Endogenous Market Segmentation for Lemons*

(Job Market Paper)

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Abstract

I explore the possibility of endogenous segmentation in a decentralized market where asymmetric information about the quality of goods may result in only low-quality goods trading (the lemons problem). Endogenous segmentation alleviates information asymmetry, potentially improving market efficiency, without employing costly signalling or screening devices. The incentives for sellers to sort themselves are endogenously generated by buyers' behavior, and vice versa. I show that endogenous segmentation is supported by a monotone market arrangement: if a market is segmented into multiple submarkets, higher-quality submarkets entail more quality uncertainty and attract relatively fewer buyers. The result has implications for applied work, including the structure of multiple marketplaces or platforms, the informativeness of costless advertisement for experience goods, and the role of non-binding list prices in decentralized markets.

JEL Classification Numbers: C72, D82, D83.

Keywords : Lemons Problem; Market Segmentation.

1 Introduction

This paper studies the resolution of information asymmetry in a market where sellers possess private information about the quality of their goods. It is now well-understood that various market and non-market innovations can alleviate the inefficiencies implied by adverse selection in general, and the lemons problem in particular. These innovations fall into two categories: signalling from informed agents (for example, warranties for durable goods, licensing for credence goods, and schooling in the labor market) and screening by uninformed agents (for example, discounting for early purchase in the airline industry and deductibles in the insurance industry). A common feature of these two

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approaches is that they rely on *payoff-relevant* devices. For example, warranties and deductibles are effective because they directly affect consumers' values, while schooling can signal the ability of workers because it is more costly for less able workers. These devices sort different types by directly affecting the incentives of informed agents.

In this paper, I explore the possibility of endogenous market segmentation. Endogenous segmentation alleviates information asymmetry, as other sorting devices do, potentially improving market efficiency. However, it does not rely on costly signalling or screening, but instead relies on equilibrium incentives. The incentives for sellers to sort themselves are endogenously generated by buyers' behavior, and vice versa.

I consider a model in which there are potentially multiple submarkets, agents *costlessly* choose a submarket to participate in, and exchange takes place in each submarket. In a submarket, each buyer randomly selects a seller and makes a take-it-leave-it offer to the selected seller. Each seller decides whether to accept or reject the highest offer she received, if any. Though buyers have all bargaining power, they take into account the fact that a seller may be matched with multiple buyers.

The two key components in the model are uncertainty about the quality of goods and agents' concern for the probability of trading. First, quality uncertainty results in a lemons problem, as in Akerlof (1970). Each seller accepts an offer only when it is greater than or equal to her cost (reservation utility). Buyers allow for the fact that their value of getting a good is determined by the average quality accepted, not by the highest quality accepted. This may induce buyers to offer lower than the cost of the highest quality in the market, resulting in some high-quality goods not trading. Second, the search frictions imply that agents have concern for trading probability. A seller may not be selected by any buyer. Several buyers may be matched with the same seller but only one can trade.

I show that endogenous market segmentation is supported by a monotone market arrangement: if a market is segmented into multiple submarkets, higher-quality submarkets entail more quality uncertainty and attract relatively fewer buyers. To separate sellers, submarkets must provide different combinations of trading probability and transaction price, where higher trading probability is associated with lower transaction price. Lower-quality sellers, due to their lower costs, self-select into submarkets with higher trading probabilities and lower transaction prices. Since sellers' trading probability is increasing in the ratio of buyers to sellers,[1] this implies that relatively more buyers must participate in lower-quality submarkets. This happens only when lower-quality submarkets contain less quality uncertainty.

Figure 1 illustrates the monotone market arrangement in a two-quality example where $\widehat{q}$ is the proportion of lemon sellers in a market. Measure q^* of lemon sellers and measure α^* of buyers join submarket L, while all other agents go to submarket H. There is no quality uncertainty in submarket L, while a positive amount of quality uncertainty remains in submarket H. In addition, relatively more buyers participate in submarket L than in submarket H. In this structure, agents

[1] When the ratio of buyers to sellers is higher, a seller is more likely to be selected by buyers and buyers bid more aggressively.

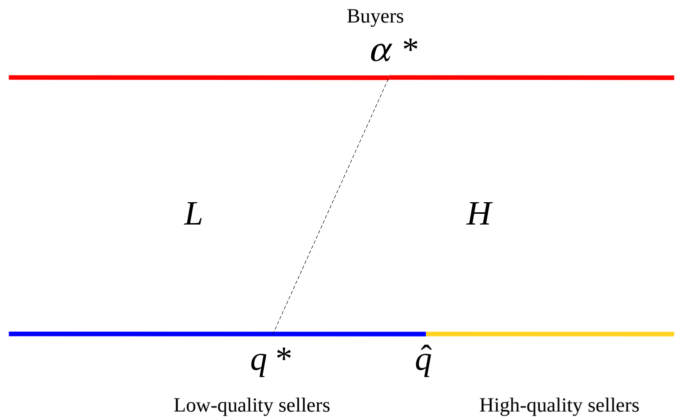

Figure 1: Equilibrium in which a market is segmented into two distinct submarkets.

face the following trade-offs. Low-quality sellers enjoy higher trading probability but suffer from lower transaction price in submarket L than in submarket H. Buyers face less quality uncertainty but more severe competition in submarket L than in submarket H. In equilibrium, these trade-offs of buyers and low-quality sellers are exactly balanced, i.e., they are indifferent between H and L. The reduction of information asymmetry through endogenous market segmentation manifests in the fact that high-quality goods, that do not trade without segmentation, trade with positive probability.

There are two requirements for endogenous market segmentation. First, the ratio of buyers to sellers must be small. If the ratio is large, trading probability is relatively less important than transaction price. Then sellers have a weak incentive to join lower-quality submarkets that provide higher trading probabilities but lower transaction prices. Second, social surpluses from trading higher-quality goods cannot be too much greater than those from lower-quality goods. If they are, buyers are much more willing to trade with higher-quality sellers. Then sellers have a strong incentive to mimic higher-quality sellers, and consequently, there cannot exist multiple submarkets.

The ratio of buyers to sellers in the grand market determines how finely a market can be segmented (how many submarkets are sustainable). The smaller the ratio is, the more important trading probability is relative to transaction price. Therefore, when the ratio is smaller, sellers have stronger incentive to join lower-quality submarkets with higher trading probabilities. This allows more submarkets to be sustainable. In the limit as the ratio of buyers to sellers tends to zero, buyers' uncertainty about the quality of goods can be eliminated through endogenous market segmentation.

The results naturally apply to markets with multiple marketplaces or platforms. Another important interpretation of submarkets is cheap-talk messages. Consider the following market structure. Sellers costlessly advertise their goods. Buyers observe those advertisements before selecting a trading partner. This market structure is essentially equivalent to the one in my model. Cheap-talk messages serve as an instrument that creates submarkets. Under the cheap-talk interpretation, the results provide a new insight for the advertisement literature in industrial organization. In his classic papers (1970, 1974), Nelson argued that quality of experience goods cannot be revealed through costless advertisement.[2] As opposed to that, my results suggest that even costless advertisement can be informative about the quality of experience goods.

