Communication with a Possibly Honest Expert*

KyungMin Kim and Jonathan Pogach †

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Abstract

This paper studies a variant of the standard cheap talk game in which the sender is possibly honest. The honest sender always recommends the ex-post optimal policy to the receiver. The presence of the honest type introduces intrinsic value for messages, necessitating a different characterization than Crawford and Sobel (1982). We characterize two classes of equilibria that satisfy two natural properties. In addition, we compare this model to a model in which the sender may be an advocate. The advocate may manipulate information, but only for the sake of the principal (i.e. the sender with no bias). We show that the receiver is often better off with a possibly honest sender than with a sender who may be an advocate.

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1 Introduction

This paper studies behavioral consequences of honesty on economic outcomes. Honesty is valued in many contexts. Elections often feature candidate who emphasize their willingness to tell the truth, even when their views are unpopular.[1] Similarly, stock analysts or journalists known for integrity enjoy more attention from the public. So is it in any employee-employer or contract relationship that a principal seeks agents who will speak honestly. The value of honesty is often understood in the context of a repeated relationship, in which building a reputation for honesty increases future prospects (Sobel (1985), Benabou and Laroque (1992)).

In this paper, we examine how the principal (she) communicates with a possibly honest agent (he). An agent's honesty can enhance the quality of communication, potentially improving efficiency. However, the principal cannot typically be certain that the agent is honest. This possibility

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†Department of Economics, University of Pennsylvania, McNeil Bldg, 3718 Locust Walk, Philadelphia, PA 19104. Kim : kim37@econ.upenn.edu, Pogach: pogachj@econ.upenn.edu.

[1] A recent strand of the political economy literature considers character to be exogenously important to otherwise policy-oriented voters. See, for example, Kartik and McAfee (2007).

introduces a non-trivial inference problem of the principal. On the one hand, the principal can make a better judgement by following the information reported by the honest agent. On the other hand, the principal must take into account the fact that the strategic agent may try to exploit the trust of the principal.

We study a variant of the standard cheap game in which the sender is honest with a positive probability. The honest sender is one who always recommends the ex post optimal policy to the receiver. The presence of the honest type introduces a new feature to the cheap talk game itself: messages are endowed with an intrinsic meaning. Each message might be sent by the honest sender and, therefore, the receiver must reflect this in her posterior.

In the first part of the paper, we characterize two classes of equilibria whose strategy profiles satisfy two natural properties. We identify two new equilibrium properties that are absent in the canonical cheap talk game. As in other contexts, incentive compatibility suggests that the biased agent induce either a locally constant action or his own most preferred action. First, suppose the receiver takes a constant action if she receives a message in some interval. For this to happen, the strategic sender must send the messages in that interval with sufficiently large probability to offset the informativeness of the messages sent by the honest type. This property restricts the behavior of the strategic sender, and provides a new necessary condition for equilibrium. This condition, which we call mass balance, is present in both classes of equilibria that we identify. Second, when the probability of the honest type is sufficiently large, the strategic sender induces his most preferred policy for a positive measure of states. This property distinguishes between the two classes of equilibria we identify, and results from a combination of incentive compatibility and mass balance. Mass balance implies that for high enough probability of the honest type, the induced action cannot be locally constant almost everywhere. Together with incentive compatability, this implies that there must exist a positive measure of states on which the strategic sender induces his most preferred action.

In the second part of the paper, we compare honesty to a closely related characteristic, advocacy. An advocate is a strategic agent who may manipulate information, but only for the sake of the principal. In the cheap talk framework, an advocate is the sender with no bias. There are two opposing arguments as to which type of agent is preferable. On the one hand, the principal may benefit from the advocate's flexibility. If an advocate is not honest in equilibrium, it is because both the principal and he are better off by not being honest. On the other hand, as we know in other contexts, the principal may benefit from an honest agent being committed to telling the truth.

We show that the receiver is often better off with a possibly honest sender than with a sender who may be an advocate. More precisely, we compare the maximum utilities the receiver can achieve in the honesty model (the sender is honest with probability μ, and is strategic with bias $b > 0$ with probability $1 - \mu$), and in the advocacy model (the sender has no bias with probability μ and has bias $b > 0$ with probability $1 - \mu$). We find that for μ sufficiently close to 0 or 1, a possibly honest sender is typically preferable to a sender who may be an advocate.[2] In addition, we explain by some numerical examples that the same conclusion holds for intermediate values of μ.

Our benchmark is the classic cheap talk game of Crawford and Sobel (1982) (hereafter, CS). Their game coincides with the degenerate version of our game ($\mu = 0$). We show that in a uniform-quadratic environment, all equilibrium outcomes in CS are supported as equilibrium outcome if and only if the probability of the honest type is less than or equal to 1/2. In addition, we identify other equilibria for $\mu \leq 1/2$, and show the necessity of alternative structures for $\mu > 1/2$.

Besides CS, this paper is related to three branches of literature about strategic information transmission. The first strand includes Sobel (1985), Benabou and Laroque (1982), and Morris (2001). In our terminology, Sobel and Morris study the advocacy case, while Benabou and Laroque consider the honesty case. The strategic sender in Morris prefers a single action independent of the true state (biased), while the strategic sender has a completely opposite preference to the receiver in the other two (enemy). The focus of these works is the dynamic incentive of the sender to maintain credibility as well as manipulate information. To highlight the intertemporal perspective, they consider simple stage games in which there are only two possible states. We study a static setting with a continuum of possible states.

Another branch is cheap talk games with uncertain bias. In Morgan and Stocken (2003), a stock analyst may or may not have bias in his recommendation. Their problem is equivalent to our advocacy model. Li and Madarasz (2008) consider more general games in which both types of sender may have non-zero bias and study whether requiring the sender to disclose his own bias is necessarily welfare-improving.

The last branch includes Olszewski (2004) and Chen, Kartik, and Sobel (2008) which study the honest sender in the cheap talk game and a selection criterion than includes an honest sender, respectively. In Olszewski, there are finite number of possible states, the receiver has uncertainty over the sender's type, and the strategic sender prefers to be perceived as the honest type. He shows that if the sender's honesty concern is sufficiently strong, information can be fully revealed. Chen, Kartik, and Sobel introduce both the honest sender and the naive receiver (who always follows the sender's recommendation) to the standard cheap talk game. They restrict attention to the class of equilibria in which the sender uses a non-decreasing strategy (in the sense that the strategic sender sends weakly higher messages in higher states) and show that only the most informative equilibrium in CS survives as the perturbation vanishes. We analyze a broader class of equilibria and are not interested in equilibrium selection.

The remainder of the paper is organized as follows. We present the formal model in Section 2 and characterize equilibrium in Section 3. Section 4 briefly examine the advocacy model. We compare the honesty model and the advocacy model in Section 5. We conclude by discussing some relevant issues in Section 6.

[2] We also show that this result is not general. For some values of μ and b the receiver is strictly better off in the advocacy model than in the honesty model.

2 Setup

The basic setup is the standard cheap talk game with one informed player (sender, agent) and one uninformed player (receiver, principal). The sender observes a random variable θ drawn from a uniform distribution with support on $\Theta = [0, 1]$ and strategically transmits information on θ to the receiver. Upon getting a message, the receiver takes an action, denoted by y, from the real line. The receiver's utility function is $U^R(y, \theta) \equiv -(y - \theta)^2$, while the sender's is $U^S(y, \theta, b) \equiv -(y - (\theta + b))^2, b > 0$ (b is interpreted as the "bias" of the sender). Both players maximize their expected utility. Without loss of generality, we can restrict the spaces of policies (Y) and messages (M) to $[0, 1]$.

We study a perturbation of this game. Now the sender is honest with probability μ, and the same type as in the original game with complementary probability. The honest sender is behavioral. He always recommends the receiver's ex-post optimal policy. In our specification, this is equivalent to always reporting the observed state, θ. The receiver's strategy is her policy choice rule $y : M \to Y$ where $y(m)$ is the action the receiver takes after getting a message m. We denote by $r_b(\cdot)$ the biased sender's strategy. $r_b(\theta)$ is a probability measure over M conditional on observation of the true state θ. An equilibrium concept is a Perfect Bayesian equilibrium.

