

# The Coase Conjecture with Incomplete Information on the Monopolist's Commitment\*

KyungMin Kim <sup>†</sup>

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## Abstract

A key to the Coase conjecture is the monopolist's inability to commit to price, which leads consumers to believe that a high current price will be followed by low future prices. This paper studies the robustness of the Coase conjecture with respect to these beliefs of consumers. In particular, there is uncertainty over whether the monopolist is committed to a price (i.e. she may be a commitment type). Consequently, consumers are no longer certain that the price will change over time. I consider two kinds of commitment types. A behavioral commitment type charges an exogenously given price, while the rational commitment type optimally chooses a price. I show that the Coase conjecture is robust with regard to uncertainty over the monopolist's commitment. When the probability of behavioral types is sufficiently small, as in the original Coase conjecture, the monopolist earns the competitive profit. When the probability of behavioral types is positive, unlike in the original Coase conjecture, there will be positive delay. But the delay disappears as the probability approaches zero. When the commitment type is rational, unless the probability of the commitment type is sufficiently high, both the normal and committed monopolists charge the competitive price, and thus there is no delay.

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<sup>†</sup>Department of Economics, University of Pennsylvania, <kim37@econ.upenn.edu>

# 1 Introduction

A durable goods monopolist has an incentive to lower the price over time to make further sales. But consumers, anticipating lower prices in the future, delay purchases unless the monopolist offers a reasonable price. When consumers are patient, the monopolist charges the competitive price immediately, and thus there is no delay. In a dynamic environment, the presence of monopoly creates no distortion in allocations. This is the well-known *Coase conjecture*.

A key to this result is that consumers believe that the price is falling toward the competitive price sufficiently quickly. These beliefs are generated by a lack of commitment by the monopolist to price and make consumers reluctant to accept a high price. As a contrast, if consumers could be convinced that the price would not change, they would accept a high price immediately. The monopolist would charge the static monopoly price and earn the static monopoly profit.

This paper studies the robustness of the Coase conjecture with respect to consumers' beliefs about a lack of commitment by the monopolist to price. I perturb the standard durable goods monopoly environment so that consumers are no longer certain that the price will change over time. Formally, I introduce incomplete information on the monopolist's commitment to price. A *commitment type* of the monopolist is one who is committed to a price, that is, who cannot adjust the price over time. In the presence of commitment types, consumers face non-trivial decision problems. Without uncertainty, they were certain that the price would drop soon. With uncertainty, the price may not fall, in which case consumers prefer an early purchase. Consumers' decision problems are more complicated by the fact that the normal monopolist has an incentive to build a reputation as a commitment type.

I consider two kinds of commitment types. A *behavioral commitment type* (shortly, behavioral type) has no payoff concern and always offers an exogenously given price. The *rational commitment type* optimally chooses a price, knowing that she cannot adjust it later. The former type corresponds to a simple commitment type in the reputation literature, while the latter type is rather new.<sup>1</sup> The introduction of behavioral types highlights the normal type's incentive to build a reputation as a commitment type, while studying the model with the rational commitment type is of interest for the following reasons. First, the rational commitment type is the monopolist who, if her type is known to consumers, charges the static monopoly price and achieves the static monopoly profit. In this sense, the rational commitment type can be thought of as the monopolist with commitment power, whose behavior has

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<sup>1</sup>To the best of my knowledge, Kambe (1999) is the only one that considers this type and studies a related strategic problem in a dynamic environment.

long been studied in the literature. Second, the rational commitment type provides a way to endogenize the behavior of the commitment type. It enables me to capture more refined beliefs of consumers that if the monopolist knew that she would be committed to a price, she must have set the price carefully in the first place. Last, some institutional features may induce bargainers to behave like the rational commitment type. For instance, a monopolist may hire a salesman without giving him the authority to bargain over the price. In addition, in a wage bargaining (an international trade negotiation), the firm (the opposite party) may be uncertain over the union leader's (the bargaining official's) level of delegation from the rank and file (the government or the voters).

I find that the Coase conjecture is robust with regard to uncertainty over the monopolist's commitment. I distinguish between the two implications of the Coase conjecture: (1) the monopolist earns the competitive profit (profit implication), and (2) there is no delay (efficiency implication). While these two are indistinguishable in the original Coase conjecture, the current setting shows that they are distinct.

When the probability of behavioral types is small, the normal monopolist charges more than the competitive price, but earns only the competitive profit. Therefore, the profit implication of the Coase conjecture is robust in a strong sense, while the efficiency result (no delay) does not hold whenever the probability of behavioral types is positive. Since delay becomes negligible as the probability of behavioral types vanishes, the efficiency implication is also robust, though not as strong as the profit implication. To see how this happens, suppose there is a single behavioral type who charges more than the competitive price. The normal monopolist offers the same price because she has an incentive to build a reputation as the commitment type. When the probability of the behavioral type is small, consumers believe that the price is likely to drop in the future and, therefore, delay purchases. If the seller keeps offering the high price, some consumers accept the price, but a majority of consumers wait, hoping for the price to fall. When the probability of the behavioral type is small, and consequently, consumers have a strong incentive to wait, the loss of profit from delayed purchases completely offsets the gain from selling to some consumers at the high price.<sup>2</sup> The conclusions carry over to the case with multiple behavioral types. This is because in equilibrium consumers' beliefs over the seller's type are determined so that the normal monopolist is indifferent over all prices she may offer. This ensures that when the

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<sup>2</sup>Another way to understand this result is to consider the problem in the reputation context. The normal monopolist establishes a reputation as the commitment type, while consumers pretend to have lower valuations. Since this is a two-sided reputation model, who is better off depends on each player's amount of uncertainty over the opponent's type. The monopolist is better off only when the possibility of commitment (the seller's bargaining power) outweighs the natural tendency of consumers to pretend to have lower valuations (the buyer's bargaining power).

unconditional probability of behavioral types is small, the probability of the behavioral type (consumers' beliefs over the seller's type) conditional on a price is small uniformly over all prices.

When the commitment type is rational, unless the probability of the commitment type is sufficiently high, both the normal and committed monopolists offer the competitive price immediately and earn the competitive profit. Therefore, both implications of the Coase conjecture are robust in a strong sense. Though the rational commitment type chooses a price, conditional on her choosing any given price, the situation amounts to the one with a single behavioral type. Consequently, when the probability of the commitment type is small, both the rational commitment type and the normal type earn only the competitive profit. The reason why the efficiency implication is also strongly robust is as follows. A monopolist generally has an incentive to avoid the delay cost by lowering the price. This incentive is particularly large to the rational commitment type who must endure all delay cost. When the rational commitment type expects no more than the competitive profit, it is optimal for her to offer the competitive price. Then the normal type has no choice but to offer the competitive price as well.

Stokey (1981), Bulow (1982), Fudenberg, Levine, and Tirole (1985), Gul, Sonnenschein, and Wilson (1986), and Ausubel and Deneckere (1989) are classic papers on the Coase conjecture. They are concerned about the conjecture itself, while the interest of this paper is its robustness. McAfee and Wiseman (2008) consider the Coase conjecture with a capacity constraint. They show that, no matter how small the capacity cost is, the monopolist achieves at least 29.8% of the static monopoly profit, and thus the Coase conjecture does not hold. The crucial idea is that, with a capacity constraint that is observable to consumers, the monopolist can effectively commit to quantity instead of price.

Another branch this paper builds upon is the reputational bargaining literature. Chatterjee and Samuelson (1988) study a bilateral bargaining game with two-sided incomplete information over fundamentals (the buyer's valuation and the seller's production cost) and characterize the equilibrium that features a war of attrition.<sup>3</sup> Myerson (1991) first introduces the commitment (irrational or obstinate) type which had been widely studied in repeated games following the seminal works of Kreps and Wilson (1982) and Milgrom and Roberts (1982) into the bargaining context. He shows that in an alternating offer bargaining game with one-sided incomplete information, as bargaining friction becomes negligible, agreement occurs immediately with the outcome that would prevail when a party is known to be stub-

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<sup>3</sup>A war-of-attrition structure is assumed in Chatterjee and Samuelson (1987), while it is derived as an equilibrium property in their 1988 paper.

born. Abreu and Gul (2000) generalize Myerson's insight and study a two-sided reputational bargaining game. In their setting, there are multiple behavioral types and each normal type chooses a behavioral type to mimic. They demonstrate that the equilibrium in continuous time is unique and features a war-of-attrition structure. Furthermore, they show that the equilibrium in discrete time converges to the unique equilibrium in continuous time, independent of the exact bargaining protocol so long as both players make offers frequently. Kambe (1999) considers the setting in which players become stubborn with positive probability after making their initial demands. He also analyzes the case in which players know their types before making their demands. The second case is analogous to the rational commitment type in this paper. Compte and Jehiel (2002) investigate the role of outside options in the reputational bargaining environment. They show that outside options may cancel out the effect of obstinacy, in the sense that players reveal their rationality as soon as possible.

The closest to this paper is Inderst (2005). He studies the case in which there is a single behavioral type. This case serves as the building block in this paper, but has the following limitations. First, with a single behavioral type, all prices other than the competitive price and the price offered by the behavioral type are irrelevant. My model with multiple behavioral types endogenizes the set of equilibrium prices and yields results that do not rely on the specific price offer. Second, in the model with a single behavioral type, the probability for the seller to be the commitment type conditional on the price offer is equal to its unconditional probability, which is exogenously given. My model with multiple behavioral types endogenizes the conditional probabilities and allows me to address an interesting question in the presence of commitment types: which price is more credible? Do consumers believe that the price is more likely to drop when the initial price is higher or lower? Last, it turns out that many equilibria in the game with the rational commitment type are not robust to the introduction of a small probability of multiple behavioral types. This equilibrium selection is not possible with a single behavioral type.

The remainder of the paper is organized as follows. The next section studies the game with a single behavioral type. In addition, I provide some definitions and facts that are useful in the subsequent sections. Section 3 and Section 4 examine the game with multiple behavioral types and the game with the rational commitment type, respectively. I conclude by discussing some relevant issues in Section 5. I also analyze the full-fledged game in which both behavioral types and the rational commitment type are active. The full-fledged game is necessary for the equilibrium selection in Section 4, but rather complicated. I provide an intuition for the selection in Section 4 and defer the analysis of the full-fledged game to the Appendix.

