1. Find the linearization $L(x, y)$ of the function

$$f(x, y) = x^y = e^{y \ln x}$$

at the point $(x_0, y_0) = (4, \frac{1}{2})$.

What is the approximate value for the power $4.1^{0.51}$ that you obtain by means of this linearization $L(x, y)$? (In your calculation you can make use of the approximate value $\ln 2 \approx 0.693$.)

A.) 2.024 B.) 2.053 C.) 2.076 D.) 2.101 E.) 2.124 F.) 2.165

$L(x,y) = f_x \, \Delta x + f_y \, \Delta y + f(x_0, y_0)$

$f(x,y) = x^y$ @ $4, \frac{1}{2}$ approx @ $4.1, 0.51$

$\Rightarrow \Delta x = \frac{1}{10}$ $\Delta y = \frac{1}{100}$

$f_x = y x^{y-1}$ $f_y = x^y \ln x$

$L(x,y) = y x^{y-1} \frac{1}{10} + x^y \ln x \cdot \frac{1}{100} + 4^{1/2}$

$= \frac{1}{2} 4^{-1/2} \frac{1}{10} + 4^{1/2} \ln 4 \frac{1}{100}$ $- \sqrt{4}$

$= \frac{1}{2} \frac{1}{\sqrt{4}} \frac{1}{10} + \sqrt{4} \ln 2^2 \frac{1}{100}$ $+2$

$= \frac{1}{40} + 2 \cdot 2 \ln 2 \frac{1}{100} + 2$

$= 1/40 + 4 \cdot (0.693) \frac{1}{100} + 2$

$= 0.025 \quad 0.024 + 2$

$\simeq 2.05$

$\frac{0.6}{100} = 0.006$

$4 \cdot 0.006 = 0.024$

2. If $\mathbf{u}$ is a unit vector (i.e., $|\mathbf{u}| = 1$), then what is the correct formula for the maximum value of the directional derivative $D_u f(a,b)$ of the function

$$f(x,y) = \sqrt{x^2 + 2y^2}$$

at the point (a,b)?

A.) $\sqrt{\frac{a^2+4b^2}{2a^2+b^2}}$

B.) $\sqrt{\frac{4a^2+b^2}{2a^2+b^2}}$

C.) $\sqrt{\frac{a^2+2b^2}{a^2+4b^2}}$

D.) $\sqrt{\frac{2a^2+b^2}{a^2+4b^2}}$

E.) $\sqrt{\frac{a^2+4b^2}{a^2+2b^2}}$

F.) 1

$$D_u f(x,y) = \nabla f \cdot \hat{u}$$

$$\nabla f = \left(\frac{x}{\sqrt{x^2+2y^2}}, \frac{2y}{\sqrt{x^2+2y^2}} \right)$$

$$|\nabla f| = \sqrt{\frac{x^2+4y^2}{x^2+2y^2}} \quad @\ (a,b) \Rightarrow \sqrt{\frac{a^2+4b^2}{a^2+2b^2}}$$

3. Consider the following three limits,

$$(I) \quad \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^4}$$

$$(II) \quad \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$$

$$(III) \quad \lim_{(x,y)\to(0,0)} \frac{x^2}{\sqrt{x^2+y^2}}$$

Which of the following statements is true? (Select one answer.)

A.) All three limits converge.
B.) None of these limits converges.
C.) Only limit (I) converges.
D.) Only limit (II) converges.
E.) Only limit (III) converges.
F.) Limits (I) and (II) converge, (III) does not.

using polar

$I \rightarrow \lim_{r\to 0} \frac{r^2\cos^2}{r^2\cos^2 + r^4\sin^4} = \lim \frac{\cos^2}{\cos^2 + r^2\sin^4} = \frac{\cos^2}{\cos^2} = \emptyset$

@ $\theta = \frac{\pi}{2}$

$II \rightarrow \lim_{r\to 0} \frac{r^2\cos^2 r^2\sin}{r^4\cos^4 + r^4\sin^4} = \lim_{r\to 0} \frac{\cos^2\sin^2}{\cos^4+\sin^4}$ = dependent on θ

$III \rightarrow \lim_{r\to 0} \frac{r^2\cos^2}{\sqrt{r^2}} = \lim \frac{r^2\cos^2}{r} = \lim r\cos = 0$ ✓

4. Find the equation for the tangent plane to the quadric surface

$$4x^2 + y^2 + 2yz = 7$$

at the point $(1, 1, 1)$. What is the equation of the line of intersection between this tangent plane and the xy-plane?

A.) $8x + 2y = 10,\ z = 0$
B.) $8x + 2y = 12,\ z = 0$
C.) $8x + 2y = 14,\ z = 0$
D.) $8x + 4y = 12,\ z = 0$
E.) $8x + 4y = 14,\ z = 0$
F.) $8x + 4y = 16,\ z = 0$

$f_x = 8x \rightarrow 8$

@ $(1,1,1)$

$f_y = 2y + 2 \rightarrow 4$

$f_z = 2y \rightarrow 4$

plane $8(x-1) + 4(y-1) + 4(z-1) = 0$

$\cap$ w/ xy plane $\Rightarrow z = 0$ $\quad 8x + 4y + 2z = 14$

$\therefore 8x + 4y = 14$

5. For a cylinder with circular base the formulas for total surface area S and volume V are as follows,

$$S = 2\pi r^2 + 2\pi r h,$$
$$V = \pi r^2 h.$$

Here r denotes the radius of the base circle, and h the height of the cylinder.

Find a relation between r and h for a cylinder with fixed surface area and maximal volume. In other words, among all cylinders that have the same surface area S, which one has maximal volume V?