Definition 1 *The strategy profile (r_b^*, y^*) constitutes an equilibrium if*

(1) given y^*, if m' is sent by the biased sender (m' is in the support of $r_b(\theta)$), then

$$m' \in \arg\max_{m \in M} U^S(y^*(m), \theta, b) = -(y^*(m) - (\theta + b))^2, \text{ and}$$

(2) given r_b^*, for all $m \in M$,

$$\begin{aligned} y^*(m) &\in \arg\max_y E_{\mu, r_b^*}[U^R(y, \theta)|m], \\ &\Leftrightarrow y^*(m) = E_{\mu, r_b^*}[\theta|m], \end{aligned}$$

where E_{μ, r_b^*} is the conditional expectation operator generated by μ, the honest sender's behavior and r_b^*.

Before analyzing the game, we present a useful lemma that addresses the incentive compatibility of the strategic sender. An outcome of the game is represented by its policy choice rule, $z : \Theta \times T \to R_+$ where T is the set of the sender's type and $z(\theta, i)$ is the policy implemented when the true state is θ and the sender has type i.[3] We denote by b the biased sender and define the indirect utility function of the strategic sender with bias b by $V^S(\theta, b) = U^S(z(\theta, b), \theta, b)$.

[3] There is no loss of generality in considering only pure outcomes. Since the receiver moves later and has a quadratic utility function, her optimal response is the conditional expectation of θ, which is unique given any information.

Lemma 1 *For any $b \in R$, $z(\cdot, b)$ is incentive compatible if and only if (i) $z(\cdot, b)$ is nondecreasing, (ii) $V^S(\theta, b)$ is absolutely continuous, and (iii) if $z_1(\theta, b) = \partial z(\theta, b)/\partial\theta$ exists, then $U_1^S(z(\theta, b), \theta, b) \cdot z_1(\theta, b) = \partial U^S(z(\theta, b), \theta, b)/\partial y \cdot \partial z(\theta, b)/\partial\theta = 0$.*

Proof. See Appendix. ■

This lemma is useful in interpreting previous results on the cheap talk game. Part (iii) of the lemma says that for almost all states,[4] the biased sender induces either a constant action around each state or induces his most preferred policy at that state. For example, CS show that any equilibrium can be characterized by a partition of Θ and a single action induced for all states in the same partition element. In terms of Lemma 1, this means that $\partial z(\theta, b)/\partial\theta = 0$ almost everywhere. On the contrary, delegation in the sense of Dessein (2002) is the case where $\partial U^S(z(\theta, b), \theta, b)/\partial y = 0$ everywhere.

3 Equilibrium Characterization

3.1 CS Equilibrium Outcomes

Given that messages have intrinsic value in our model, it is not obvious as to which, if any, CS equilibrium outcomes survive. Perhaps surprisingly, we find that all CS equilibrium outcomes survive so long as $\mu \leq 1/2$.

We first consider a no communication equilibrium in which the receiver makes no meaningful inference from the sender's message. In CS, this equilibrium is called a "babbling" equilibrium because the sender essentially randomizes over the entire message space independently of the true state. Since the honest sender places positive mass only on the true state, a babbling equilibrium does not exist in a perturbed game.

A no communication equilibrium still exists in a different form as long as $\mu \leq 1/2$. Consider the following strategy profile.

$$\begin{aligned} r_b(\theta) &= \begin{cases} 1-\theta, & \text{with probability } \frac{\mu}{1-\mu}, \\ m \sim U[0,1], & \text{with probability } \frac{1-2\mu}{1-\mu}, \end{cases} \\ y(m) &= 1/2, \forall m. \end{aligned}$$

In this profile, at state θ, the biased sender sends message $1-\theta$ with probability $\mu/(1-\mu)$ and randomizes over the entire message space with complementary probability. Having received any message m, the conditional probability that the message came from the honest agent is equal to the unconditional probability of the honest sender (μ). Similarly, the conditional probability that the message came from the biased sender when reporting $1-\theta$ and the conditional probability that it came from the biased sender when uniformly randomizing over $[0,1]$ are μ and $1-2\mu$, respectively.

[4] The derivative $z_1(\theta, b)$ generically exists, because $z(\cdot, b)$ is nondecreasing and bounded.

As such, the conditional expectation of the receiver on the true state is

$$\begin{aligned} E[\theta|m] &= \mu m + (1-\mu)\left(\frac{\mu}{1-\mu}(1-m) + \frac{1-2\mu}{1-\mu}\int_0^1 \theta d\theta\right) \\ &= \mu m + \mu(1-m) + (1-2\mu)\frac{1}{2} = \frac{1}{2} \text{ for any } m. \end{aligned}$$

Therefore, the receiver takes a single action independent of the message. This in turn makes the biased sender indifferent over all messages. While the sender is indifferent over all messages, his reporting strategy must place sufficient weight on low messages given a high state, and vice versa, in order to ensure that the receiver does not draw higher inferences for higher messages.

We note that the receiver updates her belief nontrivially. In the babbling equilibrium in CS, the posterior distribution is equal to the prior distribution. Under the strategy above, the principal's posterior puts mass on the message she received, m, and its counterpart, $1-m$. Therefore, her posterior is different for each message received even though all messages induce the same action. It is only the conditional expectation of the true state that is constant across messages.

This equilibrium construction can be generalized to all equilibria in CS. Given b, consider any equilibrium outcome in CS. We can use the same trick interval by interval: the biased sender reports exactly the "opposite" state[5] in each partition element with probability $\mu/(1-\mu)$ and randomizes over the interval with the remaining probability. This allows us to establish the following proposition.

Proposition 1 *Any equilibrium outcome in CS can be supported as an equilibrium outcome if and only if $\mu \leq 1/2$.*

If $\mu > 1/2$, no equilibrium outcome in CS is supported as an equilibrium outcome. For $\mu > 1/2$, the sender strategy above is not well defined and cannot be used as part of a "no communication" equilibrium or other CS equilibria. The following lemma shows that for $\mu > 1/2$ no other strategy profile induces "no communication." More generally, the following lemma provides a necessary and sufficient condition for the biased sender to be able to induce a single inference (and, consequently, a single optimal action for the receiver) over an interval of messages.

Lemma 2 ***(Mass Balance Condition)*** *Suppose $0 \leq \theta' < \theta'' \leq 1$, $0 \leq m' < m'' \leq 1$ and*

$$\begin{aligned} \overline{y} &= B(\mu, m', m'', \theta', \theta'') \\ &\equiv \frac{\mu(m''-m')}{\mu(m''-m') + (1-\mu)(\theta''-\theta')}\frac{m'+m''}{2} + \frac{(1-\mu)(\theta''-\theta')}{\mu(m''-m') + (1-\mu)(\theta''-\theta')}\frac{\theta'+\theta''}{2} \end{aligned}$$

There exists a collection of probability measures $\{r(\theta), \theta \in [\theta', \theta'']\} \subset \Delta([m', m''])$ such that

$$E_{\mu,r}[\theta|\mathcal{M}] = \overline{y}, \text{ for any Borel set } \mathcal{M} \text{ in } [m', m''],$$

[5] If the partition element is $[\theta', \theta'']$, then the opposite state to $\theta \in [\theta', \theta'']$ is $\theta' + \theta'' - \theta$.

if and only if

$$\mu(m'' - m')^2 \leq (1-\mu)(\theta'' - \theta')^2,$$

where $E_{\mu,r}$ is the conditional expectation operator generated by μ and r.

Proof. See Appendix. ■

Given an interval of messages, the Mass Balance Condition (hereafter, MB) establishes a lower bound on the weight that the biased agent must place on those messages to outweigh the information being conveyed by the honest type. When μ is small, the biased sender can counterbalance the truthful messages by placing small weight on those messages. When μ is large, the biased sender must place more mass on the interval in order to keep the receiver's inference constant.

