Summary of Research

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Favor-Trading with Incomplete Information (Dissertation: Chapter 2)

This paper studies whether and how individuals who stand to gain from trading favors can best form cooperative relationships in an environment with private information about each player's ability and willingness to do favors. Previous models in the favor-trading literature focused on optimizing favor-trading relationships under complete information. This paper introduces incomplete information about player types. The central question addressed is whether cooperation can be maintained in favor-trading relationships after the introduction of non-cooperative players into the pool of potential trading partners, and if so how can the cooperative players separate themselves most efficiently from the non-cooperative types.

For the purposes of this paper favor-trading is considered to be non-monetary trade in goods, services or opportunities and favors are assumed to be efficient. The model contains a positive measure of players with a low discount factor (low types) who do not find cooperation beneficial, and a positive measure of players with a high discount factor (high types) who do. Players receive opportunities to do favors for each other (favor opportunities) according to either a mutually exclusive or independent distribution, but these opportunities are private information.

As an example, consider a firm with several parallel divisions that function independently under separate managers. Suppose two new managers have been recruited to head the marketing and finance divisions, respectively. Each manager's job is to maximize productivity within her own division, but every once in a while one of the managers receives a new idea or opportunity that would be beneficial for her division but even more beneficial for the neighboring division. Monetary side payments are not allowed, but reciprocation in similar favors can provide a basis for mutual gains if both managers are sufficiently patient. However, the managers do not know each other's discount factor, which in this example could be interpreted as the likelihood of staying with the firm long-term. So how should the managers proceed?

The main result in this paper is that the high type players are almost always able to separate themselves from the low type players at the first available opportunity by using an "equality matching" (EM) mechanism if opportunities to do favors arrive independently. EM simply means that each agent waits for reciprocation of a previous favor before doing the next one. In the case of mutually exclusive favor opportunities, separation is still guaranteed for high types with probability one if one of the players is designated to do the first favor, and separation will occur as soon as the designated player receives a favor opportunity, assuming she is a high type.

However, such strategies induce separation only half as quickly (roughly speaking) as strategies that call for the first player to receive a favor opportunity to do the first favor, and hence separate if she is a high type. In equilibria based on such symmetric strategies, separation is only guaranteed with probability one under independent favor opportunities, but not under mutually exclusive favor opportunities. The paper establishes a bound on the number of periods in the mutually exclusive favor opportunity case during which the low types will not mimic the high types with positive probability. An important consequence of this result is that more information (mutually exclusive favor opportunities) leads to a worse outcome.

The paper also compares the equilibria involving a designated first favor maker (DFFM equilibria) to equilibria characterized by symmetric strategies (SS equilibria) and finds that either may dominate

depending on the parameter values. Numerical results suggest that SS equilibria dominate DFFM equilibria in cases involving relatively impatient agents likely to receive favor opportunities early on, whereas DFFM equilibria dominate when agents are very patient, but the probability of receiving favor opportunities is low. To see why, consider the case of mutually exclusive favor opportunities and suppose the probability of receiving an opportunity to do a favor is approximately one half. Then it is almost certain that one of the two agents will receive a favor opportunity during the first period, and therefore under a symmetric strategy for signaling type, separation will almost certainly occur during the first period, which is all the more important for relatively impatient high type players eager to gain from the benefits of an EM endgame. Designating one of the two players to do the first favor would halve the chance of separation in the first period, which is all the more costly when the high type players are impatient to begin a cooperative relationship.

Favor-Trading with Concave Utility Functions (Dissertation: Chapter 3)

We present this work in part as a discussion paper and in part as an empirical (numerical simulations) component for solving favor-trading equilibria. Our motivating observation is that to date prominent models of favor-trading have assumed agents have linear preferences (linear favor-trading models). In this paper informal favor-trading is considered to be a form of insurance. We study agents with concave utility functions (concave favor-trading models), and favors that derive their value from risk sharing. With concave utility functions that offer sufficiently high marginal returns at the start and low marginal returns at the end, agents can beneficially exchange favors at some level for any discount factors. This is in contrast to the linear favor-trading models, in which the incentive compatibility of favors traded in certain equilibria is independent of the size of the favors because agents are essentially risk-neutral with respect to favors.

Furthermore, we show qualitative differences in some types of equilibria. For example, if utility functions are linear and agents' discount factors are just large enough to satisfy the incentive compatibility constraint for the simplest form of favor-trading called equality matching (an agent does one favor, waits until reciprocation, then another, and so forth), the best the agents can do is to equality match full favors. If the same agents have concave utility functions, we show that the equivalent equality matching equilibria are dominated by equilibria involving a smaller than full first favor, which will be followed by a small second favor even if reciprocation has not yet been received (multi-state equality matching). Consequently, the assumption of linear preferences drives some of the results in prominent favor-trading models. The rest of chapter generalizes multi-state equality matching equilibria. We also define infinite state equality matching, but argue against their efficiency.

A key component of this paper is the construction of two parametric models to numerically analyze multi-state equilibria specifically, but these techniques may be of interest for investigating other favor-trading problems or applied game theory problems in general. The first model simulates a large sample of games for a given set of parameters, derives payoff functions from the simulation results, then optimizes these functions to solve for the optimal strategy. The second model solves directly the system of simultaneous payoff equations associated with an equality matching game, and uses the results to find the optimal strategy. We use these models to compute a large set of solutions spanning the parameter space and interpolate the general equilibrium characteristics from these data points. We also construct a model with favor-depreciation and compare our preliminary results to multi-state equality matching. Most of the results are presented as conjectures because they are based on numerical results rather than closed-form proofs.