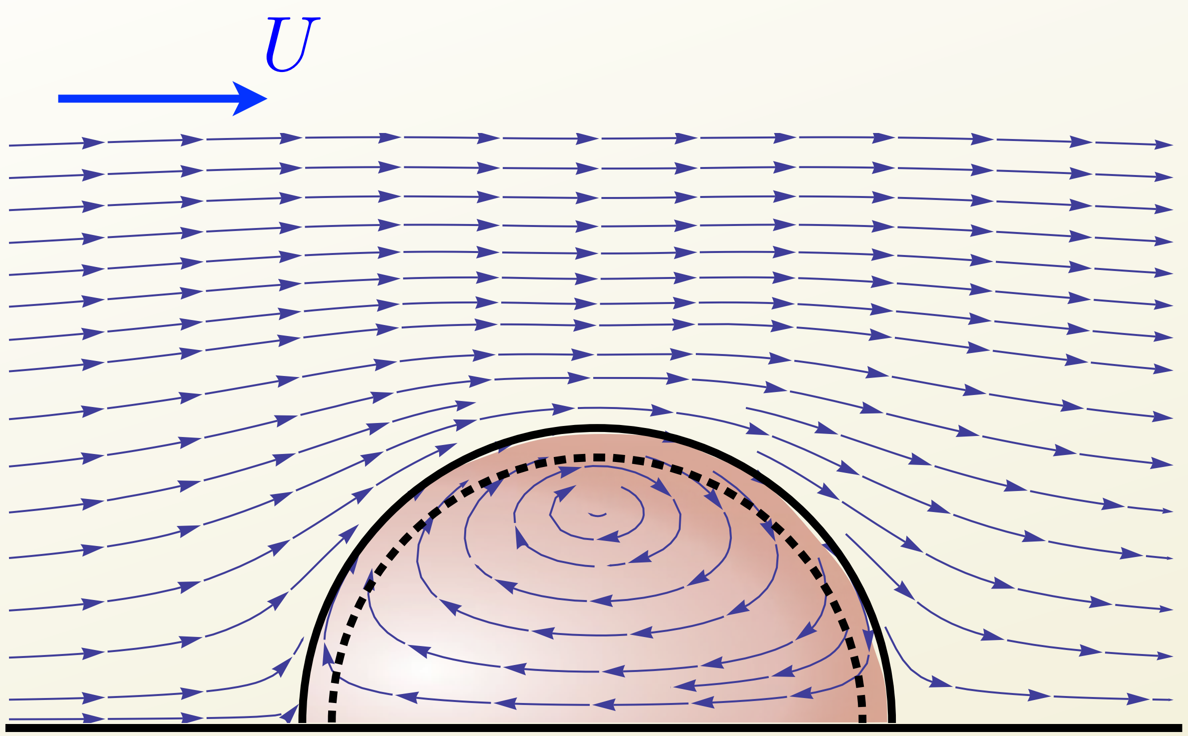
Surface tension gradients



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ratio $$\frac{\text{Marangoni}}{\text{diffusiophoresis}} \sim \frac{\mu+\tilde{\mu}}{\mu}\frac{D\gamma_1}{k_B T\alpha_1 a\delta}$$

can this be made small?

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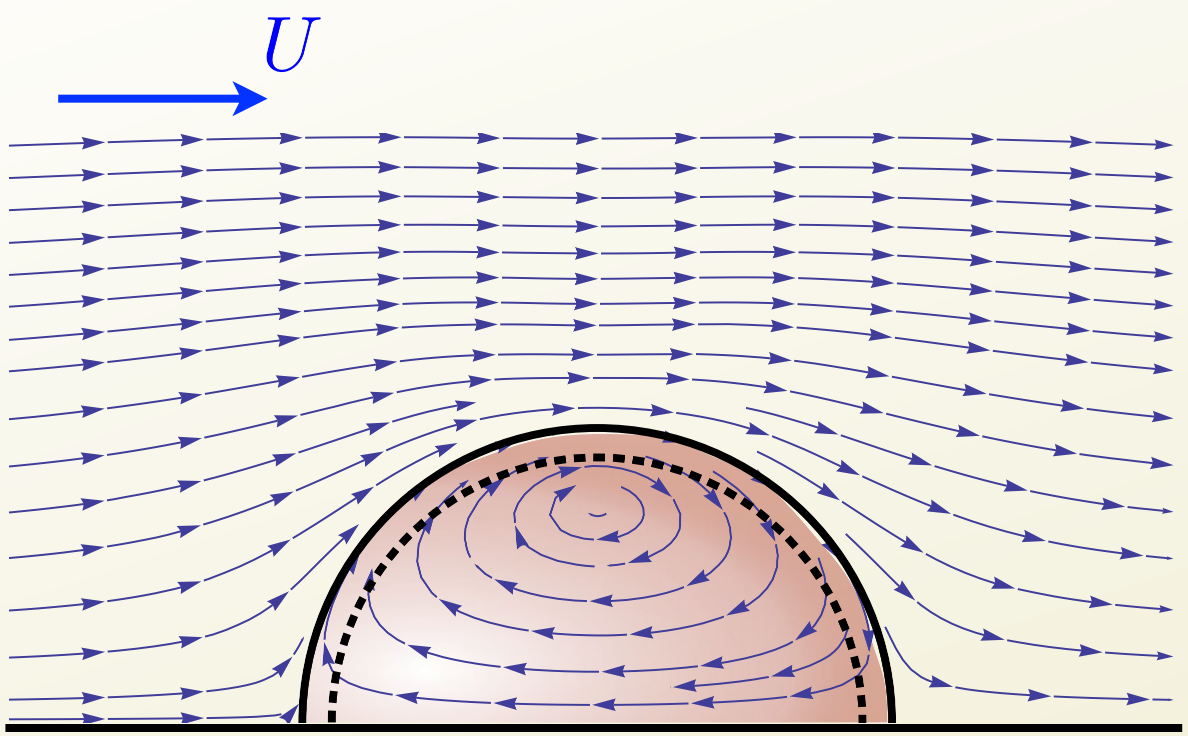
Droplet deformation