We will make use of the following reporting strategy of the biased sender throughout the paper:

$$r_b(\theta) = \begin{cases} \frac{m''-m'}{\theta''-\theta'}(\theta'' - \theta) + m', & \text{with probability } \frac{\mu}{1-\mu}\frac{(m''-m')^2}{(\theta''-\theta')^2}, \\ m \sim U[m', m''], & \text{with probability } 1 - \frac{\mu}{1-\mu}\frac{(m''-m')^2}{(\theta''-\theta')^2}. \end{cases}$$

We denote this strategy by "$r_b(\theta) = \tilde{r}([m', m''])$ if $\theta \in [\theta', \theta'']$".

3.2 Type I Equilibrium

The following strategy profile is a natural extension of no communication equilibrium for $\mu > 1/2$: for some $m_0 > 0$,

$$\begin{aligned} r_b(\theta) &= \tilde{r}([m_0, 1]), \forall \theta \\ y(m) &= \begin{cases} m, & \text{if } m < m_0, \\ B(\mu, m_0, 1, 0, 1), & \text{if } m \geq m_0. \end{cases} \end{aligned}$$

In this strategy profile, the biased sender sends messages above m_0. The receiver believes that any message lower than m_0 is sent by the honest type, and so perfectly trusts its content. This strategy profile is specifically designed to overcome the binding MB. MB is satisfied if m_0 is high enough: $\mu(1 - m_0)^2 \leq 1 - \mu$. Of course, we must ensure that the biased sender does not want to deviate to some message below m_0. Therefore,

$$m_0 \leq b \text{ and } |B(\mu, m_0, 1, 0, 1) - b| \leq b - m_0.$$

The first inequality guarantees that the biased sender cannot implement her most preferred policy by deviating to below m_0 at any state. The second inequality ensures that the biased sender prefers $B(\mu, m_0, 1, 0, 1)$ to m_0 at state 0. By the single crossing property of the sender's utility function, the biased sender does not deviate at any other state.

This strategy profile also constitutes an equilibrium when $\mu < 1/2$. Since the mass balance condition is satisfied vacuously, the strategy profile is an equilibrium as long as the incentive compatibility condition is satisfied.

This strategy profile can also be generalized for the case with more than one partition element. Formally, consider a strategy profile that is represented by three strictly increasing sequences in a unit interval, $\{m_0, m_1, ..., m_n = 1\}, \{\theta_0 = 0, \theta_1, ..., \theta_n = 1\}$ and $\{y_1, ..., y_n\}$ such that[6]

$$\begin{aligned} r_b(\theta) &= \tilde{r}([m_{k-1}, m_k]), \text{ if } \theta \in [\theta_{k-1}, \theta_k], \forall k = 1, .., n, \\ y(m) &= \begin{cases} m, & \text{if } m < m_0, \\ y_k, & \text{if } m \in [m_{k-1}, m_k], \forall k = 1, .., n. \end{cases} \end{aligned}$$

The biased sender sends $[m_{k-1}, m_k]$ on $[\theta_{k-1}, \theta_k]$ and does not send any message below m_0. The following conditions are necessary and sufficient for the strategy profile to be an equilibrium.

$$|y_1 - b| \leq b - m_0 \text{ and } m_0 \leq b \text{ if } m_0 > 0, \tag{IC}$$
$$y_k + y_{k+1} = 2(\theta_k + b), \forall k = 1, ..., n-1, \tag{NA}$$
$$y_k = B(\mu, m_{k-1}, m_k, \theta_{k-1}, \theta_k), \forall k = 1, ..., n, \tag{BR}$$
$$\mu(m_k - m_{k-1})^2 \leq (1-\mu)(\theta_k - \theta_{k-1})^2, \forall k = 1, ..., n. \tag{MB}$$

IC is the incentive compatibility condition for the biased sender to not deviate to below m_0. NA and BR are the same conditions as in CS. BR is modified to reflect uncertainty over the sender's type. MB is the mass balance condition in Lemma 2. As $\mu \to 0$, NA and BR converge to equilibrium conditions in CS, and MB becomes negligible.

This strategy profile cannot be an equilibrium for μ close to 1. As μ increases, m_0 should increase so that MB holds. But then IC binds because b is an upper bound of m_0 in this equilibrium.

3.3 Type II Equilibrium

For μ sufficiently large we need an alternative way to consume an excess of mass with the honest sender in order to satisfy both IC and MB. The key to this issue is in Lemma 1. According to Lemma 1, $\Theta = [0, 1]$ can be decomposed into three subsets such that

$$\begin{aligned} \Theta_1 &= \left\{\theta \in \Theta : z(\theta, b) = y^S(\theta, b) = \theta + b\right\}, \\ \Theta_2 &= \left\{\theta \in \Theta : z_1(\theta, b) = \frac{\partial z(\theta, b)}{\partial \theta} = 0\right\}, \\ \Theta_3 &= \left\{\theta \in \Theta : z_1(\theta, b) \text{ does not exist}\right\}. \end{aligned}$$

We (and the cheap talk literature in general) have focused on equilibria in which Θ_2 is full measure; equilibria feature a partitioning of Θ with which a constant action is induced on each partition element. However, for μ sufficiently high, Θ_2 cannot have full measure because of the conflict

[6] We do not explicitly specify what the sender's reporting policies are at the boundary points of each partition element. They do not affect both players' ex ante utilities because the set has zero measure.

between MB and IC. Then, Θ_1 is the only alternative of use, as the non-decreasing property of $z(\cdot, b)$ implies that Θ_3 has zero measure. On Θ_1, the biased sender induces his own optimal policy. This possibility does not arise in CS and many other contexts. However, Θ_1 is *necessary* in our problem for μ sufficiently large.

Consider the following strategy profile which is again represented by three strictly increasing sequences in a unit interval, $\{m_0, ..., m_n\}$, $\{\theta_0, ..., \theta_n\}$, and $\{y_1, ..., y_n\}$ such that

$$r_b(\theta) = \begin{cases} \theta + b/\mu, & \text{if } \theta \in [0, \theta_0], \\ \tilde{r}\left([m_{k-1}, m_k]\right), & \text{if } \theta \in [\theta_{k-1}, \theta_k], \forall k \geq 1, \end{cases}$$

$$y(m) = \begin{cases} m, & \text{if } m \leq b/\mu, \\ \mu m + (1-\mu)\left(m - b/\mu\right), & \text{if } b/\mu < m \leq m_0, \\ y_k, & \text{if } m \in [m_{k-1}, m_k], \end{cases}$$

In this strategy profile, the biased sender induces his own optimal policy on $[0, \theta_0]$. For example, suppose $\theta = 0$. Then the biased sender sends message b/μ. When the receiver gets this message, her inference on θ is $\mu(b/\mu) + (1-\mu)0 = b$, which is optimal to the biased sender. The conditions required for this strategy profile to be an equilibrium are similar to the previous ones.

$$\begin{aligned} & b/\mu \leq \theta_0 + b, && \text{(IC)}, \\ & m_0 = \theta_0 + b/\mu, && \text{(EL)}, \\ & y_1 = \theta_0 + b, && \text{(NA0)}, \\ & y_k + y_{k+1} = 2(\theta_k + b), \forall k \geq 1, && \text{(NA)}, \\ & y_k = B\left(\mu, m_{k-1}, m_k, \theta_{k-1}, \theta_k\right), \forall k \geq 1, && \text{(BR)}, \\ & \mu(m_k - m_{k-1})^2 \leq (1-\mu)(\theta_k - \theta_{k-1})^2, \forall k \geq 1, && \text{(MB)}. \end{aligned}$$

NA, BR, and MB are the same as before. NA0 is the condition required to prevent the biased sender from deviating to $[b/\mu, m_0]$ for $\theta > \theta_0$. IC guarantees that the deviation to $[0, b/\mu)$, where the receiver perfectly trusts messages, is not profitable. EL (equal length) is an obvious requirement from the structure of equilibrium.

The following is an example of a Type II Equilibrium with a particularly simple structure: the sender induces his own optimal policy on $\theta = [0, \theta_0]$ and induces $\theta_0 + b$ for all states above θ_0.

