1. Find the angle between the curves

$$\mathbf{r}_1(t) = (t, t^2, t^3)$$
$$\mathbf{r}_2(s) = (2 + s, 3 + 2s, s^2)$$

at their point of intersection. (If necessary, you may use a calculator to estimate the angle.)

A) The angle is close to 5°

B) The angle is close to 35°

C) The angle is close to 45°

D) The angle is close to 65°

E) The angle is close to 95°

F) The curves do not intersect

Find pt $\cap$

$t = 2+s$

$t^2 = 3+2s$

$(2+s)^2 = 3+2s$

$4+4s+s^2 = 3+2s$

$1+2s+s^2 = 0$

$(s+1)^2 = 0$

$\Rightarrow s = -1$

$\Rightarrow t = 1$

$t = 1 \Rightarrow r_1 = (1, 1, 1)$

$s = -1 \Rightarrow r_2 = (1, 1, 1)$ ✓

$r_1'(t) = \langle 1, 2t, 3t^2 \rangle$

$r_1'(1) = \langle 1, 2, 3 \rangle = \bar{a}$

$r_2'(s) = \langle 1, 2, 2s \rangle$

$r_2'(-1) = \langle 1, 2, -2 \rangle = \bar{b}$

$\vec{a} \cdot \vec{b} = \cos\theta\, |\bar{a}||\bar{b}|$

$1 + 4 + -6 = \cos\theta \sqrt{1+4+9} \cdot \sqrt{1+4+4}$

$\dfrac{-1}{3\sqrt{14}} = \cos\theta$

$\theta \approx 95°$

2. Find the arc length of the segment of the curve

$$\mathbf{r}(t) = (\sqrt{2}\ln t,\ t,\ t^{-1})$$

for the parameter values $1 \leq t \leq 2$.

A) $\frac{1}{2}$ B) 1 C) $\frac{3}{2}$ D) 2 E) $\frac{5}{2}$ F) 3

$$\int_1^2 |r'(t)|\, dt$$

$$r'(t) = \left(\frac{\sqrt{2}}{t},\ 1,\ -t^{-2}\right)$$

$$|r'(t)| = \sqrt{\frac{2}{t^2} + 1 + \frac{+1}{t^4}} = \sqrt{\left(\frac{1}{t^2} + 1\right)^2} = (t^{-2} + 1)$$

$$\int_1^2 t^{-2} + 1\, dt$$

$$-t^{-1} + t\Big|_1^2 = \left(\frac{-1}{2} + 2\right) - \left(\frac{-1}{1} + 1\right)$$

$$= \frac{3}{2}$$

3. Find the x-coordinate of the point of greatest curvature for the graph of $f(x) = \ln x$. (Make sure to simplify the formula for curvature κ before you try to maximize its value.)

Vizually verify your solution by plotting the graph on a calculator or in Maple.

A) e^{-1} B) $\frac{1}{2}\sqrt{2}$ C) 1 D) $\sqrt{3}$ E) e F) $e+e^{-1}$

$$f(x) = \ln x \; ; \; x > 0$$

$$K(x) = \frac{|f''(x)|}{\left[1+(f'(x))^2\right]^{3/2}}$$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = \frac{-1}{x^2}$$

$$K(x) = \frac{1/x^2}{\left(1+1/x^2\right)^{3/2}} = \frac{1/x^2}{\left[(x^2+1)/x^2\right]^{3/2}} = \frac{1/x^2}{(x^2+1)^{3/2} \cdot \frac{1}{x^3}} = \frac{x}{(x^2+1}$$

$$K'(x) = (x^2+1)^{3/2} - x\,\tfrac{3}{2}(x^2+1)^{1/2} \cdot 2x \;/ (x^2+1)^3$$

$$K'(x) = 0 \Rightarrow (x^2+1)^{3/2} = 3x^2 (x^2+1)^{1/2}$$

$$(x^2+1) = 3x^2$$

$$1 = 2x^2$$

$$x = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$