A.) $h = r$ B.) $h = r^2$ C.) $h = 2r$ D.) $h = 2r^2$ E.) $2h = r$ F.) $2h = r^2$

max $V(r, h)$

$$\nabla V = \lambda \nabla S$$

$$2\pi r h = \lambda (4\pi r + 2\pi h)$$
$$\pi r^2 = \lambda (2\pi r)$$

S

$rh = 2\lambda r + \lambda h$ $\quad r \neq 0$

$r = 2\lambda \; ; \; \frac{r}{2} = \lambda$

$rh = r^2 + \frac{r}{2} h$

$\frac{r}{2} h = r^2$

$h = 2r$ ✓

6. Find the distance from the origin $(0,0,0)$ to the quadric surface

$$2x^2+2y^2+2z^2+3x-z-10=0.$$

A.) $\sqrt{2}$ B.) $\frac{1}{2}\sqrt{2}$ C.) $\sqrt{5}$ D.) $\frac{1}{2}\sqrt{5}$ E.) $\sqrt{10}$ F.) $\frac{1}{2}\sqrt{10}$

$d^2 = x^2+y^2+z^2$ $\qquad y^2 = \dfrac{-2z^2-3x+z+10-2x^2}{2}$

$d^2(x,y) = \cancel{x^2} + -\cancel{z^2} - \frac{3x}{2} + \frac{z}{2} + 5 - \cancel{x^2} + \cancel{z^2}$

$= \frac{-3}{2}x + \frac{z}{2} + 5$?

$$\boxed{\begin{array}{l}\sqrt{(-3)^2+0^2+1^2} = \sqrt{10} \\ \text{but } \left(\frac{3}{2}\right)^2 \; 0^2 \; \left(\frac{-1}{2}\right)^2 = \frac{1}{2}\sqrt{10}\end{array}}$$

f: d^2

g: given

$2x = \lambda(4x+3)$

$2y = \lambda(4y) \Rightarrow$ $\quad y=0$

$\qquad y\neq 0 \Rightarrow \lambda = \frac{1}{2} \Rightarrow 2z = \frac{1}{2}(4z-1)$

$2z = \lambda(4z-1)$

$\qquad 2z = 2z - \frac{1}{2} \Rightarrow \Leftarrow$

$2x^2 + \cancel{2y^2} + 2z^2 + 3x - z - 10 = 0.$

$\therefore \neg(y\neq 0)$, $y=0$

Suppose $4x+3\neq 0$, and $4z-1\neq 0$

if they did you find a contratiction eg

$\frac{2x}{4x+3} = \frac{2z}{4z-1}$

$2x = \lambda(4x+3) = 0$

$\cancel{8xz} - 2x = \cancel{8xz} + 6z$

$\Rightarrow x=0$ but $4x+3=0$

$x = -3z$

$\Rightarrow\Leftarrow$

$3x = -9z$

$x^2 = 9z^2 \Rightarrow 18z^2 + 2z^2 - 9z - z - 10 = 0 \Rightarrow 10(2z+1)(z-1)$

$\therefore z = 1 \Rightarrow x=-3$ or $-\frac{1}{2} = z$, $x = \frac{3}{2}$

7. Analyze the critical points of the function

$$f(x, y) = x^2 + y^3 - y.$$

Which of the following is true?

A.) The function has one local minimum and one saddle point.
B.) The function has two local minima and one local maximum.
C.) The function has one local maximum and one local minimum.
D.) The function has three saddle points.
E.) The function has one local maximum and one saddle point.
F.) The function has one local minimum and one critical point at which the second derivative test is inconclusive.

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 3y \end{vmatrix} = 6y$$

$f_x = 2x$ $\quad\quad f_y = 3y^2 - 1$

$2x = 0$ $\quad\quad 3y^2 - 1 = 0$

$x = 0$ $\quad\quad y = \pm\sqrt{\frac{1}{3}}$

$\left(0, \sqrt{\frac{1}{3}}\right) \longrightarrow H[\ ] \Rightarrow 6\sqrt{\frac{1}{3}} > 0 \Rightarrow$ extrema

w/ $2 = f_{xx} > 0 \Rightarrow$ min.

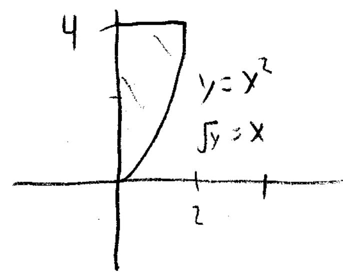
$\left(0, -\sqrt{\frac{1}{3}}\right) \longrightarrow H[\ ] \Rightarrow -6\sqrt{\frac{1}{3}} < 0 \Rightarrow$ saddle

8. Evaluate the iterated integral

$$\int_0^2 \int_{x^2}^4 xe^{(y^2)}\,dy\,dx.$$

A.) $e^{16}-1$ B.) $\frac{1}{2}(e^{16}-1)$ C.) $\frac{1}{3}(e^{16}-1)$ D.) $\frac{1}{4}(e^{16}-1)$ E.) $\frac{1}{6}(e^{16}-1)$ F.) $\frac{1}{15}(e^{16}-1)$

$$\int_0^2 \int_{x^2}^4 x e^{(y^2)}\,dy\,dx$$



$$\int_0^4 \int_0^{\sqrt{y}} x\, e^{(y^2)}\,dx\,dy$$

$$\int_0^4 \frac{x^2}{2} e^{y^2} \Big| \; dy$$

$$\int_0^4 \frac{y}{2} e^{y^2}\,dy \qquad \begin{aligned} u &= y^2 \\ du &= 2y\,dy \end{aligned}$$

$$\frac{1}{4}\int e^u\,du$$

$$\frac{1}{4} e^{y^2}\Big| = \frac{1}{4}(e^{16}-1)$$