## 2 A Single Behavioral Type : The Building Block

### 2.1 Setup

The basic setup is the standard durable goods monopoly problem. This problem is identical to the bilateral sequential bargaining game in which the buyer has private information about his valuation and the seller makes all the offers. The seller's production cost is known and normalized to 0. The buyer's valuation,  $v$ , is drawn from a distribution function  $F$  with support  $[\underline{v}, \bar{v}]$ ,  $0 < \underline{v} < \bar{v} < \infty$ .<sup>4</sup>  $F$  has a positive and continuously differentiable density  $f$ . At each date  $t \geq 0$ , the seller makes an offer and the buyer decides whether to accept the offer or not. The common discount factor is  $\delta = e^{-r\Delta}$  where  $\Delta$  is the offer interval and  $r$  is the discount rate. This game has a unique sequential equilibrium in which the price falls to  $\underline{v}$  in a finite time (See Fudenberg, Levine, and Tirole (1985) or Gul, Sonnenschein, and Wilson (1986)). As  $\delta$  approaches 1, the seller offers a price close to  $\underline{v}$  and delay disappears in the limit.

I study a perturbation of this game. The seller is committed to a price  $p \in (\underline{v}, \bar{v})$  with probability  $\mu$ . With complementary probability, the seller is the same type as in the original game (the normal type). For analytical tractability, I focus on the continuous-time game. In continuous time, the Coasian dynamics forces the seller to immediately lower the price to  $\underline{v}$  in case her type is revealed to be normal. Since any price revision reveals the seller's type to be normal, I can assume that at each time and history, the normal type has only two options: continuing to offer the same price as the commitment type or offering the competitive price,  $\underline{v}$ . For completeness, I assume that once the seller offers  $\underline{v}$ , she keeps offering  $\underline{v}$  after any history. For the convergence of the discrete-time game to the continuous-time game, see Inderst (2005).

### 2.2 Characterization

To understand the equilibrium structure, suppose there is a completely separating equilibrium. In such an equilibrium, the buyer accepts  $p$  immediately whenever his valuation is higher than  $p$ . Since the normal type achieves only  $\underline{v}$ , it is a profitable deviation for the normal type to offer  $p$  first and lower to  $\underline{v}$  an instant later. Now suppose there is a completely pooling equilibrium. The buyer again accepts  $p$  immediately whenever his valuation is higher

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<sup>4</sup>The "gap" assumption  $\underline{v} > 0$  makes the seller immediately lower the price to  $\underline{v}$ , once the buyer's valuation is known to be close to  $\underline{v}$ . With no gap ( $\underline{v} = 0$ ), the Coase conjecture may not hold (See Ausubel and Deneckere (1989)). The gap assumption plays another role in my model, which is to make the normal type exhaust her reputation. The latter role will be clarified in the last section when I examine the limit equilibrium as  $\underline{v} \rightarrow 0$ .

than  $p$ . Then the normal type lowers the price to  $\underline{v}$  an instant later. Therefore, as is common in many reputation games, the normal type plays a mixed strategy in equilibrium.

Characterizing equilibrium in this type of reputation game is difficult in general. Abreu and Gul (2000), however, show that the equilibrium analysis is tractable in the bargaining context. Their analysis builds on the following two observations. First, once a party is revealed to be normal, the Coasian dynamics forces him to accept the opponent's demand immediately in continuous time. Second, given the first observation, the only strategic issue is finding players' optimal concession times to reveal their rationality and the underlying structure of this concession game is a war of attrition.

Though Abreu and Gul study a variant of the Rubinstein bargaining game, this paper and their paper have similar equilibrium structures. Indeed, if the buyer's valuation is drawn from a binary distribution in my problem, the equilibrium characterization is straightforward from Abreu and Gul. Given a higher price than the low valuation, the normal type seller and the high valuation buyer mix between conceding and waiting over a time interval. Their concession probabilities are determined so that the opponent is indifferent between conceding and waiting at each time.

With a continuum of buyer valuations, the analysis is not immediate from Abreu and Gul. Before presenting the results, I note that this extension is different from an extension to the case with multiple behavioral types. With multiple behavioral types, an additional issue is which behavioral type each player would choose to mimic. Conditional on those choices, the game reduces to the one with a single behavioral type. With a continuum of buyer valuations, a different type of war-of-attrition arises.

An equilibrium is characterized by  $(G_S, \tau)$  where  $G_S : R_+ \rightarrow [0, 1]$  is the seller's unconditional concession probability distribution and  $\tau : [\underline{v}, \bar{v}] \rightarrow R_+ \cup \{\infty\}$  is the buyer's concession function.  $G_S(t)$  is the cumulative probability that the seller concedes (offers  $\underline{v}$ ) by time  $t$ , conditional on that the buyer has not accepted the previous offers.  $\tau(v)$  is the optimal concession time of the type  $v$  buyer, conditional on that the seller has not lowered the price. Since the commitment type seller and the buyer with a lower valuation than  $p$  never concede,  $G_S(t) \leq 1 - \mu, \forall t$  and  $\tau(v) = \infty, \forall v < p$ . In addition, define  $G_B : R_+ \rightarrow [0, 1]$  by  $G_B(t) = \Pr\{v : \tau(v) \leq t\}$ .  $G_B$  is the buyer's unconditional concession probability distribution.

The following measure can be interpreted as the buyer's bargaining power and plays an important role in this paper. Given a distribution function  $F$  and  $p \in [\underline{v}, \bar{v}]$ , let

$$\phi(F, p) = \exp\left(-\frac{1}{\underline{v}} \int_p^{\bar{v}} \frac{(v-p)f(v)}{F(v)} dv\right).$$

This is one way to measure how left-skewed the distribution is relative to  $p$ . If  $F$  first-order stochastically dominates  $F'$ , then  $\phi(F, p) \leq \phi(F', p)$ .<sup>5</sup> Intuitively, the more likely the buyer has a lower valuation than  $p$ , the more easily he can pretend to have a lower valuation than  $p$ , and consequently, the stronger his bargaining power is.  $\phi(F, p)$  is strictly increasing in  $p$  because the buyer has greater incentive to wait when facing a higher price.

The characterization results are summarized in the following proposition, which is a rewriting of Proposition 1 in Inderst (2005).

**Proposition 1** *There exists a unique sequential equilibrium  $(G_S, \tau)$  which is characterized by*

$$\tau(v) = \begin{cases} 0, & \text{if } v > F^{-1}(c_B), \\ -\frac{p-v}{\underline{v}r} \ln\left(\frac{F(v)}{c_B}\right), & \text{if } p \leq v \leq F^{-1}(c_B), \\ \infty, & \text{if } v < p, \end{cases}$$

$$G_B(t) = \begin{cases} 1 - c_B \exp\left(-\frac{v}{p-v}rt\right), & \text{if } t \leq -\frac{p-v}{\underline{v}r} \ln\left(\frac{F(p)}{c_B}\right), \\ 1 - F(p), & \text{otherwise,} \end{cases}$$

$$G_S(t) = \min\left\{1 - \mu \exp\left(\frac{1}{\underline{v}} \int_p^{F^{-1}(c_B \exp\{-\frac{\underline{v}}{p}rt\})} \frac{(v-p)f(v)}{F(v)} dv\right), 1 - \mu\right\},$$

where  $c_B = 1$  if  $\mu \leq \phi(F, p)$ , and

$$\frac{1}{\underline{v}} \int_p^{F^{-1}(c_B)} \frac{(v-p)f(v)}{F(v)} dv + \ln \mu = 0, \text{ if } \mu > \phi(F, p).$$

The equilibrium changes qualitatively at  $\mu = \phi(F, p)$ . If  $\mu > \phi(F, p)$ , then the buyer concedes (accepts the offer  $p$ ) with positive probability at date 0. If  $\mu < \phi(F, p)$ , then the seller concedes (offers  $\underline{v}$ ) with positive probability at date 0. If  $\mu = \phi(F, p)$ , then no player immediately concedes with positive probability.

I provide the idea of the proof, highlighting similarities and differences between the two-type case (Abreu and Gul) and the continuum type case (Inderst and this paper).

As in Abreu and Gul, the normal type randomizes over a time interval. Therefore, she is indifferent between waiting and conceding at time  $t$  such that  $0 < G_S(t) < 1 - \mu$ . From this

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<sup>5</sup>Notice that

$$\phi(F, p) = \exp\left(-\frac{1}{\underline{v}} \int_p^{\bar{v}} \frac{(v-p)f(v)}{F(v)} dv\right) = \exp\left(\frac{1}{\underline{v}} \int_p^{\bar{v}} \ln F(v) dv\right).$$

Since  $F(v) \leq F'(v)$  for all  $v$ ,  $\phi(F, p) \leq \phi(F', p)$ .

indifference,

$$r\underline{v} = \frac{dG_B(t)/dt}{1 - G_B(t)}(p - \underline{v}).$$

The left-hand side is the marginal cost of waiting an instant more, while the right-hand side is the corresponding marginal benefit. The latter consists of the conditional concession rate of the buyer at time  $t$  and the seller's additional benefit by selling at  $p$  instead of  $\underline{v}$ . This ordinary differential equation has a closed-form solution,  $G_B(t) = 1 - c_B \exp(-\underline{v}rt/(p - \underline{v}))$  where  $c_B \in [F(p), 1]$  is a constant.

Different from Abreu and Gul, each buyer type has his own optimal concession time. Since  $\tau(v)$  is the optimal concession time of the type  $v$  buyer,

$$r(v - p) = \frac{dG_S(\tau(v))/dt}{1 - G_S(\tau(v))}(p - \underline{v}) \text{ for } v > p \text{ and } \tau(v) > 0.$$

That is, the marginal cost (the left-hand side) and the marginal benefit (the right-hand side) of delay for the type  $v$  buyer are equated at his optimal concession time,  $\tau(v)$ . By the skimming property ( $v > v' \Rightarrow \tau(v) < \tau(v')$ ),<sup>6</sup>  $\tau^{-1}$  is well-defined in the interior. Then,

$$r(\tau^{-1}(t) - p) = \frac{dG_S(t)/dt}{1 - G_S(t)}(p - \underline{v}).$$

The solution to this ordinary differential equation is

$$G_S(t) = 1 - c_S \exp\left(-\int_0^t \frac{r(\tau^{-1}(s) - p)}{p - \underline{v}} ds\right),$$

where  $c_S \in [\mu, 1]$  is another constant. Using

$$1 - F(\tau^{-1}(t)) = G_B(t) = 1 - c_B \exp\left(-\frac{\underline{v}}{p - \underline{v}}rt\right),$$

I can substitute for  $\tau^{-1}(s)$ . After arranging terms,

$$G_S(t) = 1 - c_S \exp\left(-\frac{1}{\underline{v}} \int_{F^{-1}(c_B \exp\{-\frac{\underline{v}}{p - \underline{v}}rt\})}^{F^{-1}(c_B)} \frac{(v - p)f(v)}{F(v)} dv\right).$$

The two unknowns,  $c_B$  and  $c_S$ , are found similarly to Abreu and Gul. First, a player prefers waiting an instant to conceding immediately, if the opponent concedes with positive

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<sup>6</sup>The marginal benefit of waiting more is independent of the buyer's valuation and decreases over time, while the marginal cost is strictly increasing in the buyer's valuation and is constant across time.

probability. Therefore, either  $c_B = 1$  or  $c_S = 1$ . Second, once the opponent turns out to be the strong type (the commitment type and the buyer with a lower valuation than  $p$ ), a player concedes immediately. Therefore,  $G_S(t^*) = 1 - \mu \Leftrightarrow G_B(t^*) = 1 - F(p)$ . These two facts uniquely pin down  $c_B$  and  $c_S$ .