The model can be interpreted as a theory for the role of non-binding list prices in decentralized markets. In some markets (for example, housing, used cars, and online posting sites), sellers post prices (list prices) but those prices are often non-binding. Nevertheless, correlations between list prices and allocations have been observed: sale prices are typically lower than list prices, and the higher the list price is, the sale price is higher, the number of interested buyers is smaller, and the duration on the market is longer.[3] Relative to previous work, my model emphasizes the information transmission role of list prices and provides an intuitive reason for the aforementioned stylized facts. The higher the list price is, the more uncertain the quality of the good is, and hence there are fewer interested buyers. This prolongs the duration of the good on the market, but once there is an interested buyer, he bids high and thus the sale price is higher.

The remainder of the paper is organized as follows. The next section reviews related literature. Section 3 studies the two-quality case in detail. Sections 4 and 5 consider the case where there is a continuum of qualities. Section 4 focuses on the environment where quality uncertainty plays a crucial role. Section 5 supplements Section 4 by providing results on the general continuum quality case and by studying another extreme case where buyers' values are independent of sellers' costs (the constant value case). I conclude by discussing several relevant issues in Section 6.

2 Related Literature

Bargaining with Interdependent Values

The key idea of this paper is closely related to the results in bargaining with interdependent values. Evans (1989) and Vincent (1989) are early references, while Deneckere and Liang (2006) provided the general characterization. They showed that in the infinite-horizon bargaining game where the uninformed player (buyer) makes all the offers, efficiency (delay) depends on the relationship between the expected valuation of the buyer and the highest cost of the seller. If the former is greater than the latter, then as bargaining friction becomes negligible delay disappears, as in the Coase conjecture. Otherwise, there is a real-time delay and partial separation between different

[2] He reasoned that sellers' advertisements are unverifiable and thus cannot be punished, resulting in all sellers claiming to have high-quality goods. His reasoning has motivated many researchers to find several payoff-relevant mechanisms. His own suggestions were repeated purchase (1970) and costly advertisement (1974). Kihlstrom and Riordan (1984) refined Nelson's idea on costly advertisement. Another prominent device considered in the literature is price (or pricing schedule). Wolinsky (1983), Bagwell and Riordan (1991), and Taylor (1999) showed that prices can serve as signals of quality in various contexts. Milgrom and Roberts (1986) considered both price and costly advertisement.

[3] See, for example, Horowitz (1992) and Merlo and Ortalo-Magné (2004) for the housing market, and Farmer and Stango (2004) for the online used computer market.

types occurs over time. My paper shows that, in a static market setting, partial separation can occur across submarkets.

Hörner and Vielle (2006) studied the setting where a seller with a unit of an indivisible good faces a sequence of buyers. They demonstrated that bargaining typically ends up in an impasse if past offers are publicly observable, while agreement is eventually reached if previous offers are not observable to subsequent buyers. The intuitive reason is that the seller has too strong a signaling incentive in the former case. One of my results - that endogenous market segmentation is impossible if social surpluses from trading higher-quality goods are sufficiently larger than those from lower-quality goods - is related to their point.

Decentralized Market

Satterthwaite and Shneyerov (2007) employed the same exchange process as my model -random matching (in a submarket) and each seller running a first-price auction with unknown reservation price and unknown number of bidders- in an environment where agents have private values. They demonstrated that equilibrium outcomes in the decentralized market converge to Walrasian outcomes as the period length approaches zero. In a different context, my paper shows that the exchange process captures competitive forces in a market successfully. In some sense, this exchange process provides more intuitive results than the competitive market process because it does not exhibit a discontinuity of equilibrium outcomes with respect to the ratio of buyers to sellers.

Methodologically, this paper belongs to the growing directed search literature. Essentially, my model is a directed search model with interdependent values and without commitment to price. The combination of the two features generates a unique effect that is absent in the previous models. With price commitment or private values,[4] the search behavior of uninformed agents is determined solely by the trade-off between trading probability and deterministic gain in match.[5] My model introduces risk to the considerations of uninformed agents. This unique feature is highlighted by comparing two extreme cases in my model, the constant surplus case (Section 4) and the constant value case (Section 5.5).

Endogenous Market Segmentation

In my model, the incentives for both sides in the market are endogenously generated. The incentives for sellers to sort themselves are created by buyers' behavior, and vice versa. A similar idea has been employed in other contexts. Mailath, Samuelson, and Shaked (2000) considered the labor market where both workers and firms search for each other. They showed that "color" can create inequality endogenously. Firms search "green" workers because they are more likely to acquire skill than "red" workers. On the other hand, "green" workers are more willing to acquire skill than "red" workers because it takes less time for them to be matched with firms, and therefore, their return on skill investment is greater. Fang (2001) considered an economy where the informational

[4] The classic directed search models (Peters (1991), Moen (1997), and Burdett, Shi, and Wright (2001)) considered the model with price posting and private values. Riordan (1985) and Inderst and Müller (2002) studied the case of price posting and interdependent values, while Menzio (2007) studied the case of communication and private values.

[5] With interdependent values, if sellers commit to prices, qualities are fully revealed through prices, resulting in deterministic gains of uninformed agents (See Inderst and Müller (2002)).

free-riding problem is so severe that a socially efficient technology cannot be adopted. He showed that in such a situation costly "social activity" can emerge as an endogenous signaling instrument. Firms pay more to workers who perform a seemingly irrelevant "social activity" because those workers are more likely to acquire new skill. On the other hand, skilled workers are more willing to do the "social activity" because they expect higher wage from firms.

Cheap talk in bargaining

Farrell and Gibbons (1989) and Matthews and Postlewaite (1989) studied whether cheap talk can be informative in a bilateral bargaining situation in which each party has private information about their value. They showed that allowing for cheap-talk communication enlarges the set of equilibria in a double auction. If the seller possesses private information about the quality of the good, cheap talk cannot be informative in a bilateral setting. In the standard problem, it is impossible to provide an incentive for the low-quality seller to reveal her quality.

The role of list prices

Despite the fact that many empirical efforts have been made to identify the determinants of list prices, there has been only one theoretical explanation for the correlations between list prices and allocations, which was provided by Arnold (1999), Chen and Rosenthal (1996), and Yavas and Yang (1995). They focus on the fact that sale prices are typically lower than list prices and postulate that list prices are ceiling prices that sellers commit to accept. The crucial idea is that if a seller commits to a lower list price, buyers expect greater gains in the event their valuations turn out to be high, and consequently, they are more interested in the property.