Example 1 *For $\mu \geq 1/2$ and $b \leq \mu / \left(1 + \sqrt{\mu(1-\mu)}\right)$, the following strategy profile is a Type II equilibrium.*

$$r_b(\theta) = \begin{cases} \theta + b/\mu, & \text{if } 0 \leq \theta < \theta_0, \\ [\theta_0 + b/\mu, 1], & \text{if } \theta \in [\theta_0, 1], \end{cases}$$

$$y(m) = \begin{cases} m, & \text{if } m \leq b/\mu, \\ m - \frac{1-\mu}{\mu} b, & \text{if } b/\mu < m < \theta_0 + b/\mu, \\ \theta_0 + b, & \text{if } m \in [\theta_0 + b/\mu, 1], \end{cases}$$

where $\theta_0 = 1 - b - b\sqrt{(1-\mu)/\mu}$. *More specifically, if* $\mu = 1/2$ *and* $b = 1/8$*, the following is a Type II equilibrium.*

$$r_b(\theta) = \begin{cases} \theta + 1/4, & \text{if } 0 \le \theta < 3/4, \\ 1, & \text{if } \theta \in [3/4, 1], \end{cases}$$

$$y(m) = \begin{cases} m, & \text{if } m \le 1/4, \\ m - \frac{1}{8}, & \text{if } 1/4 < m < 1, \\ 7/8, & \text{if } m = 1. \end{cases}$$

3.4 Other Possibilities

The two equilibrium structures we have introduced do not exhaust all equilibria in this game. Unfortunately, we are unaware of a way to characterize the set of all equilibria. Instead, we introduce two natural properties of the strategy profile, one for each player and show that any equilibrium in which the strategy profiles satisfy these properties is either Type I or II.

Definition 2 *(Convexity) The biased sender's reporting strategy is convex if there exists* $m_0 \in [0,1]$ *such that the biased sender never sends messages below* m_0 *and sends all messages above* m_0*.*

That is, if the biased sender's strategy is convex, then there is a critical point in a message space from which all messages greater than that point are contaminated by the biased sender. This is intuitive because the biased sender has a positive bias and thus a smaller incentive to deviate to lower messages. The following is an example of an equilibrium in which convexity is violated.

Example 2 *Suppose* $\mu = 1/8$ *and* $b = 2/5$*. Then the following strategy profile is an equilibrium.*

$$r_b(\theta) = [0, 1/8] \cup [1/4, 1], \forall \theta,$$

$$y(m) = \begin{cases} m, & \text{if } m \in [1/8, 1/4], \\ 509/1008, & \text{otherwise.} \end{cases}$$

Definition 3 *(Monotonicity) The receiver's strategy* y *is monotone if for all* $m' > m$ *such that* $m \in$ *supp* $r_b(\theta)$ *and* $m' \in$ *supp* $r_b(\theta')$ *for some* θ *and* θ'*,* $y(m') \ge y(m)$*.*

In words, for those messages that the biased sender may send, the higher message the receiver gets, the weakly higher action she implements. This restriction makes the biased sender's strategy weakly monotone in the sense that a strictly higher action can be induced only by sending a message higher than any message that would induce a lower action. The following is an example of an equilibrium in which monotonicity is violated.

Example 3 *Suppose* $\mu = 1/8$ *and* $b = 87/448$*. Then the following strategy profile is an equilibrium.*

$$r_b(\theta) = \begin{cases} [1/8, 1/4], & \text{if } \theta \in [0, 1/8] \\ [0, 1/8] \cup [1/4, 1], & \text{otherwise,} \end{cases}$$

$$y(m) = \begin{cases} 5/64, & \text{if } m \in [1/8, 1/4], \\ 251/448,, & \text{otherwise.} \end{cases}$$

Proposition 2 *Any equilibrium in which the biased sender's strategy is convex and the receiver's strategy is monotone is either Type I or II.*

Proof. See Appendix. ■

4 The Advocacy Model

This section studies the cheap talk game in which the receiver is uncertain about the bias of the sender. More precisely, the sender has no bias with probability μ, and has bias $b > 0$ with probability $1 - \mu$. This game is essentially identical to that of Morgan and Stocken (2003). We present only necessary results and intuitions behind them.

The sender's strategy is similar to that of the biased sender in the previous section. We denote by r_b the biased sender's strategy and by r_0 the advocate's strategy. The receiver's strategy is again her policy choice rule $y : M \to Y$.

Definition 4 (r_0^*, r_b^*, y^*) *constitutes an equilibrium if*

(1) given y^**, if* m' *is sent by the biased sender at state* θ *(in the support of* $r_b^*(\theta)$*), then*

$$m' \in \arg\max_{m \in M} U^S(y^*(m), \theta, b) = -(y^*(m) - (\theta + b))^2,$$

(2) given y^**, if* m' *is sent by the advocate at state* θ *(in the support of* $r_0^*(\theta)$*), then*

$$m' \in \arg\max_{m \in M} U^S(y^*(m), \theta, 0) = -(y^*(m) - \theta)^2, \text{ and}$$

(3) given (r_0^*, r_b^*)*, for all* $m \in M$*,*

$$\begin{aligned} y^*(m) &\in \arg\max_y E_{\mu, r_0^*, r_b^*}[U^R(y, \theta) | m] \\ &\Leftrightarrow y^*(m) = E_{\mu, r_0^*, r_b^*}[\theta | m]. \end{aligned}$$

where E_{μ, r_0^*, r_b^*} *is a conditional expectation operator generated by* μ*,* r_0^* *and* r_b^**.*

Different from the honesty case, a babbling equilibrium always exists. It is enough that both types of sender randomize over the entire message space so that the receiver cannot make any meaningful inference. This reasoning also shows that MB has no role in the advocacy model. Given a partition element, it suffices that both advocate and bias sender randomize over the interval.

A new restriction in the advocacy model is a no arbitrage condition for the advocate (NAA): at the boundary state of two partition elements, the advocate must be indifferent between the two induced actions. This implies that if two actions are induced in equilibrium, and no action in between is induced, then both the advocate and the biased sender are indifferent between the two actions at their own boundary states.

Proposition 3 *For $\mu > 0$, only the no communication equilibrium outcome in CS is supported as an equilibrium outcome in the advocacy model.*

Proof. This follows from the observation that at states where the biased sender is indifferent between two distinct policies, the advocate is not indifferent. ■

Together with an appropriate updating rule for the receiver and Lemma 1, we can pin down all equilibria with the advocate for any μ. The structure of equilibrium is similar to the Type I equilibrium with the honest sender. The biased sender's strategy is a partitioning of the state space and the receiver's best response is locally constant for those reports sent by the biased sender. In addition, as in Type I equilibria, it is possible that some low enough equilibrium actions are induced only by the advocate.

Proposition 4 *Any equilibrium in the advocacy model is characterized by three strictly increasing sequences in a unit interval, $\{\theta_0^0, \theta_1^0, ..., \theta_n^0 = 1\}, \{\theta_0^b = 0, \theta_1^b, ..., \theta_n^b = 1\}$ and $\{y_1, ..., y_n\}$ such that*

$$
\begin{aligned}
& y_1 + \theta_0^0 \leq 2b \text{ if } \theta_0^0 > 0, && \text{(ICB)} \\
& \theta_0^0 = y_1 \text{ if } \theta_0^0 > 0, && \text{(ICA)} \\
& y_k + y_{k+1} = 2(\theta_k^b + b), && \text{(NAB)} \\
& y_k + y_{k+1} = 2\theta_k^0, && \text{(NAA)} \\
& y_k = B(\mu, \theta_{k-1}^0, \theta_k^0, \theta_{k-1}^b, \theta_k^b). && \text{(BR)}
\end{aligned}
$$

Proof. Each case where $\theta_0^0 = 0$ and $\theta_0^0 > 0$ correspond to "Categorical ranking system" equilibrium and "Semiresponsive" equilibrium in Morgan and Stocken (2003). ■

There are many strategy profiles that implement the same equilibrium outcome as the messages carry no intrinsic value. However, all equilibrium profiles are essentially equivalent to the following strategy profile.