One apparent difference from Abreu and Gul is the concession rate of the seller. In Abreu and Gul, both players have constant concession rates. In my model, the buyer has a constant concession rate, while the concession rate of the seller decreases over time. The constant concession rate of the buyer is due to the fact that there is a single seller type who needs to be provided with an incentive. The reason for the decreasing concession rate of the seller is that each buyer type has a different marginal cost of waiting. The intuition is as follows. By the skimming property, a buyer with a higher valuation concedes earlier. But a buyer with a higher valuation has a higher marginal cost of waiting and, therefore, must be provided with a higher marginal benefit. This can be created only through a higher concession rate of the seller.

### 2.3 The Coase Conjecture with a Single Behavioral Type

Given  $p$  and  $\mu$ , there exists a unique sequential equilibrium. Hence, the indirect expected utility of the normal type is well-defined as a function of  $p$  and  $\mu$ . I denote by  $U_N(p, \mu)$  the normal type's indirect expected utility.

**Corollary 1** (*The normal type's indirect expected utility*)

$$U_N(p, \mu) = (1 - c_B)p + c_B \underline{v},$$

where  $c_B = 1$  if  $\mu \leq \phi(F, p)$ , and

$$\frac{1}{\underline{v}} \int_p^{F^{-1}(c_B)} \frac{(v - p)f(v)}{F(v)} dv + \ln \mu = 0, \text{ if } \mu > \phi(F, p).$$

Once a war of attrition starts, the normal type is indifferent between waiting and conceding, and thus her expected payoff is equal to  $\underline{v}$ . The normal type achieves more than  $\underline{v}$  only when the buyer concedes with positive probability at date 0.  $(1 - c_B)p$  reflects that possibility.<sup>7</sup>

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<sup>7</sup>As  $\mu$  tends to 1,  $c_B$  approaches  $F(p)$ , and thus the first term in  $U_N(p, \mu)$  converges to the expected payoff of the static monopolist who charges  $p$ . Intuitively, if the seller is the commitment type almost for sure, the buyer accepts  $p$  immediately whenever his valuation is greater than  $p$ , whose probability is  $1 - F(p)$ .

What is the probability that delay occurs? Suppose  $\mu < \phi(F, p)$ . Then the probability that the seller offers  $p$  is

$$\mu + (1 - \mu) \left( 1 - \frac{1 - \mu/\phi(F, p)}{(1 - \mu)} \right) = \frac{\mu}{\phi(F, p)}.$$

The first term is the probability that the seller is the behavioral type, while the second term is the probability that the seller is the normal type and she offers  $p$ .<sup>8</sup> It is immediate that delay occurs whenever  $\mu$  is positive, but disappears as  $\mu$  vanishes.

**Corollary 2** (*The Coase conjecture with a single behavioral type*) *When there is a single behavioral type, the profit implication of the Coase conjecture is robust in a strong sense (it holds for  $\mu$  small but positive), while the efficiency implication is robust in a weak sense (it holds only when  $\mu$  tends to 0).*

## 2.4 Some Useful Facts and Isoprofit Curves

Suppose  $\mu < \phi(F, p)$ , but the seller offered  $p$ . What is the buyer's posterior belief that the seller is the commitment type? Since the normal type concedes with probability  $(1 - \mu/\phi(F, p)) / (1 - \mu)$  at date 0, conditional on that the seller did not offer  $\underline{v}$  (she did offer  $p$ ), the buyer's posterior belief is equal to

$$\frac{\mu}{\mu + (1 - \mu) \left( 1 - \frac{1 - \mu/\phi(F, p)}{(1 - \mu)} \right)} = \phi(F, p).$$

Intuitively, at the time when a war of attrition starts, players' bargaining powers must be balanced. When  $\mu < \phi(F, p)$ , the only way to establish the balance is that the normal type concedes with positive probability so that the buyer's posterior belief (the seller's bargaining power) increases up to  $\phi(F, p)$ .

**Corollary 3** *When  $\mu < \phi(F, p)$ , the buyer's belief conditional on that the seller offered  $p$  at date 0 is equal to  $\phi(F, p)$ . Therefore, the buyer's posterior belief conditional on  $p$  is bounded below by  $\phi(F, p)$ .*

Next, consider the game with the rational commitment type. Suppose the rational commitment type offers  $p$  and the buyer's belief is given by  $\mu$  after observing  $p$ . Then the subsequent game proceeds exactly in the same way as the one with a single behavioral type.

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<sup>8</sup>The probability that the seller offers  $\underline{v}$  at date 0 is  $1 - \mu/\phi(F, p)$ . Since the behavioral type does not concede, the normal type concedes with probability  $(1 - \mu/\phi(F, p)) / (1 - \mu)$ .

Therefore, I can interpret the behavioral type in the previous game as the rational commitment type and calculate the indirect expected utility of the rational commitment type as a function of  $p$  and  $\mu$ . I denote by  $U_R(p, \mu)$  the rational commitment type's indirect expected utility. From now on, subscripts N and R refer to the normal type and the rational commitment type, respectively.

**Corollary 4** (*The rational commitment type's indirect expected utility*)

$$U_R(p, \mu) = (1 - c_B)p + c_B \underline{v} \left( 1 - \left( \frac{F(p)}{c_B} \right)^{p/\underline{v}} \right)$$

where  $c_B = 1$  if  $\mu \leq \phi(F, p)$ , and

$$\frac{1}{\underline{v}} \int_p^{F^{-1}(c_B)} \frac{(v-p)f(v)}{F(v)} dv + \ln \mu = 0, \text{ if } \mu > \phi(F, p).$$

Relative to the normal type's indirect expected utility,  $U_R(p, \mu)$  contains an additional term. This can be interpreted as the cost to the rational commitment type for not being able to adjust the price.

I maintain the following assumption from now on.  $p_i^*$  is the price that maximizes  $U_i(p, 1)$ .

**Assumption 1**  $p_R^* > \underline{v}$ , and

$$\begin{aligned} \frac{1-F(p)}{f(p)} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} p, & \quad \text{if } p \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right\} p_R^*, \\ \underline{v} + \frac{1-F(p)}{f(p)} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} p, & \quad \text{if } p \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right\} p_N^*. \end{aligned}$$

This assumption states that two functions  $U_R(p, 1) = (1 - F(p))p$  and  $U_N(p, 1) = (1 - F(p))p + F(p)\underline{v}$  are single-peaked. A sufficient condition is that the inverse hazard ratio  $((1 - F(p))/f(p))$  is decreasing, which is frequently employed in the mechanism design literature.

The following results are immediate from Proposition 1.

**Lemma 1** *Suppose  $\mu > \phi(F, p)$ . Then*

- (1)  $\lim_{\mu \rightarrow 1} c_B(p, \mu) = F(p)$ ,
- (2)  $\partial c_B(p, \mu) / \partial p > 0$ ,  $\partial c_B(p, \mu) / \partial \mu < 0$ ,

(3)  $\partial U_i(p, \mu) / \partial \mu > 0$ .

For each  $p' \in [\underline{v}, p_i^*]$ , let  $\rho_i(p') \in [p_i^*, \bar{v}]$  be the value such that  $U_i(\rho_i(p'), 1) = U_i(p', 1)$ . This function  $\rho_i, i = N, R$  is well-defined under Assumption 1. Now let  $I_i^{p'}$  be the level set on which the type  $i$  seller is indifferent. Formally, for  $p' \in (\underline{v}, p_i^*]$ ,

$$I_i^{p'} = \{(p, \mu) \in (\underline{v}, \bar{v}) \times (0, 1] : U_i(p, \mu) = U_i(p', 1)\}.$$

**Lemma 2** *Given  $p' \in (\underline{v}, p_i^*]$ , for each  $p \in (p', \rho_i(p'))$ , there exists a unique  $\mu_i \in (\phi(F, p), 1)$  such that  $(p, \mu_i) \in I_i^{p'}$ .*

**Proof.** Suppose  $p \in (p', \rho_i(p'))$ . Since  $U_i(p, \phi(F, p)) \leq \underline{v} < U_i(p', 1) < U_i(p, 1)$  and  $U_i(p, \cdot)$  is continuous, there exists  $\mu_i \in (\phi(F, p), 1)$  such that  $(p, \mu_i) \in I_i^{p'}$ . It is also unique because  $U_i(p, \cdot)$  is strictly increasing when  $\mu > \phi(F, p)$ . ■

With Lemma 2, for each  $p' \in (\underline{v}, p_i^*]$ , the associated isoprofit curve is represented by a function  $\psi_i^{p'} : [p', \rho_i(p')] \rightarrow [0, 1]$  where  $(p, \psi_i^{p'}(p)) \in I_i^{p'}, \forall p \in [p', \rho_i(p')]$ . The continuity of  $\psi_i^{p'}$  is implied by the continuity of  $U_i(\cdot, \cdot)$ .

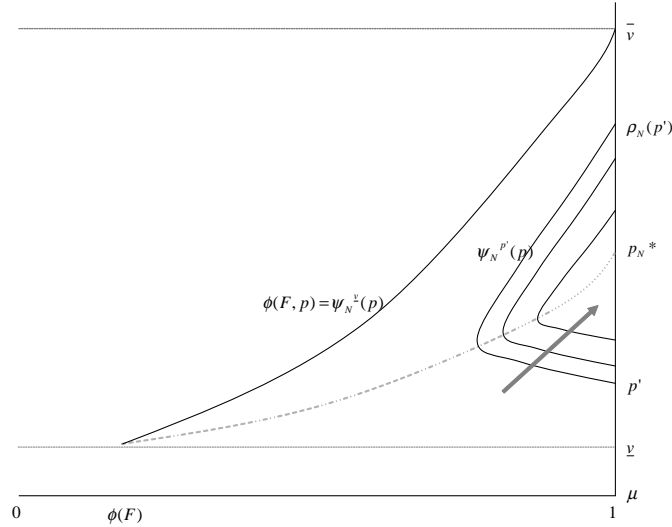


Figure 1: Isoprofit curves of the normal type.