3 The Two-Quality Case

3.1 Environment

In a market for an indivisible good, there are a continuum of sellers with measure 1 and a continuum of buyers with measure $\beta > 0$. All buyers are homogenous, while there are two types of sellers. Measure $\widehat{q} \in (0,1)$ of sellers possess a unit of low-quality good (lemon), while measure $1-\widehat{q}$ of sellers own a unit of high-quality good. A unit of low-quality good costs c_L (or reservation utility) to a seller and yields utility v_L to a buyer. The corresponding values for a unit of high-quality good are c_H ($> c_L$) and v_H ($\geq v_L$). Quality of a good is private information to each seller. Trading is socially desirable ($v_H > c_H$ and $v_L > c_L$). I assume that social surplus from trading is independent of quality, that is, $v_H - c_H = v_L - c_L$. This assumption enables me to focus on quality uncertainty, which is the central issue in the lemons problem. I later explain how equilibrium outcomes change when I relax this assumption.

The market proceeds in two steps. First, each agent chooses between submarket H and submarket L. The two submarket restriction has no loss of generality because in equilibrium there exist at most two distinct submarkets. Second, exchange takes place in each submarket. The exchange process is as follows.

1. Each buyer randomly selects a seller in the submarket.

   - I use the "urn-ball" matching technology. The probability π_k that a seller is matched with k buyers follows a Poisson distribution. Formally,

   $$\pi_k(\lambda) = \frac{\lambda^k}{k!e^{\lambda}}, k = 0, 1, ...,$$

   where λ is the ratio of buyers to sellers. To see how this is derived, suppose there are λN buyers and N sellers and each buyer selects a seller with equal probability. As N tends to infinity, by the Poisson convergence theorem (see, for example, Billingsley (1995), Theorem 23.2.), the probability that a seller is matched with k buyers converges to $\pi_k(\lambda)$.

2. Each buyer makes a take-it-or-leave-it offer to the matched seller, without observing how many competitors he is facing.

   - Under the urn-ball matching technology, the probability that a buyer is competing with k other buyers is equal to $\pi_k(\lambda)$.[6]
   - I assume that buyers use the same bidding strategy. This is natural because buyers are anonymous as well as homogeneous.

3. Sellers who received at least one offer decide whether to accept the highest offer or not. If a seller accepts an offer b, her utility is $b - c$ where c is her cost. If a seller was not matched with any buyer or the highest offer is lower than her cost, her utility is 0. If a buyer's offer b is accepted by a seller, then the buyer's utility is $v - b$ where v is the buyer valuation of the good the seller possesses.

   - Equivalently, each seller runs a first-price auction with unknown reservation price and unknown number of bidders.

3.2 Submarket Analysis

I first solve for submarket outcomes. Each submarket is characterized by (q, λ) where q is the proportion of lemon sellers and λ is the ratio of buyers to sellers in the submarket. The absolute measures of buyers and sellers do not affect equilibrium outcomes because my matching technology exhibits constant return to scale.

[6] This directly comes from the conditional independence property of Poisson distribution. For an elementary exposition, see Satterthwaite and Shneyerov (2007), p. 173.

3.2.1 Buyers' Expected Payoff

I make three observations. First, each seller accepts the highest offer if and only if it is greater than or equal to her cost. Second, in the symmetric equilibrium, buyers' bids are not deterministic. If they are, a buyer is strictly better off by bidding slightly higher than the equilibrium bid. His expected payment increases slightly but he always wins. From now on, I represent buyers' symmetric bidding strategy by a probability distribution function F over R_+. By a similar argument, F has no atom. Third, letting $\underline{b}$ be the minimum of the support of F, $\underline{b}$ is equal to the offer of the monopsonist who is facing a seller who has a low-quality good with probability q. A buyer who bids $\underline{b}$ wins only when he is the only bidder and thus behaves like a monopsonist.

Let $M(q)$ be the expected payoff of the monopsonist and $U(q, \lambda)$ be buyers' expected payoff in a submarket. Using the fact that in equilibrium buyers are indifferent over all bids in the support of F,

$$U(q, \lambda) = \pi_0 M(q).$$

That is, buyers' expected payoff is equal to the probability that they are the only bidder times the expected payoff of the monopsonist. U is strictly decreasing in λ. This is natural because λ measures the level of competition among buyers in a submarket. In addition, U inherits all the properties of M because the effects of λ and q on U are separable.

Since the monopsonistic offer is either c_L or c_H,

$$M(q) = \max \{q (v_L - c_L), E_q[v] - c_H\},$$

where $E_q[v] = qv_L + (1-q) v_H$. M is decreasing first and increasing later in q. For q small the monopsonist offers c_H, enduring the risk of paying a high price for a low-quality good. Since higher q implies higher probability of getting a low-quality good, M is decreasing in q. For q large the monopsonist makes a safe offer, c_L. In this case, higher q implies higher probability of trading. Therefore, M is increasing in q.

3.2.2 Sellers' Expected Payoffs

I derive F to calculate sellers' expected payoffs. F is found from buyers' indifference over bids in the support of F. The expected payoff of a buyer by bidding b is

$$q \sum_{k=0}^{\infty} \pi_k F(b)^k (v_L - b) = q\pi_0 e^{\lambda F(b)} (v_L - b) \text{ if } b \in [c_L, c_H),$$

and

$$\sum_{k=0}^{\infty} \pi_k F(b)^k (E_q[v] - b) = \pi_0 e^{\lambda F(b)} (E_q[v] - b) \text{ if } b \geq c_H.$$

If c_L and $b \in [c_L, c_H)$ are in the support of F, then

$$F(b) = F(c_L) + \frac{1}{\lambda} \ln\left(\frac{v_L - c_L}{v_L - b}\right).$$

Similarly, if c_H and $b\,(\geq c_H)$ are in the support of F, then

$$F(b) = F(c_H) + \frac{1}{\lambda} \ln\left(\frac{E_q[v] - c_H}{E_q[v] - b}\right).$$

Let $\overline{b}$ be the maximum of the support of F. There are three cases.

(1) Only lemons trade: $\underline{b} = c_L$ and $\overline{b} < c_H$.

From the previous result,

$$F(b) = \frac{1}{\lambda} \ln\left(\frac{v_L - c_L}{v_L - b}\right), \text{ for } b \in \left[\underline{b}, \overline{b}\right].$$

This happens when

$$E_q[v] - c_H \leq U(q, \lambda) = \pi_0 q (v_L - c_L).$$

This inequality can be interpreted as the incentive compatibility of buyers because the left-hand side is the expected payoff of buyers by deviating to c_H.

(2) High-quality goods partially trade: $\underline{b} = c_L$ and $\overline{b} > c_H$.