$$
\begin{aligned}
r_0(\theta) &= \begin{cases} \theta, & \text{if } \theta \le \theta_0^0, \\ m \sim U[\theta_{k-1}^0, \theta_k^0], & \text{if } \theta > \theta_0^0 \text{ and } \theta \in [\theta_{k-1}^0, \theta_k^0], \end{cases} \\
r_b(\theta) &= \quad m \sim U[\theta_{k-1}^0, \theta_k^0], \quad \text{if } \theta \in [\theta_{k-1}^b, \theta_k^b], \\
y(m) &= \begin{cases} m, & \text{if } m \le \theta_0^0, \\ y_k, & \text{if } m \in [\theta_{k-1}^0, \theta_k^0]. \end{cases}
\end{aligned}
$$

For the next section, we present a way to find specific equilibria. First, suppose given μ there exists an equilibrium with n partition elements and $\theta_0^0 = 0$. From all equality conditions, we find

$$
\begin{aligned}
\theta_k^b &= k\theta_1^b + (k-1)\left[2(1-\mu)kb + \mu b - \frac{\mu(1-\mu)b^2}{\theta_1^b + \mu b}\right], k = 2, .., n-1, \qquad (1) \\
\theta_k^0 &= \theta_k^b + b, \forall k = 1, ..., n-1 \\
3\theta_{n-1}^b &- \theta_{n-2}^b + 2b(2-\mu) = 2B(\mu, \theta_{n-1}^0, \theta_n^0, \theta_{n-1}^b, \theta_n^b)
\end{aligned}
$$

Combined with $\theta_n^b = \theta_n^0 = 1$, this system of equations has a unique solution for each n. The additional necessary condition to have an n-partition equilibrium is $\theta_1^b > 0$. Like in CS, this condition provides an upper bound on the possible number of partition elements.

For $\theta_0^0 > 0$, by the similar algebra, we get

$$
\begin{aligned}
\theta_k^b &= \theta_1^b(2k-1) + (k-1)\left[-\theta_0^0 + b\left[4 - \mu + 2(k-2)(1-\mu)\right]\right], k = 2, .., n-1 \qquad (2) \\
\theta_0^0 &= B(\mu, \theta_0^0, \theta_1^0, 0, \theta_1^b), \\
\theta_k^0 &= \theta_k^b + b, \forall k = 1, ..., n-1. \\
3\theta_{n-1}^b &- \theta_{n-2}^b + 2b(2-\mu) = 2B(\mu, \theta_{n-1}^0, \theta_n^0, \theta_{n-1}^b, \theta_n^b)
\end{aligned}
$$

Again using $\theta_n^b = 1$, we can identify all equilibrium values. In this equilibrium, there are two restrictions on the number of partition elements. The first one is the same as above: $\theta_1^b > 0$. This condition also converges to the one in CS and imposes an upper bound on the possible number of partition elements. The second condition is the incentive compatibility for the biased sender that $y_1 = \theta_0^0 \le b$, imposing a lower bound on n. Different from the case with $\theta_0^0 = 0$, the equilibrium with $\theta_0^0 > 0$ exists only when there are enough number of partition elements.

5 Comparison between Honesty and Advocacy

5.1 Equilibrium Structure Comparison

Figure 1 shows equilibrium structures in the advocacy model and in the honesty model. The bottom line is a partition of the biased sender, while the middle line is a partition of either the honest sender

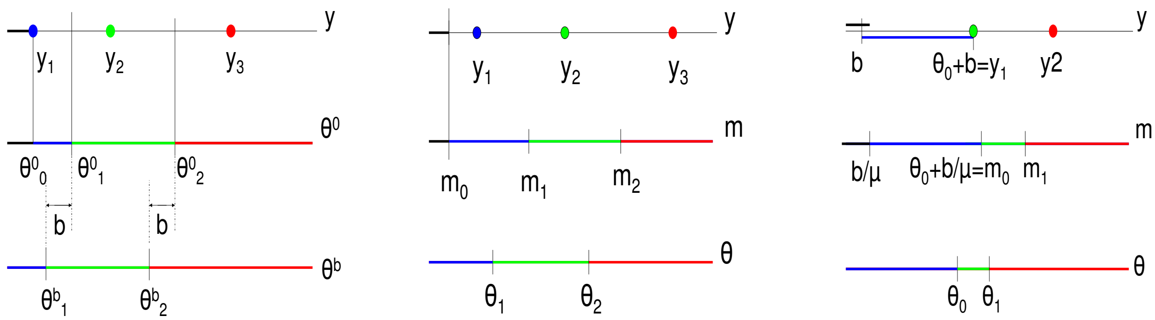

Figure 1: Left: Advocate Middle: Honest Type I Right: Honest Type II

or the advocate. The top line shows the policy choice rule of the receiver. For example, in the Type I equilibrium, the biased sender sends messages in $[m_{k-1}, m_k]$ when the true state lies in $[\theta_{k-1}, \theta_k]$. Upon getting message m, the receiver implements m if $m < m_0$, while chooses y_k if $m \in [m_{k-1}, m_k]$.

The equilibrium structure in the advocacy model and that of Type I in the honesty model are quite similar. Each may feature revelation of the advocate or the honest sender for low messages. For those messages sent by the biased sender, the induced actions are locally constant.

The Type II equilibrium has a noticeably different structure than the others. For high enough states, the Type II equilibrium resembles the others in that there is a partitioning of states and messages, with each partition element corresponding to a single induced action. However, there is an interval of states on which the biased sender induces his own optimal policy.

5.2 Welfare Comparison

The crux of the welfare comparison between honesty and advocacy relates to the familiar question of commitment and flexibility. On the one hand, the honest sender is committed to reporting the true state, which may facilitate communication between the receiver and both types of sender. On the other hand, if the advocate chooses to adjust his report, he does so in the interest of the receiver.

Almost Honest Sender vs. Almost Advocate

Suppose μ is close to 1. First, consider the honesty model. Fix any strategy for the biased sender and suppose the receiver simply follows the recommendation of the sender. That is, suppose the receiver's strategy is $y(m) = m$. Then, the receiver takes her most preferred policy with at least probability μ. Because the receiver can do no worse than this strategy, the ex-ante expected utility of the receiver approaches zero.

Next, consider the advocacy model. Equations (1) and (2) establish that partition element sizes of *both* the advocate and biased sender (other than possibly the first and last) are bounded away from zero for both cases of $\theta_0^0 > 0$ and $\theta_0^0 = 0$.[7] Therefore, the loss associated with the advocate from imperfect communication does not vanish even as the sender is the advocate with near certainty. In other words, the perfect communication equilibrium when $\mu = 1$ in the advocacy model is not lower hemi-continuous. These observations allow us to establish the following proposition.

Proposition 5 *For all $b > 0$, there exists a $\bar{\mu}$ such that for all $\mu > \bar{\mu}$ the receiver is strictly better off with a possibly honest sender than with a sender who may be an advocate.*

Almost Biased Senders

Now we consider the case where μ is small, that is, the sender is biased with a high probability. The following result shows that unless b is one of CS critical values, for μ sufficiently close to 0, the receiver can be better off with a possibly honest sender than with a sender who may be an advocate.

Proposition 6 *If $b \neq \frac{1}{2n(n-1)}$ for some natural number n, there exists $\mu(b) > 0$ such that if $\mu \leq \mu(b)$ then any equilibrium outcome in the advocacy model can be replicated in the honesty model.*

Proof. See Appendix. ■

For μ small, equilibrium is Type I in the honesty model. Then, the honesty model and the advocacy model share equilibrium conditions other than MB for the former and NAA for the latter. As μ tends to 0, MB does not restrict the set of equilibria. It is satisfied by the biased sender assigning only a small probability to each partition element. On the other hand, NAA is independent of μ and, therefore, still binds. Hence, the set of equilibrium outcomes in the honesty model contains the set of equilibrium outcomes in the advocacy model. The following example shows that the receiver can be *strictly* better off with a possibly honest sender than a sender who may be an advocate.