The isoprofit curves on which each type gets  $\underline{v}$  play crucial roles later. For the normal type, it is straightforward that as  $p'$  tends to  $\underline{v}$ ,  $\psi_N^{p'}(p)$  converges to  $\phi(F, p)$  for all  $p$ . For the rational commitment type, define  $\psi_R^{\underline{v}}(p) = \lim_{p' \rightarrow \underline{v}} \psi_R^{p'}(p), \forall p \in (\underline{v}, \bar{v})$ . Then  $\psi_R^{\underline{v}}(p) \in (\phi(F, p), 1)$ , because  $U_R(p, \phi(F, p)) = \underline{v}(1 - F(p)^{p/\underline{v}}) < \underline{v} < U_R(p, 1)$  for  $p > \underline{v}$ . In addition, as  $p$  tends to  $\underline{v}$ , for the rational commitment type to achieve  $\underline{v}$ ,  $c_B$  must be close to  $F(p)$ , and thus

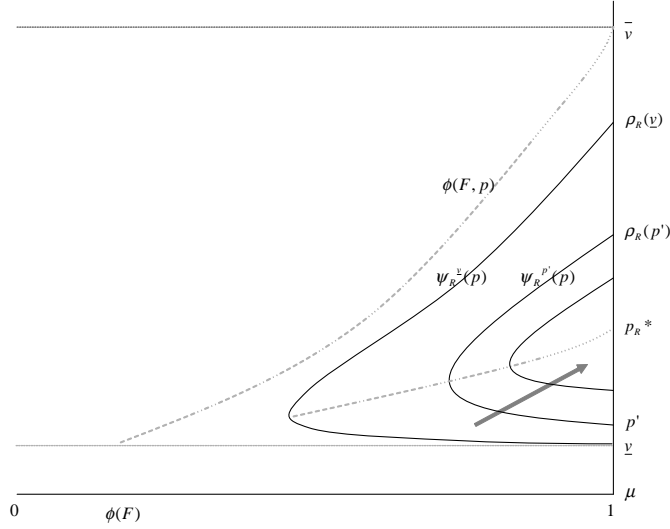


Figure 2: Isoprofit curves of the rational commitment type.

$\psi_{R^v}^v(p)$  approaches 1. Intuitively, for the rational commitment type to achieve  $\underline{v}$  by offering a price close to  $\underline{v}$ , there cannot be significant delay, which happens only when the probability of the commitment type is sufficiently large.

### 3 Multiple Behavioral Types

This section studies the game with multiple behavioral types. Let  $\Xi_B \subseteq [\underline{v}, \bar{v}]$  be the set of behavioral types.<sup>9</sup> The seller is behavioral with probability  $\mu_B$ . Conditional on that the seller is behavioral, the probability measure over the Borel  $\sigma$ -field,  $\mathcal{F}$ , of  $\Xi_B$  is given by  $\lambda$ . For notational simplicity, I assume that  $\lambda$  has no mass at  $\underline{v}$ .

Given the results in Section 2, the analysis for multiple behavioral types reduces to the following simple problem. The normal type plays a mixed strategy, denoted by a cumulative distribution function  $H_N$  over  $[\underline{v}, \bar{v}]$ . Consumers assign beliefs to prices, that is, consumers determine  $\mu : [\underline{v}, \bar{v}] \rightarrow [0, 1]$ . The solution (equilibrium) to this reduced-form problem is  $(H_N^*, \mu^*)$  such that if (i) given  $\mu^*$ , for any  $p' \in \Xi_N^* \equiv \text{supp} H_N^*$ ,

$$U_N(p', \mu^*(p')) = \max_{p \in [\underline{v}, \bar{v}]} U_N(p, \mu^*(p)),$$

and (ii) given  $H_N^*$  (and  $\lambda$ ),  $\mu^*$  is the conditional probability that the seller is a commitment type. This reduction is possible because once the seller offers a price  $p$  and consumers'

<sup>9</sup>I denote each behavioral type by the price she is committed to.

beliefs are given by  $\mu$ , the subsequent dynamics amount to the one with a single behavioral type. Furthermore, as shown in the previous section, the dynamics has a unique sequential equilibrium and the normal type's indirect expected utility is available.

I assume that whatever price is offered by the seller, there is a chance that the seller is a commitment type. This is an extreme case of a "rich" set of behavioral types.

**Assumption 2**  $\lambda([\underline{v}, p])$  is strictly increasing in  $p \in (\underline{v}, \bar{v})$ .

One important consequence of this assumption is that there is no off-the-equilibrium-path price in  $[\underline{v}, \bar{v}]$ , and therefore,  $\mu^*$  is completely determined by Bayes' rule. Once either  $\mu^*$  or  $H_N^*$  is determined, the other follows. Relying on this observation, I focus on  $\mu^*$  and do not explicitly derive  $H_N^*$ .

**Lemma 3** *The set of prices the normal type offers in equilibrium ( $\Xi_N^*$ ) is  $[p', \rho_N(p')]$  for some  $p' \in [\underline{v}, p_N^*]$ . In addition, if  $p' > \underline{v}$ , then  $\mu^*(p') = \mu^*(\rho_N(p')) = 1$ .*

**Proof.** Suppose  $p' \in \Xi_N^*$ , but  $p \notin \Xi_N^*$  for some  $p \in (p', \rho_N(p'))$ . Then the normal type deviates to  $p$ , because  $U_N(p, 1) > U_N(p', 1) \geq U_N(p', \mu^*(p'))$ . Therefore,  $[p', \rho_N(p')] \subseteq \Xi_N^*$ . This establishes the first result.

Now suppose  $\Xi_N^* = [p', \rho_N(p')]$  for some  $p' > \underline{v}$ , but  $\mu^*(p') < 1$ . Then for  $\varepsilon > 0$  sufficiently small,  $U_N(p' - \varepsilon, 1) > U_N(p', \mu^*(p'))$ , which cannot be the case in equilibrium. An analogous proof applies to  $\rho_N(p')$ . ■

Suppose in equilibrium  $\Xi_N^* = [p', \rho_N(p')]$  for some  $p' \in (\underline{v}, p_N^*]$ . A straightforward but crucial consequence is that  $\mu^*(p) = \psi_N^{p'}(p)$  on  $\Xi_N^*$ . That is, for  $\Xi_N^*$  to be the equilibrium support, the equilibrium beliefs must coincide with the isoprofit curve of the normal type starting from  $(p', 1)$ . This is simply a restatement that in equilibrium a player is indifferent over all actions in the support of his mixed strategy.

The appropriate  $p'$  is found as follows. Given  $\mu_B$  and  $\lambda$ , start from  $p_N^*$  and continuously lower the price. As  $p'$  decreases, the corresponding isoprofit curve expands to the left, encompassing previous isoprofit curves (See Figure 1). In order to make  $\mu^*$  follow  $\psi_N^{p'}$ , the normal type must play each price with a specific probability. Integrating over the probabilities that the normal type assigns to each price, the integral continuously increases as  $p'$  decreases. The integral becomes equal to 1 (the normal type must exhaust probability 1) at the proper  $p'$ . Such  $p'$  is unique because isoprofit curves do not cross each other.

If there does not exist such  $p' > \underline{v}$ , then  $\Xi_N^* = [\underline{v}, \bar{v}]$ , that is, the normal type mixes over all prices in  $[\underline{v}, \bar{v}]$ . In this case, the equilibrium beliefs are given by  $\mu^*(p) = \phi(F, p), \forall p > \underline{v}$ . This is because consumers' posterior beliefs conditional on the initial offer  $p > \underline{v}$  are at least

$\phi(F, p)$  (See Corollary 3). When  $\Xi_N^* = [\underline{v}, \bar{v}]$ , the integral over the probabilities the normal type assigns to each price in  $(\underline{v}, \bar{v}]$  is less than 1. With the remaining probability, she offers  $\underline{v}$ .

The following quantity plays a similar role to  $\phi(F, p)$  in the previous section. Given  $\lambda$  and  $F$ , let  $\varphi(F, \lambda)$  be the value that satisfies

$$\frac{1 - \varphi(F, \lambda)}{\varphi(F, \lambda)} = \int_{(\underline{v}, \bar{v})} \frac{1 - \phi(F, p)}{\phi(F, p)} d\lambda.$$

Roughly,  $\varphi(F, \lambda)$  is an average value of  $\phi(F, p)$  with respect to  $\lambda$ . It can be interpreted as the bargaining power of the buyer facing behavioral commitment types distributed according to  $\lambda$ .  $\varphi(F, \lambda)$  inherits the property that if  $F$  first-order stochastically dominates  $F'$ , then  $\varphi(F, \lambda) \leq \varphi(F', \lambda)$ . Intuitively, the seller is relatively stronger when she faces a more favorable distribution of buyer valuations.

**Proposition 2** (1) *If  $\mu_B \leq \varphi(F, \lambda)$ , then there exists a unique sequential equilibrium in which the normal type mixes over all prices in  $[\underline{v}, \bar{v}]$  and consumers' beliefs are given by  $\mu^*(p) = \phi(F, p), \forall p > \underline{v}$ . In this case, the normal type earns the competitive profit.*

(2) *If  $\mu_B > \varphi(F, \lambda)$ , then there exists a unique sequential equilibrium in which the normal type mixes over prices in the interval  $[p', \rho_N(p')]$  for some  $p' \in (\underline{v}, p_N^*)$  and consumers' beliefs coincide with the isoprofit curve of the normal type that starts from  $p'$ . In this case, the normal type achieves more than the competitive profit.*

**Proof.** The proof uses the fact that the likelihood ratio of the seller's being the normal type to being a commitment type is a martingale with respect to the distribution of commitment types (See Doob (1953)). Formally,

$$\frac{1 - \mu_B}{\mu_B} = \int_{(\underline{v}, \bar{v})} \frac{1 - \mu^*}{\mu^*} d\lambda = \int_{\Xi_N^*} \frac{1 - \mu^*}{\mu^*} d\lambda.$$

In words, the aggregate ratio of the normal type to behavioral types (the left-hand side) must be equal to the expectation of the conditional ratio (the right-hand side).<sup>10</sup>

Suppose  $\mu^*(p) > \phi(F, p)$  for some  $p > \underline{v}$ . Since  $U_N(p, \mu^*(p)) > \underline{v}$ , for any  $p'' \in \Xi_N^*$ ,

<sup>10</sup>The case where both  $H_N$  and  $\lambda$  have densities is particularly easy. Suppose  $h_N$  and  $g$  are density functions of  $H_N$  and  $\lambda$ , respectively. Then

$$\mu^*(p) = \frac{\mu_B g(p)}{(1 - \mu_B)h_N(p) + \mu_B g(p)} \Leftrightarrow \frac{1 - \mu_B}{\mu_B} h_N(p) = \frac{1 - \mu^*(p)}{\mu^*(p)} g(p).$$

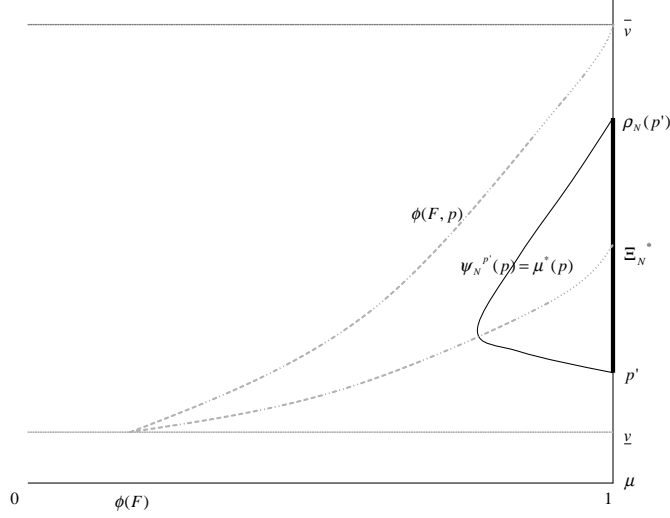


Figure 3: A typical equilibrium when  $\mu_B > \varphi(F, \lambda)$  in the model with multiple behavioral types.

