Math 114, HW 3

Due Friday, April 24

• Consider the following corollary of the compactness theorem:

If \( \Sigma \models \tau \) then there is a finite \( \Sigma_0 \subseteq \Sigma \) such that \( \Sigma_0 \models \tau \).

Give a short proof of the compactness theorem from this.

• Let \( \Sigma \) be an effectively enumerable set of wffs. Assume that for each wff \( \tau \), either \( \Sigma \models \tau \) or \( \Sigma \models \neg \tau \). Show that the set of tautological consequences of \( \Sigma \) is decidable.

• Show that \( \{ \land, \leftrightarrow, + \} \) is complete but that no proper subset is complete.

(Recall that \(+\) is the exclusive or connective: \( \alpha + \beta \) is true if exactly one of \( \alpha \) and \( \beta \) is true.)