Example 4 *Consider the case where $b = 0.075$,$\mu = 0.005$. In the advocacy model, the two equilibria with the maximum number of partition elements are*

$$\begin{aligned}\theta^0 &= \{0, 0.10997, 0.44303, 1\}, \theta^b = \{0, 0.03497, 0.36803, 1\}, y = \{0.01807, 0.20187, 0.68418\}, \text{ and} \\ \theta^0 &= \{0.01811, 0.11000, 0.44304, 1\}, \theta^b = \{0, 0.035, 0.368, 1\}, y = \{0.01811, 0.20190, 0.68419\},\end{aligned}$$

yielding utilities of -0.024097 and -0.024095, respectively. In the honesty model, there exists an equilibrium yielding utility of -0.0231 characterized by

$$m = \{0.019, 0.809, 0.917, 1\}, \theta^b = \{0, 0.065, 0.384, 1\}, y = \{0.055, 0.225, 0.692\}.$$

At CS critical values (that is, $b = \frac{1}{2n(n-1)}$ for some natural number n), the result is different.

[7] Using Equation (4) from Morgan and Stocken, $1/b$ is an upper bound on the number of partition elements when $\theta_0^0 = 0$.

Proposition 7 *Choose $n > 1$ and fix μ close to zero. If b is greater than but sufficiently close to $\frac{1}{2n(n-1)}$, there exists an equilibrium in the advocacy model whose outcome cannot be replicated in the honesty model.*

Proof. See Appendix. ■

The following example shows that the receiver can be strictly better off in the advocacy model than in the honesty model if μ is small and b is close to one of CS critical values.

Example 5 *Consider the case of $b = 0.09$ and $\mu = 0.01$. In the advocacy model, there exists a 3-partition-element equilibrium with*

$$\theta^0 = \{0.0201, 0.0913, 0.4119, 1\}, \theta^b = \{0, 0.0013, 0.3219, 1\}, y = \{0.0201, 0.1625, 0.6614\}.$$

In the honesty model, there does not exist any type II equilibrium ($b > \mu$). Furthermore, there is no equilibrium with more than two partition elements and the best 2-partition-element equilibrium is

$$\theta^b = \{0, 0.3225, 1\}, m = \{0.017, 0.530, 1\}, y = \{0.1722, 0.6647\}.$$

Honesty yields lower utility than advocacy.

Intermediate Cases

For intermediate values of μ, we cannot establish analytical results on the welfare comparison. We supplement this case by providing some numerical results, and explaining the driving forces of them. The analytical difficulty is that we are unaware of a way to characterize the set of equilibria in the honesty model. The set of equilibria is very large (due to the inequalities of MB), not convex (IC is necessary only when $m_0 > 0$), and possibly not closed (due to $m_0 < m_1 < .. < m_n$). Furthermore, we cannot fix the dimension of control variables, because the maximum number of partition elements is not analytically available.

For many values of μ, the receiver can achieve a higher utility in the honesty model than in the advocacy model. There are two primary reasons, both of which follow from the observation that the inequality property of MB does not require that partition element sizes increase as they must with NAA. In the original CS game ($\mu = 0$), partition element size increases for higher θ. This happens in the advocacy model as well, though the increase in partition element size is decreasing in μ. With the honest sender, however, this is not necessarily true. The partition elements can be adjusted (as long as all MB constraints are satisfied) so that partition elements for higher θ need not be larger than partition elements for lower θ.

The first consequence of this is that such a freedom with MB allows for more uniform partition element sizes. This is directly beneficial to the receiver due to the concavity of a quadratic utility function. This effect is highlighted in Example 6.

Example 6 *Consider the case where $b = 0.15$ and $\mu = 0.1$. In the advocacy model, the maximum number of partition elements is 2 and the two 2-partition-element equilibria are*

$$\theta^0 = \{0, 0.368, 1\}, \theta^b = \{0, 0.218, 1\}, y = \{0.121, 0.615\} \text{ for } \theta^0_0 = 0,$$
$$\theta^0 = \{0.126, 0.371, 1\}, \theta^b = \{0, 0.221, 1\}, y = \{0.126, 0.617\} \text{ for } \theta^0_0 > 0.$$

These yield receiver utility of -0.0397 and -0.0392, respectively. In the honesty model, there exists a 2-partition-element equilibrium characterized by

$$m = \{0.138, 0.575, 1\}, \theta^b = \{0, 0.2478, 1\}, y = \{0.162, 0.634\}.$$

In this equilibrium, the receiver achieves utility -0.0374 which is higher than in the advoacy model.

The second consequence of allowing for more uniform partition element sizes is that as μ increases, the maximum number of partitions increases faster in the honesty model than in the advocacy model. With advocacy, the increasing partition element sizes for both the advocate and biased sender inhibit the ability for a larger number of partition elements. With honesty, smaller mass from the honest sender may be placed on higher messages, raising the higher actions, shrinking the largest partition elements, and allowing for a greater number of intermediate equilibrium actions. This is illustrated in Example 7.

Example 7 *Consider the case where $b = 1/12$ and $\mu = 1/4$. The largest number of partition elements for the advocate is 3. Meanwhile, there exists a 5-partition honest equilibrium with*

$$m = \{0.083, 0.228, 0.539, 0.756, 0.796, 1\},$$
$$\theta^b = \{0, 0.083, 0.263, 0.387, 0.565, 1\},$$
$$y = \{0.083, 0.250, 0.443, 0.498, 0.798\}.$$

Note that the partition sizes for the biased sender are $0.083, 0.180, 0.124, 0.178$ and 0.435, respectively. If we consider any 5-partition advocate equilibrium, then it must be that the third and fourth partition elements increase in size by $4b(1 - \mu) = 1/4$ from the preceding element. Relative to honesty, this is a constraining factor in allowing partition equilibria with a greater number of elements as μ increases.

6 Discussion

6.1 Generalization

Our analyses have been done under a convenient uniform-quadratic environment. We discuss to what extent our results can be generalized.

All of our qualitative results are immediately generalized to the class of utility functions that satisfy the following three properties:

1. $U^R(\cdot,\theta)$ and $U^S(\cdot,\theta,b)$ are symmetric and strictly concave around $y^R(\theta)$ and $y^S(\theta,b)$,
2. $y^S(\theta,b) - y^R(\theta) = b$ for all θ,
3. $U^R(y,\theta) = U^R(y + (\theta' - \theta), \theta'), U^S(y,\theta,b) = U^S(y + (\theta' - \theta), \theta', b), \forall y, \forall\theta,$

where $y^R(\theta) = \arg\max_y U^R(y,\theta)$ and $y^S(\theta,b) = \arg\max_y U^S(y,\theta,b)$. One can check that we have used only these properties in our equilibrium characterization. Under common regularity assumptions, it is possible to generalize most qualitative results to a broader class of utility functions, but only with significant notational investment.

Regarding the distribution over the state space, all analyses other than the mass balance condition are generalized. A closed-form mass balance condition is not available for the general distribution. We provide a partial characterization in Appendix B.

6.2 Unbounded Support

The necessity of a locally constant equilibrium action and the Mass Balance Condition are the consequence of a bounded state space. If $\Theta \equiv [0,\infty)$ or $\Theta \equiv (-\infty,\infty)$ with an improper prior, then for any μ there exists an equilibrium in which the biased sender always induces his most preferred policy (the biased sender sends $\theta + b/\mu$ at any θ).

Appendix A: Omitted Proofs

Proof of Lemma 1. Let $z^+(\theta',b) = \lim_{\theta\to\theta'+} z(\theta,b)$, $z^-(\theta',b) = \lim_{\theta\to\theta'-} z(\theta,b)$.

($\Rightarrow$) (i) $z(\cdot,b)$ is nondecreasing.

Suppose z is strictly decreasing on (θ^1,θ^2) with $\theta^1 < \theta^2$. For $z(\cdot,b)$ to be incentive compatible, $U_S(z(\theta^1,b),\theta^1,b) \geq U_S(z(\theta,b),\theta^1,b)$ and $U_S(z(\theta^2,b),\theta^2,b) \geq U_S(z(\theta,b),\theta^2,b)$ for all $\theta \in \Omega$. Since z is strictly decreasing on (θ^1,θ^2), $z(\cdot,b)$ is continuous except countably many points. Pick some $\theta \in (\theta^1,\theta^2)$ at which $z(\cdot,b)$ is continuous. If $\partial U_S(z(\theta,b),\theta,b)/\partial y \neq 0$, then The biased sender has a profitable deviation (If $\partial U_S(z(\theta,b),\theta,b)/\partial y > (<) 0$ then The biased sender deviates to $\theta' < (>) \theta$). Hence $\partial U_S(z(\theta,b),\theta,b)/\partial y = 0$ almost everywhere on (θ^1,θ^2). But this is a contradiction to a single-crossing property of the utility function.