$\mu^*(p'') > \phi(F, p'')$  and  $\underline{v} \notin \Xi_N^*$ . For  $\mu^*$  to be the equilibrium beliefs,

$$\frac{1 - \mu_B}{\mu_B} = \int_{\Xi_N^*} \frac{1 - \mu^*}{\mu^*} d\lambda < \int_{(\underline{v}, \bar{v})} \frac{1 - \phi(F, p)}{\phi(F, p)} d\lambda = \frac{1 - \varphi(F, \lambda)}{\varphi(F, \lambda)}.$$

Hence, if  $\mu_B \leq \varphi(F, \lambda)$ , then  $\mu^*(p) = \phi(F, p), \forall p > \underline{v}$  and  $\Xi_N^* = [\underline{v}, \bar{v}]$ .

Now suppose  $\mu_B > \varphi(F, \lambda)$ . Define  $J : (\underline{v}, p_N^*] \rightarrow R_+$  by

$$J(p') = \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\lambda.$$

$J$  is strictly decreasing, because if  $p' < p''$  then  $\psi_N^{p'}(p) < \psi_N^{p''}(p)$ . In addition, as  $p'$  tends to  $\underline{v}$ ,  $\psi_N^{p'}(p)$  converges to  $\phi(F, p)$  for all  $p > \underline{v}$ . Therefore, the image of  $J$  is  $[0, (1 - \varphi(F, \lambda))/\varphi(F, \lambda)]$ . Hence, for each  $\mu_B > \varphi(F, \lambda)$ , there exists a unique corresponding  $p' \in (\underline{v}, p_N^*]$  such that  $(1 - \mu_B)/\mu_B = J(p')$ . By Lemma 3, this is the unique equilibrium. ■

Since  $h_N$  is a density function and its support is  $\Xi_N^*$ ,

$$\frac{1 - \mu_B}{\mu_B} = \frac{1 - \mu_B}{\mu_B} \int_{\Xi_N^*} h_N(p) dp = \int_{\Xi_N^*} \frac{1 - \mu^*(p)}{\mu^*(p)} g(p) dp = \int_{\Xi_N^*} \frac{1 - \mu^*}{\mu^*} d\lambda.$$

If  $\mu_B \leq \varphi(F, \lambda)$ , the probability that delay occurs is

$$\mu_B + (1 - \mu_B) \int_{(\underline{v}, \bar{v})} \frac{\mu_B}{1 - \mu_B} \frac{1 - \phi(F, p)}{\phi(F, p)} d\lambda = \frac{\mu_B}{\varphi(F, \lambda)}.$$

The first term is the probability that the seller is behavioral, while the second term comes from the mimicking behavior of the normal type. The integrand is the probability that the normal type assigns to  $p$  so that consumers' conditional beliefs are equal to  $\phi(F, p)$ . Hence, the integral is the probability that the normal type offers a higher price than  $\underline{v}$  at date 0. It is immediate that delay occurs whenever  $\mu_B > 0$ , but disappears as  $\mu_B$  approaches 0.

**Corollary 5** (*The Coase conjecture with multiple behavioral types*) *When there are multiple behavioral types, the profit implication of the Coase conjecture is robust in a strong sense, while the efficiency implication is robust in a weak sense.*

The reason why the conclusions in the previous section carry over to the case with multiple behavioral types is that when the unconditional probability of behavioral types,  $\mu_B$ , is small, the probabilities conditional on the seller's initial offer are small uniformly over all prices. This relationship is generated by the fact that in equilibrium the normal type is indifferent over all prices she may offer. Intuitively, if the normal type prefers a price to others, consumers believe that the seller who offers the price is the normal type with a high probability. This reduces the normal type's expected payoff and, therefore, nullifies her preference for the price.

Now suppose the seller is likely to be committed ( $\mu_B > \varphi(F, \lambda)$ ). Which prices would consumers believe are more likely to drop? Consumers are most suspicious when the seller offers a price that is neither low nor high (See Figure 3). Only a few consumers may accept a high price. Therefore, if the price is too high, it is unlikely that the seller rationally set the price. The seller does not benefit a lot from offering a low price. Consumers believe that the opportunistic seller must have tried a higher price, and consequently, the seller offering a low price is indeed committed to the price.

Lastly, observe that if  $\mu_B$  is sufficiently high, then the normal type always offers a higher price than the static monopoly price (and earns more than the static monopoly profit). Formally, if

$$\frac{1 - \mu_B}{\mu_B} < C(F, \lambda) \equiv \int_{(p_R^*, \rho_N(p_R^*))} \frac{1 - \psi_N^{p_R^*}(p)}{\psi_N^{p_R^*}(p)} d\lambda,$$

then any price the normal type offers in equilibrium is higher than the static monopoly price. To see this, suppose  $\mu_B$  is arbitrarily close to 1. Then, whatever price was offered, consumers believe that the seller is committed almost for sure and, therefore, accept the price as long as

their valuations are greater than the price. An instant later, the normal type can lower the price and earn an additional profit. In this scenario, the effective objective function of the normal type is  $U_N(p, 1) = (1 - F(p))p + F(p)\underline{v}$ , whose maximum  $p_N^*$  is strictly greater than  $p_R^*$ . Without uncertainty, her flexibility is a curse. With uncertainty, it may be a blessing.

## 4 The Rational Commitment Type

In this section, the seller is the rational commitment type with probability  $\mu_R$  and the normal type with complementary probability. Again, I focus on the determination of equilibrium prices and beliefs. This is possible because the indirect expected utility of the rational commitment type is available as well (See Corollary 4).

It is always an equilibrium that both types offer the competitive price. This equilibrium is induced by the most skeptical beliefs of consumers that  $\mu^*(p) = \phi(F, p)$  for  $p > \underline{v}$ . Under these beliefs, it is optimal for the rational commitment type to offer  $\underline{v}$ . The normal type is indifferent over all prices in  $[\underline{v}, \bar{v}]$ , and thus it is rational for her to offer  $\underline{v}$ . Given these choices, consumers' beliefs are consistent because any higher price than  $\underline{v}$  is off the equilibrium path.

For  $\mu_R$  small, this equilibrium is the unique equilibrium. Suppose  $\mu_R \leq \phi(F) (\equiv \phi(F, \underline{v}))$ , but the rational commitment type offers  $p > \underline{v}$ . Then consumers' beliefs conditional on  $p$ ,  $\mu(p)$ , must be greater than  $\phi(F, p)$ , because  $\underline{v}$  is the lower bound of the seller's payoff (she can always offer  $\underline{v}$ ), while  $U_R(p, \phi(F, p)) < \underline{v}$ . But this is a contradiction for the following reason. On the one hand, for consumers' beliefs to be greater than  $\phi(F, p)$ , the normal type must offer  $\underline{v}$  with positive probability, because  $\mu(p) > \phi(F, p) \geq \phi(F) \geq \mu_R$ . This implies that the equilibrium expected payoff of the normal type is  $\underline{v}$ . On the other hand, the normal type can achieve more than  $\underline{v}$  by offering  $p$ , because  $U(p, \mu(p)) > U(p, \phi(F, p)) = \underline{v}$ . Consequently, when  $\mu_R \leq \phi(F)$ , the rational commitment type offers  $\underline{v}$  in equilibrium. Given this, the normal type has no choice but to offer  $\underline{v}$  as well.

The cutoff value for  $\mu_R$  below which it is the unique equilibrium that both types offer  $\underline{v}$  is greater than  $\phi(F)$ . When  $\mu_R$  is slightly greater than  $\phi(F)$ , the normal type prefers a slightly higher price than  $\underline{v}$ . On the contrary, the rational commitment type is not better off by offering a higher price than  $\underline{v}$ . The normal type's flexibility ensures her  $\underline{v}$  under any circumstance, while the rational commitment type must endure all delay cost from a war of attrition. Let  $\tilde{\phi}(F)$  be the minimum value of  $\mu$  above which the rational commitment type can achieve at least  $\underline{v}$  by offering some higher price than  $\underline{v}$ . Formally,

$$\tilde{\phi}(F) = \min_p \psi_R^{\underline{v}}(p) = \min_p \lim_{p' \rightarrow \underline{v}} \psi_R^{p'}(p).$$

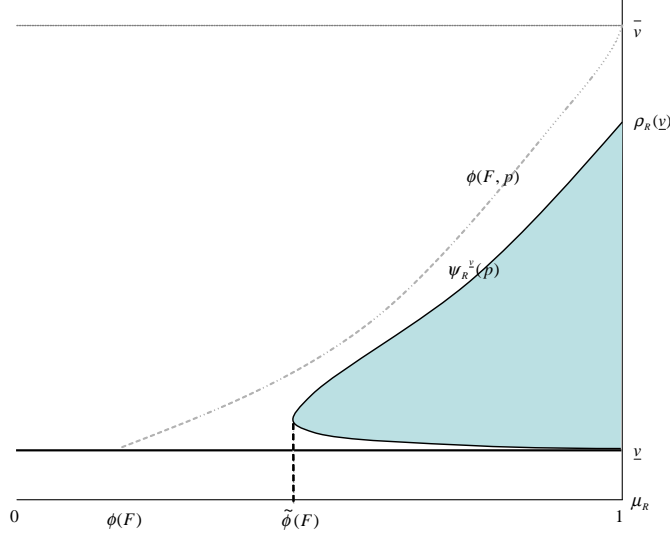


Figure 4: The set of equilibria in the model with the rational commitment type.

