The final exam will take place on 12/12 from 8:00-10:00 in MSB 403. The exam will cover all the material we have covered in class: 11.1-11.4, 11.5, 11.6, 12.2-12.9, Chapter 13, Chapter 14. The material covered after the second midterm (sections 14.3-14.7) will be more important, but the final exam is cumulative.

You should be able to:

1. Define and give examples of vectors, parallel vectors, scalars.
2. Perform basic operations (scalar multiplication, addition, subtraction, dot product, cross product) on vectors *and explain their geometric meaning.*
3. Find scalar and orthogonal projections and explain their geometric meaning.
4. Give examples of vector valued functions.
5. Translate between curves described geometrically or as solutions to an equation and parametric representations of a curve.
6. Find derivatives, integrals, unit tangent vectors, and unit normal vectors to a paramaterized curve, and give geometric interpretations of these notions.
7. Interpret functions $z = f(x, y)$ as graphs. Understand, draw, and interpret level curves of a 3D function.
8. Limits
   (a) Define a limit in multiple dimensions.
   (b) Explain why limits in more than 1 dimension are more complicated than 1 dimensional limits.
   (c) Find a limit that exists using limit laws or by cancelling everywhere a denominator is non-zero.
   (d) Show that a limit does not exist using the two-path test.
9. Partial Derivatives
   (a) Define a partial derivative.
   (b) Give a geometric interpretation of the partial derivative.
   (c) Calculate partial derivatives using the definition.
   (d) Calculate partial derivatives using rules from Calc 1 (product rule, chain rule, etc.).
   (e) State and apply Clairaut's Theorem.
   (f) Define the tangent plane to a multivariable function.
   (g) Define what it means for a multivariable function to be differentiable.

(h) Be able to use the tangent plane to give a linear approximation to a multivariable function.

(i) Know Theorem 12.5 (sufficient conditions for differentiability).

(j) Know the difference between partial derivatives existing and differentiability.

(k) Be able to state and apply the chain rule for *any* series of functions applied to other functions, no matter how many intervening variables there are.

10. Gradients

    (a) Define the directional derivative.

    (b) Define the gradient.

    (c) Find the gradient and use it to find directional derivatives.

    (d) Give a geometric interpretation of the gradient.

    (e) Understand the relationship between the gradient and level curves of a differentiable function.

11. Minima and Maxima

    (a) Define critical points and local minima/maxima.

    (b) Find critical points using the first derivative test.

    (c) Use the second derivative test to classify critical points as local minima, local maxima, or saddle points.

12. Lagrange Multipliers

    (a) Use the method of Lagrange multipliers to find minima and maxima of constrained functions.

    (b) Explain why the method of Lagrange multipliers works, using the relationship between gradients and level functions.

13. Express a two dimensional region using rectangular or polar coordinates.

14. Find double integrals.

15. Express a three dimensional region using rectangular coordinates.

16. Find triple integrals using rectangular, polar, or cylindrical coordinates.

17. Find the center of mass of an object in two or three dimensions.

18. Carry out a change of variables in two dimensions.

19. Define and give examples of vector fields.

20. Evaluate line integrals.

21. Modify the paramaterizations of simple surfaces (sphers, cylinders, etc.) to find paramaterizations of portions of these surfaces.
22. Evaluate surface integrals.
23. Define a connected and a simply connected region, and give an example and a non-example of each.
24. State the four equivalent formulations of a vector field being conservative.
25. Prove certain directions of the equivalence:
    (a) That having a potential function implies path independence ($1 \Rightarrow 3$).
    (b) That path independence is equivalent to having 0 circulation on all closed curves ($3 \Leftrightarrow 4$).
26. Be able to find a potential function for a conservative vector field.
27. State Green's Theorem in both circulation and flux forms.
28. Explain why Green's Theorem fails if a region is not simply connected.
29. Apply Green's Theorem in either direction.
30. Understand the proof of Green's Theorem for simple regions and be able to carry out the individual steps in similar situations.
31. Define divergence and curl and explain the intuition behind these notions.
32. State Stokes' Theorem and the Divergence Theorem.
33. Be able to apply Stokes' Theorem and the Divergence Theorem in both directions.