(ii) $V^S(\theta,b)$ is continuous.

Suppose $V^S(\cdot,b)$ is not continuous at $\theta' \in (0,1)$. Then $z(\cdot,b)$ shouldn't be continuous at θ'. Since $z(\cdot,b)$ is nondecreasing, this means $z(\cdot,b)$ has jump at θ'. Pick θ^+ and θ^- sufficiently close to θ' so that $z^+(\theta',b) \leq z(\theta^+,b)$ and $z(\theta^-,b) \leq z^-(\theta',b)$. We have the following three cases: (1) $z(\theta^+,b) \leq y^S(\theta',b)$, (2) $z(\theta^-,b) < y^S(\theta',b) < z(\theta^+,b)$ and (3) $y^S(\theta',b) \leq z(\theta^-,b)$. In case (1), The biased sender has an incentive to deviate at θ^-, while he does at θ^+ in case (3). In case (3), no type has an incentive to deviate only when $\lim_{\theta\to\theta'+} V^S(\theta,b) = \lim_{\theta\to\theta'-} V^S(\theta,b)$. Hence $V^S(\cdot,b)$ is continuous.

(iii) If $z_1(\theta,b)$ exists, then $U_1^S(z(\theta,b),\theta,b)\cdot z_1(\theta,b) = 0$.

The differentiability of z at θ implies the differentiability of $V^S(\cdot, b)$ at θ, because $V^S(\theta, b) = U^S(z(\theta, b), \theta, b)$. By the Envelope theorem,

$$\frac{\partial U_S(z(\theta,b),\theta,b)}{\partial y}\frac{\partial z(\theta,b)}{\partial \theta} = 0$$

($\Leftarrow$) We want to show that $U^S(z(\theta'', b), \theta'', b) \geq U^S(z(\theta', b), \theta'', b)$ for all $\theta', \theta'' \in \Omega$.

$$\begin{aligned}
U^S(z(\theta'',b),\theta'',b) - U^S(z(\theta',b),\theta'',b) &= [U^S(z(\theta'',b),\theta'',b) - U^S(z(\theta',b),\theta',b)] \\
&\quad -[U^S(z(\theta',b),\theta'',b) - U^S(z(\theta',b),\theta',b)] \\
&= \int_{\theta'}^{\theta''} \frac{\partial V^S(\theta,b)}{\partial \theta} d\theta - \int_{\theta'}^{\theta''} U_2^S(z(\theta',b),\theta,b) d\theta \\
&= \int_{\theta'}^{\theta''} U_2^S(z(\theta,b),\theta,b) d\theta - \int_{\theta'}^{\theta''} U_2^S(z(\theta',b),\theta,b) d\theta \\
&= \int_{\theta'}^{\theta''} \left[\int_{z(\theta',b)}^{z(\theta,b)} U_{12}^S(z(t,b),\theta,b) dz \right] d\theta.
\end{aligned}$$

V^S is absolutely continuous via application of an Envelope theorem for this environment, see Milgrom and Segal (2002).

If $\theta'' > \theta'$, then $z(\theta, b) \geq z(\theta', b), \forall \theta \in [\theta', \theta'']$, and so $U^S(z(\theta'', b), \theta'', b) - U^S(z(\theta', b), \theta'', b) \geq 0$. Similarly, if $\theta'' < \theta'$, then $z(\theta, b) \leq z(\theta', b), \forall \theta \in [\theta', \theta'']$, and again $U^S(z(\theta'', b), \theta'', b) - U^S(z(\theta', b), \theta'', b) \geq 0$.

Q.E.D.

Proof of Lemma 2. ($\Leftarrow$) For each θ, consider the following probability measure.

$$r(\theta) = \begin{cases} \frac{m''-m'}{\theta''-\theta'}(\theta''-\theta) + m', & \text{with probability } \frac{\mu}{1-\mu}\frac{(m''-m')^2}{(\theta''-\theta')^2}, \\ m \sim U[m', m''], & \text{with probability } 1 - \frac{\mu}{1-\mu}\frac{(m''-m')^2}{(\theta''-\theta')^2}. \end{cases}$$

This probability measure is well-defined when $\mu(m'' - m')^2 \leq (1 - \mu)(\theta'' - \theta')^2$. Then for any $m \in [m', m'']$,

$$\begin{aligned}
E_{\mu,r}[\theta|m] &= \frac{\mu(m''-m')}{\mu(m''-m') + (1-\mu)(\theta''-\theta')} m \\
&\quad + \frac{(1-\mu)(\theta''-\theta')}{\mu(m''-m') + (1-\mu)(\theta''-\theta')} \frac{\mu}{1-\mu} \frac{(m''-m')^2}{(\theta''-\theta')^2} \left(\theta'' - \frac{\theta''-\theta'}{m''-m'}(m-m') \right) \\
&\quad + \frac{(1-\mu)(\theta''-\theta')}{\mu(m''-m') + (1-\mu)(\theta''-\theta')} \left(1 - \frac{\mu}{1-\mu} \frac{(m''-m')^2}{(\theta''-\theta')^2} \right) \frac{\theta''+\theta'}{2} \\
&= \overline{y}.
\end{aligned}$$

($\Rightarrow$) Let $m^* \in [m', m'']$ be the value such that $m^* - m' = m'' - m^*$. Let θ^* be the value such that

$$B(\mu, m', m^*, \theta^*, \theta'') = \overline{y}.$$

Such θ^* is well-defined if and only if $\mu(m'' - m')^2 \leq (1 - \mu)(\theta'' - \theta')^2$.

Suppose there exists a collection of probability measures, $\{r(\theta), \theta \in [\theta', \theta'']\}$, that satisfies the given property. Then by construction,

$$\int_{\theta'}^{\theta''} r(\theta)([m', m^*])d\theta \geq \theta'' - \theta^*.$$

Therefore, θ^* must be well-defined, which is true only when $\mu(m'' - m')^2 \leq (1-\mu)(\theta'' - \theta')^2$. **Q.E.D.**

Proof Proposition 2: The proof is straightforward from the following claim (together with Lemma 1), which establishes that there can be no partition element below the region on which the sender induces his most preferred policy.

Lemma 3 *Fix $\theta_0 \in (0, 1 - b/\mu)$ and $m_0 = \theta_0 + b/\mu$. There cannot exist $\theta \in [0, \theta_0)$ and $m \in [0, m_0)$ such that $B(\mu, m, m_0, \theta, \theta_0) = \theta_0 + b$.*

Proof. Suppose not. Then for some θ and m,

$$\theta_0 + b = \frac{\mu(m_0 - m)}{\mu(m_0 - m) + (1-\mu)(\theta_0 - \theta)} \frac{m + m_0}{2} + \frac{(1-\mu)(\theta_0 - \theta)}{\mu(m_0 - m) + (1-\mu)(\theta_0 - \theta)} \frac{\theta + \theta_0}{2}.$$

Rearranging terms,

$$\begin{aligned} & \theta_0^2 - 2(\mu m + (1-\mu)\theta - b)\theta_0 + 2b(b - \mu m - (1-\mu)\theta) - \frac{b^2}{\mu} + \mu m^2 + (1-\mu)\theta^2 \\ = \quad & \left[\theta_0^2 - (\mu m + (1-\mu)\theta - b)\right]^2 + \mu(1-\mu)\left[(m-\theta)^2 - \left(\frac{b}{\mu}\right)^2\right] = 0. \end{aligned}$$

For the solution to exist, the right-hand side should be not more than 0 when $\theta_0 = \mu m + (1-\mu)\theta - b$. But in that case,

$$(m-\theta)^2 - \left(\frac{b}{\mu}\right)^2 = \frac{1}{\mu^2}\left[(\theta_0 - \theta)^2 + 2b(\theta_0 - \theta)\right] > 0.$$

Hence there cannot exist such θ and m. ∎

Q.E.D.