Then, by the same reasoning as in the previous paragraph, the Coase conjecture holds up to  $\tilde{\phi}(F)$ .

If  $\mu_R \geq \tilde{\phi}(F)$ , then there exist another equilibrium. Let  $P(\mu_R)$  be the set of prices such that if  $p \in P(\mu_R)$  then  $p > \underline{v}$  and  $U_R(p, \mu_R) \geq \underline{v}$ . Fix  $p \in P(\mu_R)$  and find  $p_N \in (\underline{v}, p_N^*)$  and  $p_R \in (\underline{v}, p_R^*)$  such that  $\mu_R = \psi_R^{p_R}(p) = \psi_N^{p_N}(p)$ . Then take off-the-equilibrium-path beliefs so that  $\mu^*(p') \leq \min\{\psi_R^{p_R}(p'), \psi_N^{p_N}(p')\}, \forall p' \neq p$  (for example,  $\mu^*(p') = \phi(F, p), \forall p' \neq p$ ). Under these beliefs, by construction,  $p$  is the optimal price to both types. Therefore, for any  $p \in P(\mu_R)$ , there exists a corresponding sequential equilibrium.

**Proposition 3** (1) *It is always an equilibrium that both types offer the competitive price. For  $\mu_R < \tilde{\phi}(F)$ , this is the unique sequential equilibrium.*

(2) *If  $\mu_R \geq \tilde{\phi}(F)$ , then for any  $p \in P(\mu_R)$ , there exist a corresponding sequential equilibrium in which both types offer  $p$ .*

The following result is immediate.

**Corollary 6** *(The Coase conjecture with the rational commitment type) When the commitment type is rational, both the profit implication and the efficiency implication of the Coase conjecture are robust in a strong sense.*

## 4.1 Equilibrium Selection

The multiplicity of equilibrium is driven by the signalling structure of the game at the stage when the seller decides her initial offer. While the common equilibrium refinements do not have any power (for example, the intuitive criterion does not exclude any equilibrium),<sup>11</sup> a selection method using behavioral types is quite effective in this game. The procedure is (1) perturb the game by allowing a small probability of behavioral types, (2) let the probability approach zero, and (3) evaluate what equilibria survive the process.

In the full-fledged game, given the equilibrium beliefs  $\mu^*$ , each payoff type finds her own optimal price. Hence, the isoprofit curve of each type is tangent to the equilibrium belief function  $\mu^*$  at her own optimal price (if the optimal price is greater than  $\underline{v}$ ). But when the probability of behavioral types,  $\mu_B$ , is positive, the equilibrium beliefs coincide with an isoprofit curve of the normal type (See Lemma 3 and the subsequent discussion). Therefore, at the optimal price of the rational commitment type, the isoprofit curve of the rational commitment type is tangent to that of the normal type. This property holds whenever  $\mu_B > 0$  and, therefore, holds in the limit as  $\mu_B$  tends to 0.

Figure 5 shows the set of equilibria that survive as  $\mu_B$  approaches zero. It consists of two kinds of equilibria. The first type is the degenerate one in which both the normal type and the rational commitment type offer the competitive price. This equilibrium survives the selection process, because for  $\mu_B$  small, the most skeptical beliefs of consumers,  $\mu^*(p) = \phi(F, p)$ ,  $p > \underline{v}$ , induce the rational commitment type to offer  $\underline{v}$ , which in turn makes the beliefs consistent. The second type is the one in which both types offer  $p > \underline{v}$  and the isoprofit curves of the two types are tangent to each other. These are the equilibria that are limits of the equilibria in the full-fledged game in which the two payoff types pool at higher prices than  $\underline{v}$ . As discussed before, the property that the isoprofit curves of the two payoff types are tangent to each other is preserved in the limit.

Let  $\hat{\phi}(F)$  be the largest value of  $\mu_R$  below which the isoprofit curves of the two payoff types are never tangent to each other. In other words,  $\hat{\phi}(F)$  is the leftmost value of the contract curve between  $U_N$  and  $U_R$ .

**Proposition 4** (1) *If  $\mu_R < \hat{\phi}(F)$ , then the only equilibrium that survives the selection process is the one in which both the normal type and the rational commitment type offer*

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<sup>11</sup>A criterion that has some selection power is that of "undefeated" defined by Mailath, Okuno-Fujiwara, and Postlewaite (1993). Undefeated is equivalent to Pareto domination in the current game. When there are two equilibria in which  $p$  and  $p'$  are played, the equilibrium with  $p$  defeats that with  $p'$ , if both types are better off in the former. If one type is better off but the other type is worse off, the two equilibria are incomparable.

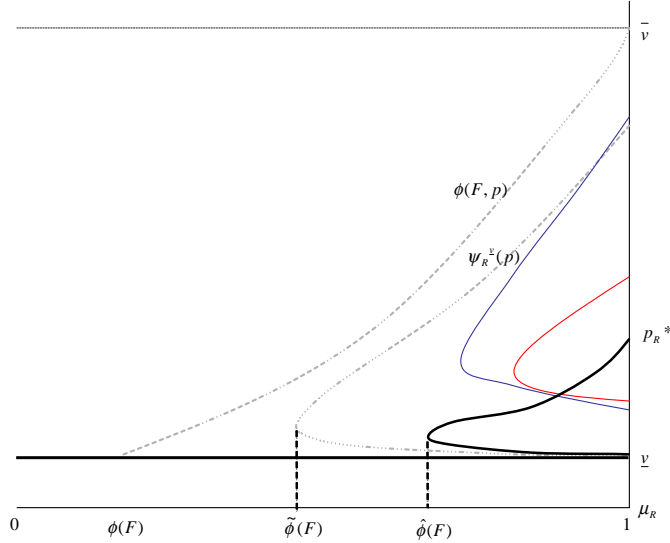


Figure 5: The thick black line is the set of equilibria that survive the selection process.

$\underline{v}$ . This remains as an equilibrium even for  $\mu_R \geq \hat{\phi}(F)$ .

(2) If  $\mu_R \geq \hat{\phi}(F)$ , then there exists another equilibrium that survives the selection process in which both types offer a higher price than  $\underline{v}$ .

$\hat{\phi}(F)$  is greater than  $\tilde{\phi}(F)$ . Therefore, restricting attention to the set of equilibria that survive the selection process, the Coase conjecture is even more robust. The mathematical reason why  $\hat{\phi}(F) > \tilde{\phi}(F)$  is that the contract curve of  $U_N$  and  $U_R$  lies in the interior of  $\psi_R^v$ . The intuitive reason is that off-the-equilibrium-path beliefs are more restricted when there are behavioral types. Without behavioral types, the only requirement on off-the-equilibrium-path beliefs was that the equilibrium beliefs must be to the left of the minimum of the isoprofit curves of the two payoff types. This allows the seller to have rather pessimistic off-the-equilibrium-path beliefs and, consequently, yields a large set of equilibria. With behavioral types, the seller loses this latitude. Now  $\mu^*$  must coincide with an isoprofit curve of the normal type and the isoprofit curve of the rational commitment type must be tangent to that of the normal type. This makes it harder for the two payoff types to coordinate on a higher price than  $\underline{v}$  and, ultimately, increases the cutoff value.

## 5 Discussion

(1) Acquired stubbornness vs. Inborn stubbornness

Kambe (1999) introduces two kinds of stubbornness. With "inborn" stubbornness an

agent knows her type before deciding the initial offer, while with "acquired" stubbornness she does not know whether she will be stuck with the initial offer. Kambe argues that acquired stubbornness can arise either psychologically or economically (reputationally).

This paper studies inborn stubbornness. Extending the model to incorporate acquired stubbornness is straightforward. The only difference is that the relevant indirect expected utility of the seller is the weighted sum of the two indirect utility functions. One feature of acquired stubbornness is that equilibrium is always unique, which is a trivial consequence from the fact that there is a single decision maker.

(2) The no gap case

With no gap ( $\underline{v}=0$ ), there are multiple equilibria in the standard problem (See Ausubel and Deneckere (1989)). Therefore, in order to analyze the no gap case, the equilibrium behavior in case the seller's type is revealed to be normal needs to be specified. I focus on the weak-Markov equilibrium. This case is particularly tractable, because it is the limit of the unique equilibrium in the Gap case, and thus the limit argument can be used in my problem as well.

Given  $p$  and  $\mu$ , as  $\underline{v} \rightarrow 0$ ,  $\tau(v) = 0$  for  $v < p$ , and  $G_S(t) = 0, \forall t$ . That is, the seller never lowers the price and all consumers with higher valuations than  $p$  purchase immediately. The driving force of this result is that the normal type becomes more reluctant to concede as the gain from doing so ( $\underline{v}$ ) becomes smaller and she has no reason to reveal her type in the limit. In the limit as  $\underline{v} \rightarrow 0$ , the seller always (whether the seller is the normal type or the rational commitment type and whether there are behavioral types or not) offers the static monopoly price and achieves the static monopoly profit.

(3) Different discount factors - relation to the standard reputation result

Suppose the buyer and the seller have different discount factors,  $r_B$  and  $r_S$ , respectively. Then for all  $p$ ,

$$\phi(F, p) = \exp \left( -\frac{1}{\underline{v}} \frac{r_B}{r_S} \int_p^{\bar{v}} \frac{(v-p)f(v)}{F(v)} dv \right).$$

All other results have straightforward extensions. For fixed  $r_B$ , as  $r_S$  becomes smaller,  $\phi(F, p)$  decreases, and thus the buyer concedes faster. The robustness results for the Coase conjecture still hold, but less strongly.

This analysis shows that the standard reputation result still holds in my problem in terms of discount rates: in the game with behavioral types, for fixed  $\mu_B > 0$ , there exists  $\bar{r}_S > 0$  such that if  $r_S \leq \bar{r}_S$  then the expected payoff of the normal type is  $\varepsilon$ -close to her maximum expected payoff  $U_N(p_N^*, 1)$ .

## 6 Appendix: The Full-fledged Game

This appendix studies the game in which both the rational commitment type and behavioral types are active. Let  $\mu_N, \mu_R$  and  $\mu_B$  be the probability for the seller to be the normal type, the rational commitment type, and one of behavioral types, respectively. I keep notations and assumptions in Section 2 and 3.

Again I focus on the determination of equilibrium prices and beliefs. A mixed strategy of the type  $i$  seller is represented by a cumulative distribution function  $H_i$  over  $[\underline{v}, \bar{v}]$ ,  $i = N, R$ .

