PRACTICE FINAL EXAM

Math 340
12/13/2018

Name: ________________________________

ID: ________________________________

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

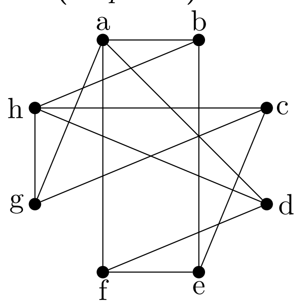
Signature: ________________________________

Read all of the following information before starting the exam:

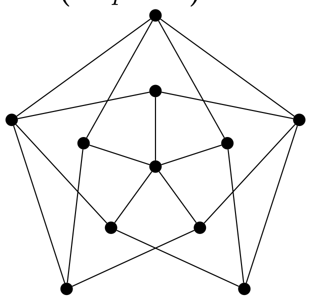
- Check your exam to make sure all pages are present.
- Leave answers unsimplified—you may include factorials, $P(n,k)$, $\binom{n}{k}$, etc. in your answers. Do not leave unevaluated $\sum_{i \le k}$ in your final answers.
- You may use writing implements and a single 3"x5" notecard.
- You may not use a calculator.
- You may use any result proved in class or in the textbook in your arguments.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	10	
3	10	
4	15	
5	12	
6	12	
7	14	
Total	100	

1. (*10 points*) Either redraw this graph so no lines cross or prove that it is non-planar.



2. (*15 points*) Show that the following graph has chromatic number exactly 3:



3. (*15 points*) (**a**) How may ways are there to rearrange the letters AEIOUCDVTTT so that none of the T's are adjacent.

(**b**) How may ways are there to rearrange the letters AEIOUBCDFGHTTT so that none of the T's are adjacent and the vowels appear in alphabetical order.

4. (*12 points*) Determine the number of integers between 1 and 1000 (including 1 and 1000) which are not divisible by 6, by 7, or by 8. ($1000/6 = 166.6\overline{6}$, $1000/7 = 142.8\ldots$, $1000/8 = 125$. You may need to do some additional division by hand.)

5. (*14 points*) Suppose we roll 8 six sided dice, each a different color, so there are 6^8 possible outcomes. In how many of these outcomes do all six numbers appear?

6. (*10 points*) We are interested in quaternary sequences (i.e. using the digits $\{0, 1, 2, 3\}$) which never contain either the subsequence 00 or the subsequence 02. Let s_n be the number of such sequences of length n.

(**a**) How many of these sequences are there of length 1?

(**b**) How many of these sequences are there of length 2?

(**c**) Verify that the numbers s_n satisfy the recurrence relation

$$s_n = 3s_{n-1} + 2s_{n-2}.$$

(**d**) Solve the recurrence relation to give a formula for s_n.

7. (*10 points*) The game of Kayles is played with a row of n coins. Two players alternate, and on their turn, a player can either remove a single coin, or two adjacent coins. In this case, adjacent means that there are no holes (places where a coin was removed previously) between them. A player wins if they remove the final coin.
(For example, the first play wins when the game starts with three coins by first removing the middle coin; the remaining coins are not adjacent, so the second player can only remove one, and the first player wins by removing the other.)

What is the Grundy number of the game which begins with three coins?