Rest frame of the particle

- Particle is a fluid droplet -- no reason why it won't deform
- Normal stress balance is really an equation for the drop shape

droplet remains approximately spherical provided $\frac{\mu U}{\gamma} \ll 1$

Stress balance at the interface

$$(\sigma_{ab} - \tilde{\sigma}_{ab})\, n_b - \gamma(2H)\, n_a + \nabla_a \gamma = 0$$

external fluid stress

internal fluid stress

surface normal

surface tension

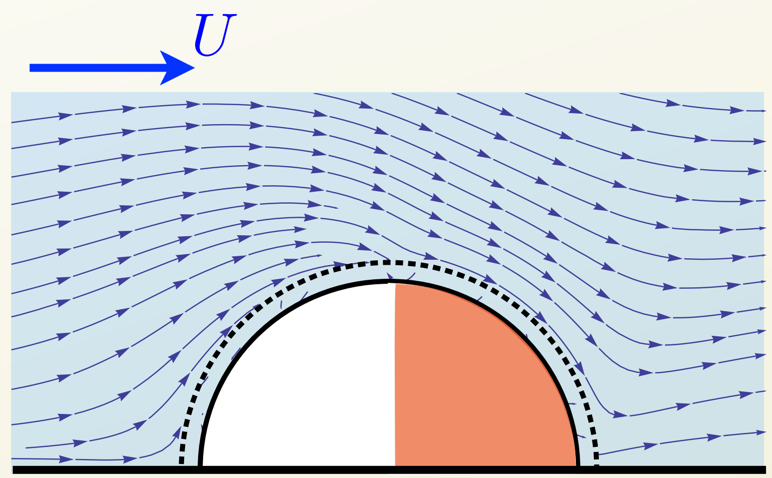
mean curvature

Marangoni stress

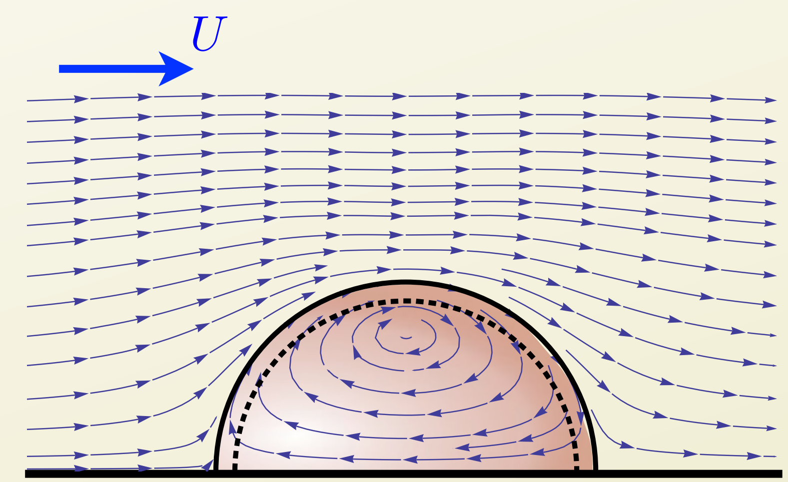
TAYLOR *Proc. Roy. Soc. Lond. A* **146**, 501–523 (1934)
TAYLOR, ACRIVOS *J. Fluid Mech.* **18**, 466–476 (1964)
STONE, LEAL *J. Fluid Mech.* **220**, 161–186 (1990)

Thanks!

Andrea Liu, Timon Idema



- High Péclet number relevant to biological motility
- Different scaling with activity, dependence on coverage and successful strategies



- Fluid drops can move due to internal motor
- "Clean" system
- Faster than a solid particle

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