**Definition 1** *A triple  $(H_N^*, H_R^*, \mu^*)$  constitutes a sequential equilibrium in the full-fledged game if (i) given  $\mu^*$ , for any  $p' \in \Xi_i^* \equiv \text{supp}H_i^*$ ,  $i = N, R$ ,*

$$U_i(p', \mu^*(p')) = \max_{p \in [\underline{v}, \bar{v}]} U_i(p, \mu^*(p)),$$

*and (ii) given  $H_N^*$  and  $H_R^*$  (and  $\lambda$ ),  $\mu^*$  is the conditional probability that the seller is a commitment type.*

Since Lemma 3 applies whenever  $\mu_B > 0$ , once  $\Xi_N^*$  is given,  $\mu^*$  is uniquely identified, and  $\Xi_R^*$  has one of the following two possibilities.

**Lemma 4** *Either  $\Xi_R^* \subseteq \Xi_N^*$  or  $\Xi_R^* = \{p_R^*\}$ .*

**Proof.** Suppose there exists  $p \in \Xi_R^* - \Xi_N^*$ . If  $p \neq p_R^*$ , for  $\varepsilon$  small enough, the rational commitment type has a profitable deviation to either  $p - \varepsilon$  or  $p + \varepsilon$ , because  $U_R(p, 1)$  is single-peaked. ■

There are three types of equilibria according to  $\Xi_N^*$ . (1)  $\Xi_N^* = [\underline{v}, \bar{v}]$ , (2)  $\Xi_R^* = \{p_R^*\} \not\subseteq \Xi_N^*$ , and (3)  $\Xi_R^* \subseteq \Xi_N^* \neq [\underline{v}, \bar{v}]$ .

*Case 1:*  $\Xi_N^* = [\underline{v}, \bar{v}]$ .

By the discussion in Section 3, if  $\Xi_N^* = [\underline{v}, \bar{v}]$ , then  $\mu^*(p) = \phi(F, p)$ ,  $\forall p > \underline{v}$ . In this case, the rational commitment type offers  $\underline{v}$ . Given this, the game is essentially equivalent to the one without the rational commitment type. The normal type plays each price  $p > \underline{v}$  to the extent that  $\mu^*(p) = \phi(F, p)$  and plays  $\underline{v}$  with the remaining probability. As in Section 3, this equilibrium exists if and only if

$$\frac{\mu_N}{\mu_B} \geq \int_{(\underline{v}, \bar{v})} \frac{1 - \phi(F, p)}{\phi(F, p)} d\lambda = \frac{1 - \varphi(F, \lambda)}{\varphi(F, \lambda)}.$$

*Case 2:*  $\Xi_R^* = \{p_R^*\} \not\subseteq \Xi_N^*$ .

At the end of Section 3, I discuss the possibility that the normal type always offers a higher price than the static monopoly price,  $p_R^*$ . It happens when the probability of behavioral types is sufficiently greater than that of the normal type. The same phenomenon happens in the full-fledged game. If  $\mu_B$  is sufficiently larger than  $\mu_N$ , then any price the normal type offers in equilibrium is greater than  $p_R^*$ . Then  $\mu^*(p_R^*) = 1$  and the rational commitment type obviously chooses  $p_R^*$ . In turn, the normal type does not offer  $p_R^*$  because she is achieving more than  $U_N(p_R^*, 1)$  already. As in Section 3, this equilibrium exists if and only if

$$\frac{\mu_N}{\mu_B} < C(F, \lambda) \equiv \int_{(p_R^*, \rho_N(p_R^*))} \frac{1 - \psi_N^{p_R^*}(p)}{\psi_N^{p_R^*}(p)} d\lambda.$$

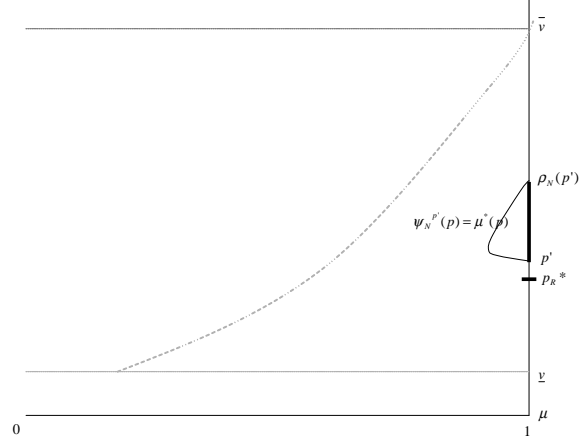


Figure 6: Case 2 equilibrium.

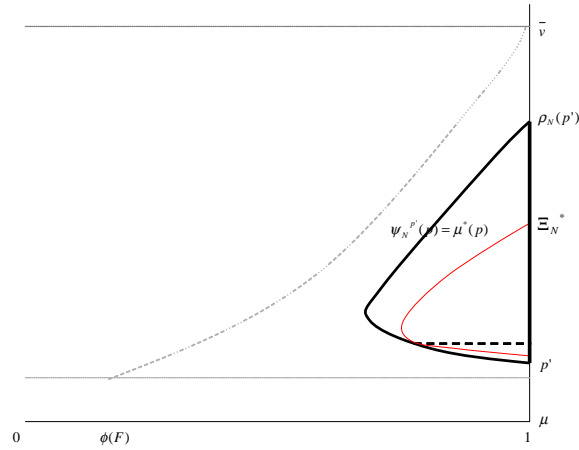


Figure 7: Case 3 equilibrium.

*Case 3:*  $\Xi_R^* \subseteq \Xi_N^* \neq [v, \bar{v}]$ .

Suppose  $\Xi_N^* = [p', \rho_N(p')]$  for some  $p' \in (v, p_R^*]$ . Since  $\mu^*(p) = \psi_N^{p'}(p)$ , for any  $p'' \in \Xi_R^*$ ,

$$p'' \in \arg \max_p U_R(p, \mu^*(p)) = \arg \max_p U_R(p, \psi_N^{p'}(p)).$$

Since  $\psi_N^{p'}(\cdot)$  is an isoprofit curve of the normal type, this implies that for  $p'' \in \Xi_R^*$ , the isoprofit curve of the rational commitment type is tangent to that of the normal type at  $(p'', \psi_N^{p'}(p''))$ .

Let  $\Upsilon(p')$  be the set of optimal prices to the rational commitment type given  $\mu^*(p) = \psi_N^{p'}(p)$ .  $\Upsilon(p')$  is well-defined because  $\psi_N^{p'}(\cdot)$  is closed and connected and  $U_R$  is continuous. The following lemma provides necessary information on  $\Upsilon(p')$ .

**Lemma 5** *For any  $p' \in (\underline{v}, p_R^*]$ , if  $p'' \in \Upsilon(p')$ , then  $p'' \leq p_R^*$ . That is, the rational commitment type never chooses a higher price than  $p_R^*$ . In addition, as  $p'$  tends to  $p_R^*$  or  $\underline{v}$ , for any  $p'' \in \Upsilon(p')$ ,  $\psi_N^{p'}(p'')$  converges to 1. That is, the associated beliefs to prices in  $\Upsilon(p')$  are close to 1, if  $p'$  is sufficiently close to  $p_R^*$  or  $\underline{v}$ .*

**Proof.** For  $p \in (\underline{v}, \bar{v})$  and  $\mu \in (\phi(F, p), 1)$ , let

$$\begin{aligned} K(p, \mu) &= \frac{\partial U_R(p, \mu)}{\partial p} \frac{\partial U_N(p, \mu)}{\partial \mu} - \frac{\partial U_R(p, \mu)}{\partial \mu} \frac{\partial U_N(p, \mu)}{\partial p} \\ &= \left( \frac{F(p)}{c_B} \right)^{p/\underline{v}} \frac{\partial c_B}{\partial \mu} (p - \underline{v}) \left( c_B \left( 1 + \ln \left( \frac{F(p)}{c_B} \right) + p \frac{f(p)}{F(p)} \right) - 1 \right). \end{aligned}$$

$K(p, \mu) = 0$  if and only if

$$k(p, \mu) = c_B \left( 1 + \ln \left( \frac{F(p)}{c_B} \right) + p \frac{f(p)}{F(p)} \right) - 1 = 0.$$

Since  $c_B < 1$ ,

$$\ln \left( \frac{F(p)}{c_B} \right) + p \frac{f(p)}{F(p)} > 0.$$

In addition, at  $(p, \mu)$  that satisfies the above inequality,

$$\frac{\partial k(p, \mu)}{\partial \mu} = \frac{\partial c_B}{\partial \mu} \left( \ln \left( \frac{F(p)}{c_B} \right) + p \frac{f(p)}{F(p)} \right) < 0.$$

This implies that for each  $p$ , if exists, there is a unique  $\mu$  such that  $k(p, \mu) = 0$ . Now notice that

$$\lim_{\mu \rightarrow 1} k(p, \mu) = pf(p) - (1 - F(p)) \begin{cases} < \\ = \\ > \end{cases} 0 \text{ if } p \begin{cases} < \\ = \\ > \end{cases} p_R^*.$$

Hence for  $K(p, \mu) = 0$ ,  $p$  must be less than  $p_R^*$ . This establishes the first result.

If  $p'$  is sufficiently close to  $p_R^*$ , the rational commitment type chooses  $(p, \mu)$  sufficiently close to the global maximizer  $(p_R^*, 1)$ , and thus  $\mu$  is close to 1 (If  $p'$  is equal to  $p_R^*$ , then the rational commitment type obviously chooses  $(p_R^*, 1)$ ).

Now suppose  $p'$  is close to  $\underline{v}$ . Since  $U_R(p, \mu) < \underline{v}$  for  $\mu < 1$  and  $p$  close to  $\underline{v}$  (See Figure 2), the rational commitment type chooses  $p$  such that the associated belief  $\psi_N^{p'}(p)$  is sufficiently close to 1. ■

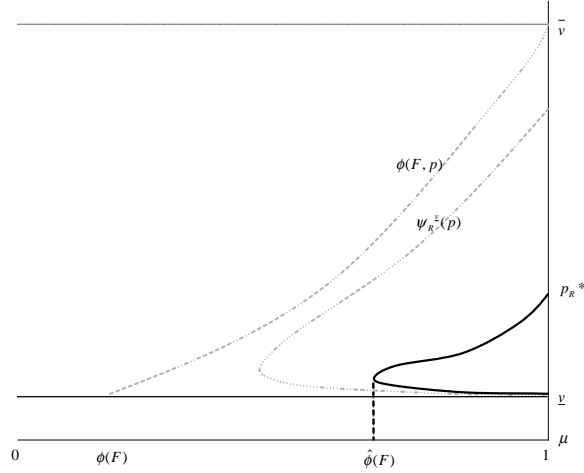


Figure 8: A typical shape of the contract curve.

