

# MIDTERM 1

Math 340  
10/4/2012

Name: \_\_\_\_\_

ID: \_\_\_\_\_

“I have adhered to the Penn Code of Academic Integrity in completing this exam.”

Signature: \_\_\_\_\_

**Read all of the following information before starting the exam:**

- Check your exam to make sure all pages are present.
- You do not need to simplify answers. You may include factorials,  $P(n, k)$ ,  $\binom{n}{k}$ , etc. in your answers. Do not use  $\sum$  in your answers.
- You may use writing implements and a single 3"  $\times$  5" notecard.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Good luck!

1	15	
2	10	
3	20	
4	15	
5	20	
6	10	
7	10	
Total	100	

Name: \_\_\_\_\_

**1.** (*15 points*) 10 people go to a party.

(a) If all 10 people arrive separately, how many possible orders are there for them to arrive in?

(b) If instead the people arrive in 3 groups of 3 and one single person, how many possible orders are there? (We do not make further distinctions about the three people in a group arriving together.)

(c) If instead the people arrive in 2 groups of 3 and 2 groups of 2, how many possible orders are there?

**2.** (*10 points*) I deal out 13 red cards and 13 black cards in a long line of 26 cards. How many ways are there so that the red cards are split into at least two runs?

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**3.** (20 points) A bag contains red, blue, and green marbles (and many marbles of each color). Marbles are indistinguishable except for color.

(a) I select 8 marbles from the bag and arrange them in a line. How many possible arrangements are there?

(b) I place 8 marbles from the bag in a cup. Paying attention only to how many marbles of each color there are in the cup, how many possibilities are there?

(c) If I place *at most* 8 marbles in the cup, how many possibilities are there?

(d) If I place exactly 8 marbles in the cup, but the bag has only 4 green marbles available (but plenty of the other colors), how many possibilities are there?

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4. (15 points) Prove by induction that for all  $n \geq 0$ ,  $\sum_{i=0}^n 3^i = \frac{3^{n+1}-1}{2}$ .

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**5.** (*20 points*) Five people will each perform one chore. The people are Alan, Betsy, Chris, David, and Ellen. The chores are taking out the trash, washing the dishes, sweeping, cleaning the bathroom, and vacuuming. However Alan is afraid of the vacuum, Betsy cannot be trusted to either clean the bathroom or vacuum. Chris, David, and Ellen all refuse to take out the trash, and David also refuses to sweep or wash dishes.

(a) Draw a chessboard corresponding to this problem.

(b) Consider the portion of the chessboard corresponding to the disallowed combinations of people and chores. What is the rook polynomial of this board?

(c) How many ways are there to assign chores so that everyone gets a chore they can actually do?

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- 6.** (*10 points*) We distribute 15 (identical) pieces of candy to 6 children.
- (a) How many ways are there to distribute the candy?
- (b) In how many of those ways does *no* child receives *exactly* 4 pieces?

**7.** (*10 points*) A director needs to cast a play from the 40 people, 20 male and 20 female, who auditioned. The cast will consist of a male lead, a female lead, two secondary roles that could be either male or female, and a chorus of 15 people of either gender. Furthermore Jane and John are two of the people who auditioned, and since their divorce, it would be a bad idea to cast them as the two leads. How many possible casts are there?