Proof of Proposition 6: Notice that all partition element sizes for the advocate are less than or equal to the partition element sizes for the biased agent except the first partition element, because $\theta_k^0 = \theta_k^b + b, \forall k = 1, .., n-1$. Therefore, given an advocate equilibrium, MB trivially holds for all but the first partition element as long as $\mu \leq 1/2$.

Suppose that as $\mu \to 0$, $\theta_1^b \to \bar{\theta} > 0$. Then the proof is immediate as the advocate equilibrium outcome $\{\theta^b, \theta^0, y\}$ can be replicated as an honest equilibrium for μ sufficiently small.

Now consider the case in which $\theta_1^b \to 0$. First, suppose $\theta_0^0 = 0$. It is straightforward to show that $\theta_1^b \to \frac{1-2n(n-1)b}{n}$. Therefore, $\theta_1^b \to 0$ only when b is a critical CS value. Second, suppose $\theta_0^0 > 0$. Let $\theta_0^0 \to \bar{\theta}_0$ and consider the case where $\theta_1^b \to 0$. Then $\theta_n^b \to 2(nb - \bar{\theta}_0)(n-1) = 1$ so that $\bar{\theta}_0 = \frac{2(n-1)nb-1}{2(n-1)}$. Respecting $\theta_0^0 \leq y_2$ and noting that $\lim_{\mu \to 0} y_2 = \theta_2^b/2$ and $\lim_{\mu \to 0} \theta_2^b = \frac{1-2n(n-1)b}{n}$ it must be that $\frac{2n(n-1)b-1}{2(n-1)} \leq \frac{1-2n(n-1)b}{2n}$. Combined with the fact that $\bar{\theta}_0 \geq 0$, this implies that, $b = \frac{1}{2n(n-1)}$. **Q.E.D.**

Proof of Proposition 7: For small but positive μ, there exists an n-partition advocate equilibrium with θ_1^b small but positive. MB requires that $\mu(\theta_1^b + b - \theta_0^0)^2 \leq (1-\mu)(\theta_1^b)^2$. We show that this condition does not hold for b sufficiently close to $\frac{1}{2n(n-1)}$.

Fix μ close to 0, and suppose b is slightly greater than $\frac{1}{2n(n-1)}$. From Equation (1), we know that θ_1^b satisfies

$$1 = n\theta_1^b + (n-1)\left[2(1-\mu)nb + \mu b - \frac{\mu(1-\mu)b^2}{\theta_1^b + \mu b}\right].$$

As b converges to $\frac{1}{2n(n-1)}$, this expression becomes

$$1 = n\theta_1^b + (1-\mu) + \frac{\mu}{2n} - \frac{\mu(1-\mu)}{2n(n-1)\theta_1^b + \mu}\frac{1}{2n}.$$

Arranging terms,

$$\mu\left[2n(n-1)(2n-1)\theta_1^b + \mu(2n-1) + (1-\mu) - 2n^2\theta_1^b\right] = 4n^3(n-1)\left(\theta_1^b\right)^2.$$

When μ is close to 0, θ_1^b is also close to 0 because b is near a critical value, and

$$\mu \approx 4n^3(n-1)\left(\theta_1^b\right)^2.$$

Therefore,

$$\begin{aligned}\mu(\theta_1^b + b - \theta_0^0)^2 &\approx 4n^3(n-1)\left(\theta_1^b\right)^2\frac{1}{4n^2(n-1)^2} = \frac{n}{n-1}\left(\theta_1^b\right)^2 \\ &> \left(\theta_1^b\right)^2 > (1-\mu)(\theta_1^b)^2.\end{aligned}$$

Q.E.D.

Appendix B: Mass Balance Condition for the General Distribution

Suppose $\theta \in [0,1]$ is drawn from a distribution function F with a positive and continuous density f. Fix $\mu \in (0,1)$ and $[\theta', \theta''], [m', m''] \subseteq [0,1]$. Let

$$\begin{aligned}\overline{y} &= \frac{\mu(F(m'') - F(m'))}{\mu(F(m'') - F(m')) + (1-\mu)(F(\theta'') - F(\theta'))}E\left[\theta|\theta \in [m', m'']\right] \\ &+ \frac{(1-\mu)(F(\theta'') - F(\theta'))}{\mu(F(m'') - F(m')) + (1-\mu)(F(\theta'') - F(\theta'))}E\left[\theta|\theta \in [\theta', \theta'']\right].\end{aligned}$$

We want to know under what conditions there exists a collection of probability measures $\{r(\theta), \theta \in [\theta', \theta'']\} \subset \Delta([m', m''])$ such that

$$E_{\mu,r}[\theta|\mathcal{M}] = \overline{y}, \forall m \in (m', m''), \text{ for any Borel set } \mathcal{M} \text{ in } [m', m''].$$

Suppose such a collection $\{r(\cdot)\}$ exists. Define $\gamma : \Delta([m', m'']) \to [0,1]$ by

$$\gamma(\mathcal{M}) = \int_{\theta'}^{\theta''} r(\theta)(\mathcal{M})\, dF(\theta), \text{ for all Borel set } \mathcal{M} \subset \left[m', m''\right].$$

In addition, define $\gamma_1, \gamma_2 : [m', m''] \to [\theta', \theta'']$ so that

$$\begin{aligned} \overline{y} &= \frac{\mu(F(m) - F(m'))}{\mu(F(m) - F(m')) + (1-\mu)(F(\theta'') - F(\gamma_1(m)))} E\left[\theta | \theta \in \left[m', m\right]\right] \\ &\quad + \frac{(1-\mu)(F(\theta'') - F(\gamma_1(m)))}{\mu(F(m) - F(m')) + (1-\mu)(F(\theta'') - F(\gamma_1(m)))} E\left[\theta | \theta \in \left[\gamma_1(m), \theta''\right]\right], \end{aligned}$$

and

$$\begin{aligned} \overline{y} &= \frac{\mu(F(m'') - F(m))}{\mu(F(m'') - F(m)) + (1-\mu)(F(\gamma_2(m)) - F(\theta'))} E\left[\theta | \theta \in \left[m, m''\right]\right] \\ &\quad + \frac{(1-\mu)(F(\gamma_2(m)) - F(\theta'))}{\mu(F(m'') - F(m)) + (1-\mu)(F(\gamma_2(m)) - F(\theta'))} E\left[\theta | \theta \in \left[\theta', \gamma_2(m)\right]\right]. \end{aligned}$$

Both γ_1 and γ_2 must be well-defined if a collection $\{r(\cdot)\}$ exists. In addition, $\gamma([m', m]) \geq \gamma_1(m)$ and $\gamma([m, m'']) \geq \gamma_2(m)$ for all $m \in [m', m'']$. Since $\gamma([m', m]) + \gamma([m, m'']) = F(\theta'') - F(\theta')$, this condition implies that

$$\gamma_1(m) + \gamma_2(m) \leq F\left(\theta''\right) - F\left(\theta'\right), \forall m \in \left[m', m''\right].$$

The special interest is when this condition holds with equality for all $m \in [m', m'']$, that is, $\gamma_1(m) + \gamma_2(m) = F(\theta'') - F(\theta'), \forall m \in [m', m'']$. In this case, $r(\theta)$ is a degenerate random variable and r^{-1} coincides with γ_1. Furthermore, γ_1 satisfies a first-order ordinary differential equation

$$(1-\mu) f(\gamma_1(m))(\gamma_1(m) - \overline{y})\gamma_1'(m) = \mu f(m)(m - \overline{y}), \forall m \in \left[m', m''\right],$$

with boundary conditions $\gamma_1(m') = \theta''$ and $\gamma_1(m'') = \theta'$. For the uniform distribution, this happens when $\mu(m'' - m')^2 = (1-\mu)(\theta'' - \theta')^2$, and

$$\gamma_1(m) = \theta'' - \frac{\theta'' - \theta'}{m'' - m'}\left(m - m'\right),$$

which was used in Section 3.

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