The argument from this point proceeds as follows. First, I suppose  $\Xi_N^* = [p', \rho_N(p')]$  and the rational commitment type chooses prices in  $\Upsilon(p')$ . This produces the distribution of commitment types, including both behavioral types and the rational commitment type. Given this distribution, the problem is equivalent to the one with only behavioral types. As in the proof of Proposition 2, it suffices to check whether the ratio of the normal type to commitment types is equal to the expectation of the conditional ratio under the new distribution.

Let  $p^{\Upsilon(p')} = \arg \min_{p \in \Upsilon(p')} \{\psi_N^{p'}(p)\}$  and  $p_{\Upsilon(p')} = \arg \max_{p \in \Upsilon(p')} \{\psi_N^{p'}(p)\}$ . For each  $p' \in (\underline{v}, p_R^*]$  define  $\bar{\lambda}^{\mu_B, \mu_R, p'} : \mathcal{F} \rightarrow [0, 1]$  and  $\underline{\lambda}^{\mu_B, \mu_R, p'} : \mathcal{F} \rightarrow [0, 1]$  by

$$\bar{\lambda}^{\mu_B, \mu_R, p'}(P) = \frac{\mu_R \mathbf{1}_{\{p^{\Upsilon(p')} \in P\}} + \mu_B \lambda(P)}{\mu_R + \mu_B},$$

and

$$\underline{\lambda}^{\mu_B, \mu_R, p'}(P) = \frac{\mu_R \mathbf{1}_{\{p_{\Upsilon(p')} \in P\}} + \mu_B \lambda(P)}{\mu_R + \mu_B},$$

for  $P \in \mathcal{F}$ . These are mixed distributions of commitment types, conditional on the rational commitment type playing  $p^{\Upsilon(p')}$  and  $p_{\Upsilon(p')}$ , respectively.

Similarly to the proof of Proposition 2, define  $\bar{J}^{\mu_B, \mu_R}, \underline{J}^{\mu_B, \mu_R} : (\underline{v}, p_R^*] \rightarrow R_+$  by

$$\begin{aligned}\bar{J}^{\mu_B, \mu_R}(p') &= \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\bar{\lambda}^{\mu_B, \mu_R, p'} \\ &= \frac{\mu_R}{\mu_R + \mu_B} \frac{1 - \psi_N^{p'}(p^\Upsilon(p'))}{\psi_N^{p'}(p^\Upsilon(p'))} + \frac{\mu_B}{\mu_R + \mu_B} \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\lambda, \\ \underline{J}^{\mu_B, \mu_R}(p') &= \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\underline{\lambda}^{\mu_B, \mu_R, p'} \\ &= \frac{\mu_R}{\mu_R + \mu_B} \frac{1 - \psi_N^{p'}(p^\Upsilon(p'))}{\psi_N^{p'}(p^\Upsilon(p'))} + \frac{\mu_B}{\mu_R + \mu_B} \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\lambda.\end{aligned}$$

By definition,  $\psi_R^{p'}(p^\Upsilon(p')) \leq \psi_R^{p'}(p^\Upsilon(p'))$ , and thus  $\bar{J}^{\mu_B, \mu_R}(p') \geq \underline{J}^{\mu_B, \mu_R}(p')$ . The minimum of both  $\bar{J}^{\mu_B, \mu_R}(p')$  and  $\underline{J}^{\mu_B, \mu_R}(p')$  is achieved at  $p_R^*$  because, by Lemma 5,

$$\frac{1 - \psi_N^{p_R^*}(p_R^*)}{\psi_N^{p_R^*}(p_R^*)} = 0 \leq \frac{1 - \psi_N^{p'}(p^\Upsilon(p'))}{\psi_N^{p'}(p^\Upsilon(p'))} \leq \frac{1 - \psi_N^{p'}(p^\Upsilon(p'))}{\psi_N^{p'}(p^\Upsilon(p'))},$$

and  $\psi_N^{p'}(p)$  is strictly increasing in  $p'$  (so  $(1 - \psi_N^{p'}(p)) / \psi_N^{p'}(p)$  is strictly decreasing in  $p'$ ). In addition, its value is  $\mu_B C(F, \lambda) / (\mu_R + \mu_B)$  (See the definition of  $C(F, \lambda)$  in Case 2). Now define a correspondence  $J^{\mu_B, \mu_R} : (\underline{v}, p_R^*] \rightarrow R_+$  by

$$J^{\mu_B, \mu_R}(p') = [\underline{J}^{\mu_B, \mu_R}(p'), \bar{J}^{\mu_B, \mu_R}(p')].$$

Every value in  $J^{\mu_B, \mu_R}(p')$  can be achieved by putting appropriate weights to  $p^{\Upsilon(p')}$  and  $p^\Upsilon(p')$ . In addition, by the Theorem of Maximum,  $J^{\mu_B, \mu_R}$  is upper hemi-continuous.

Case 3 equilibrium exists when the ratio of the normal type to commitment types is in the range of the correspondence  $J^{\mu_B, \mu_R}$ , that is, when

$$\frac{\mu_B}{(\mu_R + \mu_B)} C(F, \lambda) \leq \frac{\mu_N}{(\mu_R + \mu_B)} < M^{\mu_1, \mu_2} \equiv \sup\{\bar{J}^{\mu_B, \mu_R}(p') : p' \in (\underline{v}, p_R^*]\}.$$

**Proposition 5** (1) (Case 1: degenerate) If  $\mu_N / \mu_B \geq (1 - \varphi(F, \lambda)) / \varphi(F, \lambda)$ , then there exists an equilibrium in which  $\Xi_N^* = [\underline{v}, \bar{v}]$ ,  $\mu^*(p) = \phi(F, p)$ ,  $\forall p > \underline{v}$ , and  $\Xi_R^* = \{\underline{v}\}$ .

(2) (Case 2: separating) If  $\mu_N / \mu_B < C(F, \lambda)$ , then there exists an equilibrium in which  $\Xi_N^* = [p', \rho_N(p')]$  for some  $p' \in (p_R^*, p_N^*)$ ,  $\Xi_R^* = \{p_R^*\}$ , and

$$\mu^*(p) = \begin{cases} 1, & \text{if } p \notin \Xi_N^*, \\ \psi_N^{p'}(p), & \text{if } p \in \Xi_N^*. \end{cases}$$

(3) (Case 3: pooling) If  $\mu_B C(F, \lambda) \leq \mu_N < (\mu_R + \mu_B) M^{\mu_1, \mu_2}$ , then there exists an equi-

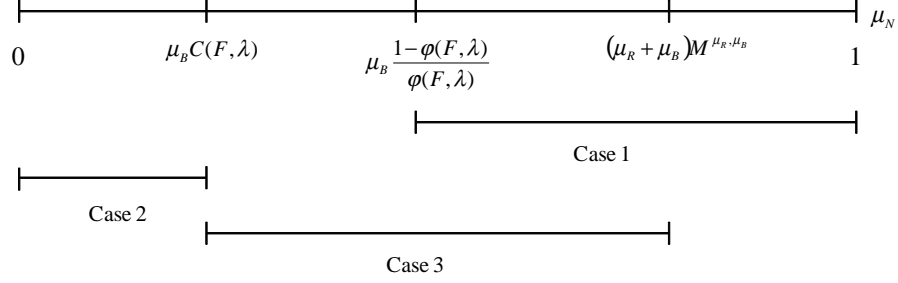


Figure 9: The summary of the full-fledged game.

librium in which  $\Xi_R^* \subset \Xi_N^* = [p', \rho_N(p')]$  for some  $p' \in (\underline{v}, p_R^*]$ , and

$$\mu^*(p) = \begin{cases} 1, & \text{if } p \notin \Xi_N^*, \\ \psi_N^{p'}(p), & \text{if } p \in \Xi_N^*. \end{cases}$$

Figure 9 summarizes all results. When  $\mu_N$  is sufficiently small, the normal type and the rational commitment type separate. The normal type offers only higher prices than the static monopoly price and the rational commitment type offers the static monopoly price. When  $\mu_N$  is sufficiently large, the normal type mixes over all prices in  $[\underline{v}, \bar{v}]$  and the rational commitment type offers the competitive price. In between, the two payoff types pool at higher prices than the competitive price.

When  $\mu_B (1 - \varphi(F, \lambda)) / \varphi(F, \lambda) \leq \mu_N < (\mu_R + \mu_B) M^{\mu_1, \mu_2}$ , there are multiple equilibria. This is when the probability of behavioral types is not sufficiently high, and thus the normal type has an incentive to mimic the rational commitment type, while the probability of the rational commitment type is not sufficiently low so that both types can be better off by coordination. If the rational commitment type offers higher prices than  $\underline{v}$ , then the normal type achieves more than  $\underline{v}$  by mimicking the rational commitment type as well as behavioral types. The rational commitment type also earns more than  $\underline{v}$ . If the rational commitment type offers  $\underline{v}$ , the normal type randomizes over  $[\underline{v}, \bar{v}]$  and  $\mu^*(p) = \phi(F, p)$ . This in turn makes  $\underline{v}$  the optimal price to the rational commitment type.

For the equilibrium selection in Section 4, let  $\hat{\phi}(F)$  be the largest value of  $\mu$  below which the isoprofit curves of the two payoff types are never tangent to each other (See Figure 5). This value is not less than  $\tilde{\phi}(F)$  because  $U_R$  is tangent to  $U_N$  only in the upper Cantor set of  $U_R(p, \psi_N^{\underline{v}}(p))$ . In fact, this value is strictly greater than  $\tilde{\phi}(F)$ , because if  $p'$  is sufficiently close to  $\underline{v}$ , then the associated beliefs to prices in  $\Upsilon(p')$  are close to 1 (Lemma 5).

As  $\mu_B$  tends to 0,

$$\begin{aligned} \bar{J}^{\mu_B, \mu_R}(p') &= \frac{\mu_R}{\mu_R + \mu_B} \frac{1 - \psi_N^{p'}(p^{\Upsilon(p')})}{\psi_N^{p'}(p^{\Upsilon(p')})} + \frac{\mu_B}{\mu_R + \mu_B} \int_{(p', \rho_N(p'))} \frac{1 - \psi_N^{p'}(p)}{\psi_N^{p'}(p)} d\lambda \\ &\rightarrow \frac{1 - \psi_N^{p'}(p^{\Upsilon(p')})}{\psi_N^{p'}(p^{\Upsilon(p')})}, \end{aligned}$$

and thus  $M^{\mu_R, \mu_B}$  converges to  $(1 - \widehat{\phi}(F)) / \widehat{\phi}(F)$ . In addition, as  $\mu_B$  tends to 0, Case 2 becomes empty, while Case 1 covers the interval  $(0, 1]$ . Given this, the equilibrium selection exercise in Section 4 is straightforward. Case 1 equilibrium corresponds to the Coase conjecture outcome, while Case 3 corresponds to the one in which the isoprofit curves of the two payoff types are tangent to each other.

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