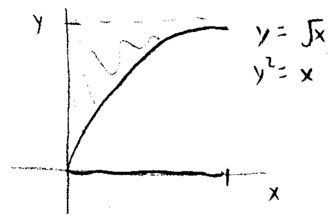
1. Evaluate the iterated integral,

$$\int_0^1 \int_{\sqrt{x}}^1 3y^3 \sin(xy)\, dy\, dx.$$

A) $\sqrt{3}$ B) 3 C) $1 - \sqrt{x}$ D) $\pi/2$ (E) $1 - \sin 1$ F) $3 \sin 1$



$$\int_0^1 \int_0^{y^2} 3y^3 \sin(xy)\, dx\, dy$$

$$\int 3y^3 \left[-\frac{\cos(xy)}{y} \right|_0^{y^2} dy$$

$$-\frac{\cos y^3}{y} + \frac{\cos(0)}{y}$$

$$\int_0^1 3y^2 \left[\frac{1}{y} - \frac{\cos y^3}{y} \right] dy$$

$$\int_0^1 3y^2\, dy - \int_0^1 3y^2 \cos y^3\, dy$$

Note $u = y^3$
$du = 3y^2\, dy$

$$\frac{3y^3}{3}\Big|_0^1 - \sin(y^3)\Big|_0^1$$

$$1 - (0) - [\sin(1) - \sin(0)]$$

$$\boxed{1 - \sin(1)}$$

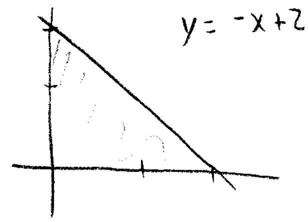
2. Evaluate the double integral

$$\int\int_R (2-x)^2 y \, dA$$

for the region

$$R = \{(x, y) \mid x + y \leq 2,\ x \geq 0,\ y \geq 0\}.$$

A) 4 B) 8 C) 16 D) $\frac{4}{5}$ E) $\frac{8}{5}$ (F) $\frac{16}{5}$



$$\int_0^2 \int_0^{2-y} (2-x)^2 y \; dx\, dy$$

$$\int y \int 4 - 4x + x^2 \; dx\, dy$$

$$\int y \left[4x - 2x^2 + \frac{x^3}{3} \Big|_0^{2-y} \right. dy$$

$$\int y \left[8 - 4y - 8 + 8y - 2y^2 + \frac{8}{3} - \frac{12}{3}y + \frac{6}{3}y^2 - \frac{y^3}{3} \right] dy$$

$$\int y \left[\frac{8}{3} - \frac{y^3}{3} \right] dy$$

$$\int \frac{8y}{3} - \frac{y^4}{3} \; dy = \frac{4y^2}{3} - \frac{y^5}{15} \Big|_0^2 = \frac{16}{3} - \frac{32}{15}$$

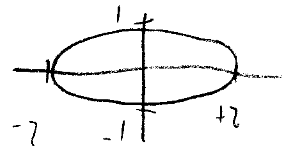
$$\frac{16 \cdot 5}{3 \cdot 5} - \frac{16 \cdot 2}{15} = \frac{16 \cdot 3}{3 \cdot 5} = \boxed{\frac{16}{5}}$$

3. Find the volume of the region under the elliptic paraboloid $x^2 + 4y^2 + z = 4$ bounded by the xy-plane.

A) π B) 2π C) 3π D) 4π E) 5π F) 6π

$f(x,y) = z = 4 - x^2 - 4y^2$

when $z = 0 \Rightarrow x^2 + 4y^2 = 0$



$\int_{-2}^{2} \int_{\sqrt{1-\frac{x^2}{4}}}^{\sqrt{1-\frac{x^2}{4}}} (z)\, dy\, dx$

since $z(x,y) = z(-x,y) = z(x,-y) = z(-x,-y)$

$= 4 \int_0^2 \int_0^{\sqrt{1-\frac{x^2}{4}}} (4-x^2) - 4y^2 \, dy\, dx$

$\sqrt{1 - \frac{x^2}{4}} = \sqrt{\frac{4}{4} - \frac{x^2}{4}} = \frac{\sqrt{4-x^2}}{2}$

$4 \int_0^2 \left[(4-x^2)\Big| - \frac{4y^3}{3}\Big| \right] dx$

$= 4 \int \frac{1}{2}(4-x^2)\sqrt{4-x^2} - \frac{4}{3}\left(\frac{4-x^2}{4}\right)^{3/2} dx$

$4 \int \frac{(4-x^2)^{3/2}}{2} - \frac{4}{3\sqrt{4}^3} (4-x^2)^{3/2}$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$(0,2) \to (0, \frac{\pi}{2})$

$\frac{4}{3} \int (4-x^2)^{3/2} dx =$

$\frac{4}{3} \int (4 - 4\sin^2\theta)^{3/2}\, 2\cos\theta\, d\theta = \frac{8}{3} \int (4\cos^2)^{3/2} \cos\theta\, d\theta$

$= \frac{8}{3} \int_0^{\pi/2} 8 \cos^3 \cos\, d\theta$: " $\int_0^{\pi/2} \cos^4\theta\, d\theta = \frac{3\pi}{16}$ "

$\frac{8}{3} \cdot 8 \cdot \frac{3\pi}{16} = 4\pi$