Let $\left[\underline{b}, \overline{b}_L\right] \cup \left[c_H, \overline{b}\right]$ be the support of F where $\overline{b}_L < c_H$. Then

$$F(b) = \begin{cases} \frac{1}{\lambda} \ln\left(\frac{v_L - c_L}{v_L - b}\right), & \text{if } b \in \left[c_L, \overline{b}_L\right], \\ \frac{1}{\lambda} \ln\left(\frac{\widehat{q}(v_L - c_L)}{E[v] - b}\right), & \text{if } b \in \left[c_H, \overline{b}\right]. \end{cases}$$

This case arises when

$$q\pi_0 (v_L - c_L) < E_q[v] - c_H < q(v_L - c_L).$$

(3) Both qualities fully trade: $\underline{b} = c_H$.

In this case, trade occurs whenever a seller is matched with at least one buyer. From the preliminary result,

$$F(b) = \frac{1}{\lambda} \ln\left(\frac{E_q[v] - c_H}{E_q[v] - b}\right), b \in \left[\underline{b}, \overline{b}\right].$$

This happens when the optimal offer of the monopsonist is c_H. Therefore,

$$q(v_L - c_L) \leq E_q[v] - c_H.$$

Let $V_L(q, \lambda)$ and $V_H(q, \lambda)$ be the expected payoffs of lemon sellers and high-quality sellers,

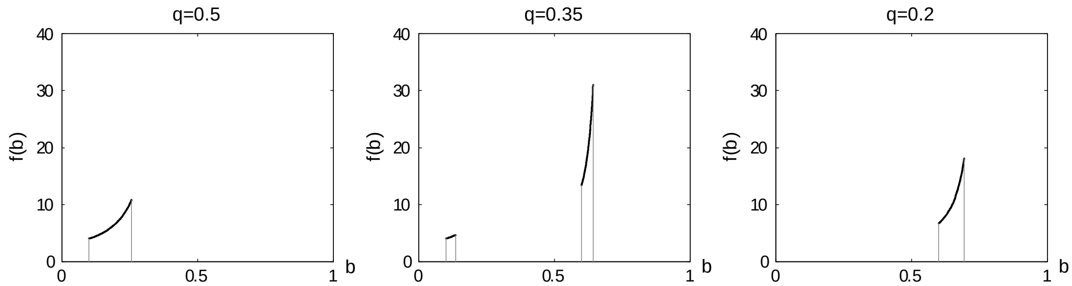

Figure 2: Buyers' bidding strategies represented by density functions over a real line for $c_L = 0.1, v_L = 0.35, c_H = 0.6, v_H = 0.85$ and $\lambda = 1$.

respectively. Then,

$$\begin{aligned} V_L(q,\lambda) &= \sum_{k=1}^{\infty} \pi_k \int_{c_L}^{\bar{b}} (b - c_L)\, dF^k(b)\,, \text{ and} \\ V_H(q,\lambda) &= \sum_{k=1}^{\infty} \pi_k \int_{c_H}^{\max\{\bar{b}, c_H\}} (b - c_H)\, dF^k(b)\,. \end{aligned}$$

Applying the previous results on F,

(1) if $E_q[v] - c_H \leq \pi_0 q (v_L - c_L)$ then

$$\begin{aligned} V_H(q,\lambda) &= 0, \text{ and} \\ V_L(q,\lambda) &= (1 - \pi_0 - \lambda\pi_0)(v_L - c_L)\,, \end{aligned}$$

(2) if $\pi_0 q (v_L - c_L) < E_q[v] - c_H < q(v_L - c_L)$ then

$$\begin{aligned} V_H(q,\lambda) &= (E_q[v] - c_H) - q\pi_0(v_L - c_L) - \pi_0(E_q[v] - c_H)\ln\frac{E_q[v] - c_H}{q\pi_0(v_L - c_L)}, \text{ and} \\ V_L(q,\lambda) &= (v_L - c_H)\frac{q\pi_0(v_L - c_L)}{E_q[v] - c_H} - \pi_0(v_L - c_L) - \pi_0(v_L - c_L)\ln\left(\frac{q(v_L - c_L)}{E_q[v] - c_H}\right) \\ &\quad + (E_q[v] - c_L) - q\pi_0(v_L - c_L) - \pi_0(E_q[v] - c_H)\ln\frac{E_q[v] - c_H}{q\pi_0(v_L - c_L)}, \end{aligned}$$

(3) if $q(v_L - c_L) \leq E_q[v] - c_H$ then

$$\begin{aligned} V_H(q,\lambda) &= (1 - \pi_0 - \lambda\pi_0)(E_q[v] - c_H)\,, \text{and} \\ V_L(q,\lambda) &= (1 - \pi_0 - \lambda\pi_0)(E_q[v] - c_H) + (1 - \pi_0)(c_H - c_L)\,. \end{aligned}$$

Unlike buyers' expected payoff, the effects of q and λ on sellers' expected payoffs are intertwined. To see this, consider the case where $0 < E_q[v] - c_H < q(v_L - c_L)$. If λ is sufficiently high, high-quality goods partially trade (Case (2)), while if λ is close to 0, only lemons trade (Case (1)).

Both V_H and V_L are increasing in λ and decreasing in q. More buyer competition (higher λ) increases sellers' matching probability and drives up winning bids. Higher q implies lower average quality of goods, which leads to buyers bidding lower. Conditional on only lemons trading, V_H and V_L are constant in q because buyers' bidding strategy is independent of q.

3.3 Equilibrium Characterization

Subsequently, I assume that only lemons trade without market segmentation. Formally, $E_{\widehat{q}}[v]-c_H < \widehat{q}\pi_0(v_L - c_L)$. This is the environment where the role of market segmentation can be highlighted. In addition, I focus on the case in which all high-quality sellers participate in submarket H. This is without loss of generality. First, switching the roles of H and L makes no difference. Second, if both types of sellers participate in both submarkets, then it is one of trivial equilibria. Both submarkets are essentially identical to the grand market. Third, in equilibrium it never happens that all lemon sellers go to the same submarket, while high-quality sellers join both submarkets.

I use the following notations.

$$\widetilde{E}_q[v] \equiv \frac{\widehat{q}-q}{1-q}v_L + \frac{1-\widehat{q}}{1-q}v_H \text{ for } q \leq \widehat{q},$$

and

$$\lambda_L(\alpha, q) \equiv \frac{\alpha}{q},\ \lambda_H(\alpha, q) \equiv \frac{\beta-\alpha}{1-q}.$$

$\widetilde{E}_q[v]$ is the expected buyer valuation of goods in submarket H when measure q of lemon sellers join submarket L (measure $\widehat{q}-q$ of lemon sellers go to submarket H). $\lambda_L(\alpha, q)$ and $\lambda_H(\alpha, q)$ are the ratios of buyers to sellers in submarket L and in submarket H, respectively, when measure q of lemon sellers and measure α of buyers join submarket L and all other agents go to submarket H.

