Mid-Term Exam I (Make-up Version) *Math 114, February 8, 2007.*

Your name: Your Penn ID:

Lecturer: Erik van Erp

Please circle the name of your TA

James Kania Tolga Karayalya

Instructions. *You have 1 hour to complete this exam. The exam consists of ten multiple choice questions. Each question is worth 10 points. There is no penalty for incorrect answers. No partial credit is given. Answers with no supporting work will be given no points. If you change an answer, either erase or cross out the answer you do not want considered. Questions with more than one answer will be marked wrong. Clearly circle your answer in the grid below. Only your answer choice indicated on this page will be marked for credit.*

Do not detach this sheet from the body of the test!

Your Answers:

1. A B C D E F

2. A B C D E F

3. A B C D E F

4. A B C D E F

5. A B C D E F

6. A B C D E F

7. A B C D E F

8. A B C D E F

9. A B C D E F

10. A B C D E F

1. Let $ABCD$ be a parallellogram with diagonals AC and BD. Halfway the line segment AD there is a point P, such that the length of AP is equal to PD.
If we write $\mathbf{a} = \vec{AC}$, $\mathbf{b} = \vec{BD}$, then what is the correct expression for $\vec{AP}$

A.) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ B.) $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$ C.) $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ D.) $\frac{1}{3}\mathbf{a} - \frac{1}{3}\mathbf{b}$ E.) $\frac{1}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ F.) $\frac{1}{4}\mathbf{a} - \frac{1}{4}\mathbf{b}$

2. Find the area of the triangle ABC with vertices $A(1,0,0)$, $B(0,3,0)$, $C(0,0,4)$.

A.) $\sqrt{50}$ B.) $2\sqrt{10}$ C.) $\frac{25}{3}$ D.) $\frac{13}{2}$ E.) $\sqrt{60}$ F.) 8

3. The square $ABCD$ is the base of a rectangular box, while $EFGH$ is the square at the top of the box. Point E is straight above A, F above B, G above C, H above D. The sides of the square $ABCD$ (for example, AB) have length 1, while the height of the box (the length of AE) is h. Let θ be the angle between two main diagonals (AG and BH) of the box. Introduce an xyz-coordinate frame and use vectors to derive a formula for θ.

A.) $\cos\theta = \dfrac{h}{2+h}$

B.) $\cos\theta = \dfrac{h^2}{(2+h)^2}$

C.) $\cos\theta = \dfrac{h}{\sqrt{h^2+2}}$

D.) $\cos\theta = \dfrac{h^2}{h^2+2}$

E.) $\cos\theta = \dfrac{h^2}{h^2+4h+2}$

F.) $\cos\theta = \dfrac{h}{\sqrt{h^2+4h+2}}$

4. Find an equation for the plane through the points $A(3, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 3)$. What is the distance from the origin $(0, 0, 0)$ to this plane?

A.) 1 B.) $\frac{3}{2}$ C.) 3 D.) $\sqrt{2}$ E.) $\sqrt{3}$ F.) $\sqrt{6}$

5. The two curves

$$\mathbf{r}_1(t) = (t\sin t,\ t\cos t,\ t)$$
$$\mathbf{r}_2(s) = (s^2 - s,\ s - 1,\ 0)$$

intersect at the origin. What is the angle between their tangent lines at the point of intersection?

A.) 0 B.) $\pi/6$ C.) $\pi/4$ D.) $\pi/3$ E.) $\pi/2$ F.) $3\pi/4$

6. What is the equation in spherical coordinates (ρ, θ, ϕ) of the part of the cone $3x^2 + 3y^2 = z^2$ with $z \geq 0$?

A.) $\rho^2 = 3\theta$ B.) $\theta = 3\phi$ C.) $\rho = 3\cos\phi$ D.) $\rho = 3$ E.) $\theta = \pi/2$ F.) $\phi = \pi/6$

7. Find the arc length of the segment of the curve $\mathbf{r}(t) = (t^2,\ 3t,\ \frac{4}{\sqrt{3}}t^{3/2})$, for $0 \le t \le 2$.

A.) 10 B.) $\frac{31}{3}$ C.) $\frac{32}{3}$ D.) 11 E.) $\frac{34}{3}$ F.) $\frac{35}{3}$

8. Which of the following equations represents the quadric surface you see in the picture?

A.) $x^2 - y^2 = z$
B.) $x^2 - z^2 = y$
C.) $y^2 - z^2 = x$
D.) $x^2 + y^2 = z$
E.) $x^2 + z^2 = y$
F.) $y^2 + z^2 = x$

9. Find an equation for the tangent line at point $(1, 1, 1)$ to the curve $\mathbf{r}(t) = (t, t^2, t^3)$. What are the coordinates of the point of intersection of this tangent line with the plane $x + y + z = 15$?

A.) $(2, 7, 6)$ B.) $(3, 5, 7)$ C.) $(4, 6, 5)$ D.) $(3, 6, 6)$ E.) $(2, 5, 8)$ F.) $(4, 7, 4)$

10. The graph of the equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \; z = 0$$

is a hyperbola in the xy-plane. Because of the equality $\cosh^2 t - \sinh^2 t = 1$, the same hyperbola can be represented by the vector-valued function

$$\mathbf{r}(t) = (a \cosh t, b \sinh t, 0).$$

Use the general curvature formula

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3}$$

to derive a formula for the curvature of the hyperbola at the point $(a, 0, 0)$.

A.) $\kappa = \dfrac{a^{1/2}}{b^{3/2}}$

B.) $\kappa = \dfrac{a}{b^2}$

C.) $\kappa = \dfrac{a^2}{b^3}$

D.) $\kappa = \dfrac{b^{1/2}}{a^{3/2}}$

E.) $\kappa = \dfrac{b}{a^2}$

F.) $\kappa = \dfrac{b^2}{a^3}$

(This page is intentionally left blank.)

(This page is intentionally left blank.)