ECON897 Math Camp Part II **KyungMin (Teddy) Kim**
Summer 2008 **University of Pennsylvania**

Quiz II. (Friday, August 8)

1. (1.5) Prove that if $f : X \to Y$ is continuous and X is compact, then f is uniformly continuous.

2. (1.5) uppose X_i is an independent random variable with density function $f_i, i = 1, 2$. Let $f(x_1, x_2)$ be the joint density function of X_1 and X_2 (Of course, $f(x_1, x_2) = f_1(x_1)f_2(x_2)$ because X_1 and X_2 are independent). Define a new random variable Y by $Y = X_1 + X_2$ (Y is called the convolution of the independent X_1 and X_2). Show that the density function of Y is given by

$$f_Y(y) = \int_{-\infty}^{\infty} f_1(x_1)f_2(y - x_1)dx_1.$$

Hint : Find a distribution function F_Y of Y first.

3. (2) Let $f : A \subset R^n \to R$ be a function. Show that for fixed $\varepsilon > 0$, the following set is closed.

$$B_\varepsilon = \left\{ a \in A : o(f, a) \equiv \limsup_{x \to a} f(x) - \liminf_{x \to a} f(x) \geq \varepsilon \right\}.$$

4. Let $M_{2\times2}$ be the set of 2×2 matrix and $M^* \subset M_{2\times2}$ be the set of 2×2 matrix whose determinant is 0. We show that M^* has zero measure in $M_{2\times2}$.

(1) (1.5) Define a function $f : M_{2\times2} \times R^2 \to R^3$ by

$$f(A, v) = \begin{pmatrix} Av \\ v^T v - 1 \end{pmatrix} = \begin{pmatrix} a_{11}v_1 + a_{12}v_2 \\ a_{21}v_1 + a_{22}v_2 \\ v_1^2 + v_2^2 - 1 \end{pmatrix}$$

Obviously, f is C^1. Find $Df(A, v)$.

(2) (1.5) Let $c = (0, 0, 0) \in R^3$. Show that for all $(A, v) \in f^{-1}(c)$, $Df(A, v)$ is surjective. Hint : it is enough to show that there are at least three independent columns.

(3) (2) **Transversality Theorem** $\Lambda \subset R^l, U \subset R^m$: open. $f : \Lambda \times U \to R^n$: C^1-map. Let $c \in R^n$ be such that for all $(\lambda, x) \in f^{-1}(c), Df(\lambda, x) : R^l \times R^m \to R^n$ is surjective. Then the set $\Lambda' = \{\lambda \in \Lambda | D_x f(\lambda, x)$ is not surjective for some $x \in U$ with $f(\lambda, x) = c\}$ has measure zero.

By the Transversality Theorem, the set $M' = \{A \in M_{2\times2} : D_v f(A, v)$ is not surjective for some $v \in R^2$ with $f(A, v) = c\}$ has measure zero. Show that $M^* = M'$.