3.3.1 Buyers' Indifference Function

Suppose measure $q \in (0, \widehat{q}]$ of lemon sellers participate in submarket L. For each q, let $B(q)$ be the measure of buyers such that buyers are indifferent between the two submarkets if measure $B(q)$ of buyers join submarket L. Formally, let $B(q)$ be the value such that

$$U(1, \lambda_L(B(q), q)) = U\left(\frac{\widehat{q}-q}{1-q}, \lambda_H(B(q), q)\right).$$

Applying the submarket outcomes,

$$\frac{1}{e^{\lambda_L(B(q),q)}}(v_L - c_L) = \frac{1}{e^{\lambda_H(B(q),q)}} \max\left\{\widetilde{E}_q[v] - c_H, \frac{\widehat{q}-q}{1-q}(v_L - c_L)\right\}.$$

Arranging terms,

$$B(q) = \beta q + q(1-q)\ln\left(\frac{v_L - c_L}{\max\left\{\widetilde{E}_q[v] - c_H, \frac{\widehat{q}-q}{1-q}(v_L - c_L)\right\}}\right).$$

Let $\overline{q} \in (0, \widehat{q})$ be the value such that

$$\frac{\widehat{q} - \overline{q}}{1 - \overline{q}}(v_L - c_L) = \widetilde{E}_q[v] - c_H.$$

First, consider the case where $q \in [\overline{q}, \widehat{q}]$. In this case, the monopsonistic offer is c_H in submarket H, and hence, both goods fully trade. On this interval, B is not necessarily increasing in q. To see this, fix $q \in [\overline{q}, \widehat{q}]$ and $B(q)$. As q increases, λ_L decreases (relatively less buyer competition), which makes submarket L more attractive to buyers. On the other hand, in submarket H, buyer competition becomes more severe (higher λ_H) but the average quality improves (higher $\widetilde{E}_q[v]$). When the quality improvement effect outweighs that of competition, B is decreasing. For instance, since

$$B'(\widehat{q}) = \beta - \frac{\widehat{q}(v_H - v_L)}{(v_H - c_H)},$$

if β is small, B is decreasing at around $\widehat{q}$.

Now consider the case where $q < \overline{q}$. In this case, high-quality goods do not fully trade in submarket H. B is strictly increasing in q. This is because, unlike in the previous case, buyers do not benefit from the improvement of average quality in submarket H. Formally,

$$\begin{aligned} B'(q) &= \beta + (1-q)\ln\left(\frac{1-q}{\widehat{q}-q}\right) + q\left(\frac{(1-\widehat{q})}{(\widehat{q}-q)} - \ln\left(1 + \frac{1-\widehat{q}}{\widehat{q}-q}\right)\right) \\ &> \beta + (1-q)\ln\left(\frac{1-q}{\widehat{q}-q}\right). \end{aligned}$$

3.3.2 Sellers' Indifference Function

Suppose measure $\alpha \in [0, \beta]$ of buyers participate in submarket L. Let $S(\alpha) \in [0, \widehat{q}]$ be the measure of lemon sellers such that lemon sellers are indifferent between the two submarkets if measure $S(\alpha)$ of lemon sellers join submarket L. Formally,

$$V_L(1, \lambda_L(\alpha, S(\alpha))) = V_L\left(\frac{\widehat{q} - S(\alpha)}{1 - S(\alpha)}, \lambda_H(\alpha, S(\alpha))\right).$$

I let $S(\alpha) = 0$ if lemon sellers always prefer submarket H to submarket L, and let $S(\alpha) = \widehat{q}$ if the opposite is true.

Unlike B, S is strictly increasing at α if $S(\alpha) \in (0, \widehat{q})$. As more buyers join submarket L, submarket L always becomes more attractive to sellers than submarket H. Therefore, for lemon sellers to be indifferent between the two submarkets, more sellers should join submarket L.

There are three critical values of α, $0 < \beta_1 < \beta_2 < \beta_3 < \beta$. β_2 and β_3 are defined to be the values that satisfy $S(\beta_2) = \overline{q}$ and $S(\beta_3) = \widehat{q}$. These two values are well-defined in interior because sellers' expected payoffs in a submarket are equal to 0 if the measure of buyers is negligible. $\beta_2 < \beta_3$ comes from the fact that S is strictly increasing. β_1 is the value such that

$$\frac{\widehat{q} - S(\beta_1)}{1 - S(\beta_1)} \frac{1}{e^{\lambda_H(\beta_1, S(\beta_1))}} (v_L - c_L) = \widetilde{E}_{S(\beta_1)}[v] - c_H.$$

This value is also well-defined because the right-hand side is strictly greater (smaller) than the left-hand side if β is close to β_2 (0) and the right-hand side increases faster than the left-hand side.

By construction, if $\alpha > \beta_3$ then all lemon sellers prefer staying in submarket L ($S(\alpha) = \widehat{q}$). If $\alpha \in [\beta_2, \beta_3]$ then $S(\alpha) \in [\overline{q}, \widehat{q}]$ and both qualities of goods fully trade in submarket H. If $\alpha \in [\beta_1, \beta_2]$ then high-quality goods partially trade in submarket H. If $\alpha < \beta_1$ then only lemons trade in submarket H.

The case where $\alpha < \beta_1$ needs emphasis. In this case,

$$v_L - \frac{(1 + \lambda_L(\alpha, S(\alpha)))}{e^{\lambda_L(\alpha, S(\alpha))}} (v_L - c_L) = v_L - \frac{(1 + \lambda_H(\alpha, S(\alpha)))}{e^{\lambda_H(\alpha, S(\alpha))}} (v_L - c_L).$$

This equality holds when $\lambda_L(\alpha, S(\alpha)) = \lambda_H(\alpha, S(\alpha))$, which implies $S(\alpha) = \alpha/\beta$. Intuitively, when only lemons trade buyers' bidding strategy is independent of the proportion of lemon sellers. Therefore, for lemon sellers to be indifferent between the two submarkets, the arrival rates of buyers must be identical.

3.3.3 Equilibrium

An equilibrium is characterized by (α^*, q^*) such that $\alpha^* = B(q^*)$ and $q^* = S(\alpha^*)$ or by q^* that is a fixed point of the function $S(B(\cdot))$. It is trivially an equilibrium that all agents go to submarket H. There exists another equilibrium in which a market is segmented into two distinct submarkets.

Proposition 1 *When social surplus from trading is independent of quality and only lemons trade without segmentation (or in a trivial equilibrium), there always exists an equilibrium in which a market is segmented into two distinct submarkets. In such an equilibrium, high-quality goods trade with positive probability in submarket H.*

This is because $q < S(B(q))$ for $q \in (0, \beta_1/\beta)$, while $q > S(B(q))$ for q close to $\widehat{q}$.

