1. Suppose the temperature at the point (x, y) in the xy-plane is given by the formula

$$T(x,y) = \frac{1}{x^2 + 2y^2 + 2}.$$

Which of the following vectors points in the direction one should move from $(x_0, y_0) = (2, 1)$ in order to increase the temperature most rapidly?

A) $\mathbf{u} = (-1, -1)$ B) $\mathbf{u} = (-1, 0)$ C) $\mathbf{u} = (-1, -2)$

D) $\mathbf{u} = (-2, -1)$ E) $\mathbf{u} = (0, -1)$ F) $\mathbf{u} = (0, -2)$

max $D_u f(x,y) = \nabla f(x,y)\, u$.

$$f_x = \frac{-2x}{(x^2+2y^2+2)^2}$$

$$f_y = \frac{-4y}{(x^2+2y^2+2)^2}$$

$$\nabla f_{(2,1)} = \frac{-4}{(4+2+2)^2},\ \frac{-4}{(4+2+2)^2} = \frac{-4}{64},\ \frac{-4}{64}$$

$$= \left\langle \frac{-1}{16}, \frac{-1}{16} \right\rangle$$

A) $u = \langle -1, 1 \rangle$ gives the greatest increase

c, and D need to be normalized first.

$\Rightarrow \frac{c}{|c|} = \left\langle \frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle$ which yield a smaller. $D_u f$.

2. What is the directional derivative $D_{\mathbf{u}}f$ for the function $f(x,y) = 3x^2y$ at the point $(x_0, y_0) = (1,1)$ in the direction $\mathbf{u} = (\frac{3}{5}, \frac{4}{5})$?

A) 6 B) $\frac{31}{5}$ C) $\frac{32}{5}$ D) $\frac{33}{5}$ E) $\frac{34}{5}$ F) 7

$D_u f = \nabla f \cdot u$

$3^2 + 4^2 = 5^2$ ✓

$= (6xy, 3x^2)\ u$

$= \langle 6, 3 \rangle \langle \frac{3}{5}, \frac{4}{5} \rangle$

$= \frac{18 + 12}{5} = \frac{30}{5} = \boxed{6}$

3. Find the linearization of the function

$$f(x,y) = 3x^2 + 5xy - y^2$$

near the point $(x, y) = (1, 1)$.

A) $L(x,y) = 7 + 11(x-1) + 3(y-1)$

B) $L(x,y) = 7 + 12(x-1) + 3(y-1)$

C) $L(x,y) = 7 + 13(x-1) + (y-1)$

D) $L(x,y) = 7 + 9(x-1) + 4(y-1)$

E) $L(x,y) = 7 + 10(x-1) + 3(y-1)$

F) $L(x,y) = 7 + 11(x-1) + 2(y-1)$

$L = (f_x \Delta x, f_y \Delta y) + f(x,y)$

$f_x = (6x + 5y)$ @ $(1,1) \Rightarrow 11$

$f_y = 5x - 2y$ @ $(1,1) \Rightarrow 3$

$f(1,1) = 7$

$L = 7 + 11(x-1) + 3(y-1)$