1. Suppose a sufficiently small measure of lemon sellers join submarket L. Then the proportion of lemons in submarket H is still so high that only lemons trade in submarket H. Given that, for buyers to be indifferent between the two submarkets, $\lambda_H < \lambda_L$. Buyers in submarket H meet high-quality sellers, with whom they do not trade, with positive probability. Therefore, they must be compensated through lower competition. But then lemon sellers prefer submarket

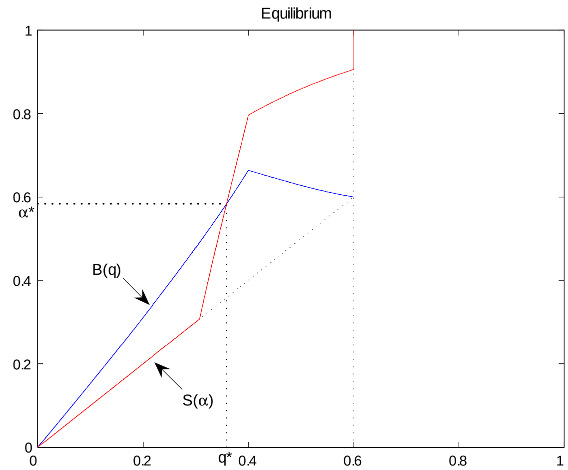

Figure 3: For parameter values $c_L = 0.1, v_L = 0.35, c_H = 0.6, v_H = 0.85, \widehat{q} = 0.6$ and $\beta = 1$.

L to submarket H. As shown before, for lemon sellers to be indifferent between the two submarkets when only lemons trade in submarket H, $\lambda_H = \lambda_L$.

2. Now suppose all lemon sellers go to submarket L, that is, sellers are fully separated. Buyers are indifferent between the two submarkets only when the ratios of buyers to sellers are identical ($\lambda_H = \lambda_L$). But then lemon sellers strictly prefer submarket H.

High-quality goods fully trade in submarket H when β is small. If β is large, trading probability is relatively less important than transaction price, which weakens lemon sellers' incentive to join submarket L. The proportion of lemons in submarket H is so large that full trade cannot occur. Conversely, if β is small, lemon sellers have less incentive to join submarket H whose ratio of buyers to sellers is smaller. Therefore, they must be compensated through higher transaction prices in submarket H, which happens when buyers bid more than c_H.

Do agents benefit from market segmentation? First of all, it is straightforward that both types of sellers are better off. Now high-quality sellers achieve a positive payoff. Lemon sellers enjoy more severe competition among buyers in submarket L.

In the two-quality case, it is non-trivial whether buyers are better off or not. In submarket L, buyers face no quality uncertainty but more severe competition. In the two-quality case, buyers are better off when β is sufficiently small. The result is not ambiguous when there is a continuum of qualities. With a continuum of qualities, buyers are strictly better off when a market is segmented. Furthermore, the more submarkets a market is segmented into, the greater payoff buyers achieve.

3.4 Relaxing Constant Surplus Assumption

Now I discuss what happens if I relax the constant surplus assumption.

Lower trading surplus with high-quality goods

Suppose $v_L - c_L > v_H - c_H$ and all lemon sellers go to submarket L (full separation). For buyers to be indifferent between the two submarkets, $\lambda_L > \lambda_H$. Buyers in submarket L enjoy higher trading surplus. Therefore, they should face more severe competition. In this case, lemon sellers have less incentive to deviate to submarket H than in the constant surplus case. When $v_L - c_L$ is sufficiently larger than $v_H - c_H$, lemon sellers' incentive to move to submarket H disappears and the full separation state persists. There are two kinds of fully separating equilibria.

(1) Fully separating equilibrium without trade in submarket H

This happens when

$$\frac{1}{e^{\beta/\widehat{q}}}(v_L - c_L) \geq (v_H - c_H),$$

and

$$v_L - \frac{1}{e^{\beta/\widehat{q}}}(v_L - c_L) \leq c_H.$$

The first inequality is buyers' incentive compatibility condition that they should prefer submarket L, even though they can extract the full trading surplus from high-quality sellers. The second inequality is high-quality sellers' incentive compatibility condition. It states that the maximum bid in submarket L (the left-hand side) should be less than the cost of high-quality sellers. Since the first inequality implies the second one, a fully separating equilibrium without trade in submarket H exists if and only if

$$\frac{1}{e^{\beta/\widehat{q}}}(v_L - c_L) \geq (v_H - c_H).$$

That is, this equilibrium exists when β is sufficiently small. The intuition behind this result is as follows. Suppose β is close to 0 and sellers are fully separated. Then buyers strictly prefer submarket L to submarket H because their expected payoff in submarket L is close to $v_L - c_L$, which is greater than $v_H - c_H$. Lemon sellers obviously do not deviate to submarket H. High-quality sellers also do not deviate because buyers bid lower than c_H in submarket L.

(2) Fully separating equilibrium with trade in submarket H

Let $\pi_0 = 1/e^{\lambda_L}$ and $\pi_0' = 1/e^{\lambda_H}$. A fully separating equilibrium with trade in submarket H exists if and only if

$$\pi_0 (v_L - c_L) = \pi_0' (v_H - c_H),$$

$$(1 - \pi_0 - \lambda\pi_0)(v_L - c_L) \geq \left(1 - \pi_0' - \lambda'\pi_0'\right)(v_H - c_H) + \left(1 - \pi_0'\right)(c_H - c_L),$$

and

$$\begin{aligned} & (1 - \pi_0) v_L + \pi_0 c_L - c_H - \pi_0 (v_L - c_H) \ln \frac{v_L - c_H}{\pi_0 (v_L - c_L)} \\ \leq \; & \left(1 - \pi_0' - \lambda\pi_0'\right)(v_H - c_H). \end{aligned}$$

The first condition is buyers' indifference between the two submarkets. The two inequalities are the incentive compatibility conditions of lemon sellers and high-quality sellers, respectively. These conditions hold when

$$\frac{1}{e^{\beta/\widehat{q}}}\left(v_L - c_L\right) \leq \left(v_H - c_H\right),$$

and v_L is sufficiently close to v_H. The latter guarantees that λ_L is sufficiently greater than λ_H, while the former (β is sufficiently large) ensures that trade occurs in submarket H as well.

Of particular interest is the constant value case where $v_H = v_L$. In this case, a non-trivial equilibrium is always fully separating, whether trade occurs in submarket H or not. This is not a special feature of the two-quality case. In Section 5, I show that even when there is a continuum of qualities, if buyers' values are independent of sellers' costs, a fully revealing equilibrium exists.

Higher trading surplus with high-quality goods

Now suppose $v_L - c_L < v_H - c_H$ and sellers are fully separated. For buyers to be indifferent, $\lambda_L < \lambda_H$, by the same reason as in the previous case. Then lemon sellers have stronger incentive to join submarket H than in the constant surplus case. This guarantees that $\widehat{q} > S\left(B\left(\widehat{q}\right)\right)$. Therefore, whenever only lemons trade without segmentation (this ensures $\varepsilon < S\left(B\left(\varepsilon\right)\right)$ for ε sufficiently small), a market can be segmented into two distinct submarkets.

There does not exist a separating equilibrium if $v_H - c_H$ is sufficiently larger than $v_L - c_L$. In this case, both goods trade even without segmentation and lemon sellers have a strong incentive to mimic high-quality sellers. Therefore, the condition that $\varepsilon < S\left(B\left(\varepsilon\right)\right)$ for ε sufficiently small is violated. For example, suppose $E_q\left[v\right] - c_H \geq v_L - c_L$. Since $v_L - c_L < \widetilde{E}_q\left[v\right] - c_H$, $\lambda_L\left(q, B\left(q\right)\right) < \lambda_H\left(q, B\left(q\right)\right)$ for any q. But then lemon sellers always prefer submarket H to submarket L, and hence, the market cannot be segmented. In Section 5, I show that this result is generalized into a continuum quality case.

3.5 More Finite Qualities

The insights from the two-quality case are transferrable to the general finite quality case. Figure 4 shows an example of equilibrium with three qualities of goods. The high two submarkets consist of different two qualities, whose proportions determine the amount of quality uncertainty and thus the level of buyer competition in each submarket. However, the characterization for the general finite quality case is quite involved. Instead, I turn my attention to a continuum quality case from the next section.

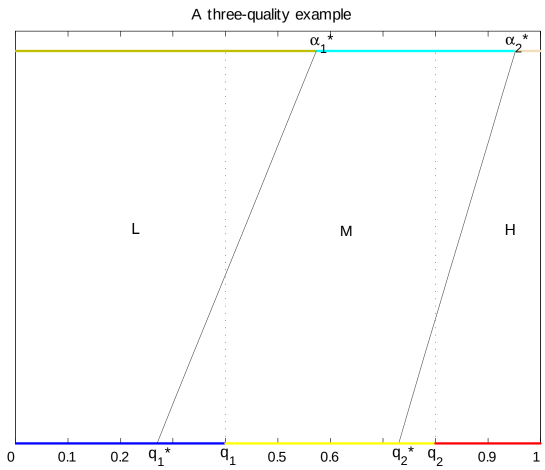

Figure 4: For parameter values, $c_L = 0, v_L = 1/8, c_M = 3/8, v_M = 4/8, c_H = 6/8, v_H = 7/8, q_1 = 0.5, q_2 = 0.9$, and $\beta = 1$. Measure q_1^* of low-quality sellers join submarket L, the other low-quality sellers and measure $(q_2^* - q_1)$ of middle-quality sellers participate in submarket M, and all other sellers go to submarket H. On the long-side of the market, measure α_1^*, $\alpha_2^* - \alpha_1^*$, and $1 - \alpha_2^*$ of buyers join submarket L, submarket M, and submarket H, respectively.

4 A Continuum Quality: Constant Surplus Case

4.1 Environment

Now there is a continuum of qualities uniformly distributed over $[0, 1]$. A unit of q-quality good costs $c(q)$ to a seller and yields utility $v(q)$ to a buyer. This section studies the case where $c(q) = q$ and $v(q) = q + \Delta$ for some $\Delta > 0$. These parametric assumptions incorporate two simplifications. First, trading surplus is independent of quality, that is, $v(q) - c(q) = \Delta$ for all $q \in [0, 1]$. Second, the distribution of seller's costs is uniform. These properties enable me to focus on the quality uncertainty aspect of the problem. In addition, they allow me to apply a recursive method in characterizing the set of equilibria and thus provide more understanding on equilibrium.

The market proceeds as in the previous section except that the set of submarkets is not fixed ex ante. I examine how many distinct submarkets are sustainable in equilibrium.

Definition 1 *An equilibrium with n submarkets is characterized by a strictly increasing sequence $\{q_0 = 0, q_1, ..., q_n = 1\}$ and a sequence $\{\lambda_1, ..., \lambda_n\}$ such that*

(1) (Sellers' optimality) if $q \in [q_{k-1}, q_k]$ then q-quality seller prefers the k-th submarket to the other submarkets, $\forall k = 1, ..., n$,[7]

[7] I restrict attention to equilibria in which the set of qualities in each submarket is convex. There may exist another equilibria. In such an equilibrium, some highest-quality sellers join multiple submarkets but their goods never trade in any submarket.

(2) (Buyers' optimality) if $\lambda_k > 0$ then buyers weakly prefer the k-th submarket to the other submarkets, and

(3) (Market clearing) $\beta = \sum_{k=1}^{n} (q_k - q_{k-1}) \lambda_k$.

Remark 1 This is a reduced-form definition of market equilibrium. I did not impose the optimality conditions on buyers' bidding strategies and sellers' acceptance strategies. In addition, I did not explicitly require buyers' beliefs about sellers' qualities in each submarket to be consistent. They are straightforward.

Remark 2 The definition can be generalized for the case with infinitely many submarkets and for the case with a continuum of submarkets. But it requires unnecessarily substantial investment in notations.

4.2 Submarket Analysis

Suppose a submarket is populated by sellers in $\left[\underline{q}, \overline{q}\right]$ and its ratio of buyers to sellers is given by $\lambda > 0$. Let F be buyers' symmetric bidding strategy in the submarket and $\left[\underline{b}, \overline{b}\right]$ be the support of F.

4.2.1 Buyers' Expected Payoff

As in the two-quality case, $\underline{b}$ is equal to the offer of the monopsonist who is facing a seller whose quality is uniformly distributed over $\left[\underline{q}, \overline{q}\right]$. Let $U\left(\underline{q}, \overline{q}, \lambda\right)$ be buyers' expected payoff in a submarket and $M\left(\underline{q}, \overline{q}\right)$ be the expected payoff of the monopsonist. Then,

$$U\left(\underline{q}, \overline{q}, \lambda\right) = \pi_0 M\left(\underline{q}, \overline{q}\right).$$

U is again strictly decreasing in λ and inherits all the properties of $M\left(\underline{q}, \overline{q}\right)$.

First, $\underline{b} = \min\left\{\overline{q}, \underline{q} + \Delta\right\}$. When the monopsonist increases his offer (b) slightly, the marginal benefit is $b + \Delta$, while the corresponding marginal cost is $\left(b - \underline{q}\right) + b$. $b - \underline{q}$ is the marginal increase of payment to all of the seller types who would accept a slightly lower offer and b is the gross payment to the marginal seller type who accepts the offer. The marginal benefit and cost match when $b = \underline{q} + \Delta$, but if $\overline{q} < \underline{q} + \Delta$ then the monopsonist has no reason to offer more than $\overline{q}$.

Second, by a direct calculation,

$$M\left(\underline{q}, \overline{q}\right) = \begin{cases} \Delta - \frac{\overline{q} - \underline{q}}{2}, & \text{if } \overline{q} - \underline{q} \leq \Delta, \\ \frac{\Delta^2}{2\left(\overline{q} - \underline{q}\right)}, & \text{otherwise.} \end{cases}$$

The expected payoff of the monopsonist depends only on $\overline{q} - \underline{q}$, which is the measure of quality uncertainty in the current setting. It is natural that M is strictly decreasing in the amount of quality uncertainty ($\overline{q} - \underline{q}$).

4.2.2 Sellers' Expected Payoffs

F is again derived from buyers' indifference over $\left[\underline{b}, \overline{b}\right]$. If $\overline{q} - \underline{q} \leq \Delta$ then $\underline{b} = \overline{q}$, and trade occurs whenever a seller is matched with at least one buyer. Buyers' expected payoff by bidding $b \geq \underline{b}$ is

$$\sum_{k=0}^{\infty} \pi_k F(b)^k \left(\int_{\underline{q}}^{\overline{q}} \frac{(v(q) - b)}{\overline{q} - \underline{q}} dq \right) = \pi_0 e^{\lambda F(b)} \left(E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] - b \right),$$

where

$$E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] = \frac{1}{\overline{q} - \underline{q}} \int_{\underline{q}}^{\overline{q}} v(q)\, dq = \frac{\underline{q} + \overline{q}}{2} + \Delta.$$

Since buyers are indifferent over $\left[\underline{b}, \overline{b}\right]$,

$$e^{\lambda F(b)} = \frac{E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] - \overline{q}}{E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] - b}.$$

Using $F\left(\overline{b}\right) = 1$, I find that

$$\overline{b} = (1 - \pi_0)\, E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] + \pi_0 \overline{q}.$$

Now suppose $\overline{q} - \underline{q} > \Delta$. In this case, $\underline{b} = \underline{q} + \Delta < \overline{q}$, and trade may not occur even if a seller is matched with buyers. Buyers' expected payoff by bidding $b \geq \underline{b}$ is

$$\pi_0 e^{\lambda F(b)} \frac{b - \underline{q}}{\overline{q} - \underline{q}} \left(E_{\underline{q},b} \left[v\left(q'\right) \right] - b \right), \text{ if } b \leq \overline{q},$$

and

$$\pi_0 e^{\lambda F(b)} \left(E_{\underline{q},\overline{q}} \left[v\left(q'\right) \right] - b \right), \text{ if } b > \overline{q}.$$

From buyers' indifference over bids in $\left[\underline{b}, \overline{b}\right]$,

$$e^{\lambda F(b)} = \begin{cases} \frac{\Delta\left(E_{\underline{q},\underline{q}+\Delta}[v(q')] - \left(\underline{q}+\Delta\right)\right)}{\left(b-\underline{q}\right)\left(E_{\underline{q},b}[v(q')]-b\right)}, & \text{if } b \leq \overline{q}, \\ \frac{\Delta\left(E_{\underline{q},\underline{q}+\Delta}[v(q')] - \left(\underline{q}+\Delta\right)\right)}{E_{\underline{q},\overline{q}}[v(q')]-b}, & \text{if } b > \overline{q}. \end{cases}$$

Regarding $\overline{b}$, there are two possibilities: $\overline{b} \geq \overline{q}$ and $\overline{b} < \overline{q}$. In the former case some highest qualities ($q \in \left(\underline{q} + \Delta, \overline{q}\right]$) partially trade, while in the latter case some highest qualities ($q \in \left(\overline{b}, \overline{q}\right]$) never trade. Using $F\left(\overline{b}\right) = 1$, I find that

$$\overline{b} = \left\{ \begin{array}{cc} \Delta + \frac{\underline{q}+\overline{q}}{2} - \pi_0 \frac{\Delta^2/2}{\overline{q}-\underline{q}} & \leq \\ \underline{q} + \Delta\left(1 + \sqrt{1-\pi_0}\right) & > \end{array} \right\} \overline{q} \text{ if } \left\{ \begin{array}{l} \overline{q} - \underline{q} \leq \Delta\left(1 + \sqrt{1-\pi_0}\right), \\ \overline{q} - \underline{q} > \Delta\left(1 + \sqrt{1-\pi_0}\right). \end{array} \right.$$

Now I can calculate sellers' expected payoffs. Though the calculation is not particularly hard, the form of $V\left(q; \underline{q}, \overline{q}, \lambda\right)$ is unnecessarily complicated. Below, I present the expected payoffs of the boundary sellers, $\underline{q}$ and $\overline{q}$. Due to the single-crossing property in Lemma 1, only these payoffs are necessary for further analysis. Let $z = \overline{q} - \underline{q}$. There are three cases according to z.

(1) If $z \leq \Delta$ then

$$\begin{aligned} V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= \Delta\left(1-\pi_0-\lambda\pi_0\right)+\frac{z}{2}\left(1-\pi_0+\lambda\pi_0\right), \text{ and} \\ V\left(\overline{q};\underline{q},\overline{q},\lambda\right) &= \Delta\left(1-\pi_0-\lambda\pi_0\right)+\frac{z}{2}\left(1-\pi_0-\lambda\pi_0\right). \end{aligned}$$

(2) If $\Delta < z < \Delta\left(1+\sqrt{1-\pi_0}\right)$ then

$$\begin{aligned} V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= \frac{z}{2}+\left(1-\pi_0\right)\Delta-\pi_0\frac{\Delta^2}{2z} \\ &\quad -\pi_0\frac{\Delta}{2}\ln\frac{z}{\left(2\Delta-z\right)}-\pi_0\frac{\Delta^2}{2z}\ln\frac{z\left(2\Delta-z\right)}{\pi_0\Delta^2}, \text{ and} \\ V\left(\overline{q};\underline{q},\overline{q},\lambda\right) &= \Delta-\frac{z}{2}-\pi_0\frac{\Delta^2}{2z}-\pi_0\frac{\Delta^2}{2z}\ln\frac{z\left(2\Delta-z\right)}{\pi_0\Delta^2}. \end{aligned}$$

(3) If $z \geq \Delta\left(1+\sqrt{1-\pi_0}\right)$ then

$$\begin{aligned} V\left(\underline{q};\underline{q},\overline{q},\lambda\right) &= \Delta\left(1-\pi_0+\sqrt{1-\pi_0}\right)-\pi_0\frac{\Delta}{2}\ln\frac{\left(1+\sqrt{1-\pi_0}\right)}{\left(1-\sqrt{1-\pi_0}\right)}, \text{ and} \\ V\left(\overline{q};\underline{q},\overline{q},\lambda\right) &= 0. \end{aligned}$$

Similarly to $U\left(\underline{q},\overline{q},\lambda\right)$, both $V\left(\underline{q};\underline{q},\overline{q},\lambda\right)$ and $V\left(\overline{q};\underline{q},\overline{q},\lambda\right)$ are functions of only λ and $\overline{q}-\underline{q}$.[8] Both functions are increasing in λ. $V\left(\underline{q},\underline{q},\overline{q},\lambda\right)$ is increasing in $\overline{q}-\underline{q}$, while $V\left(\overline{q},\underline{q},\overline{q},\lambda\right)$ is not monotone in $\overline{q}-\underline{q}$.

4.3 Necessary and Sufficient Conditions for an Equilibrium

Unlike the two-quality case, it is complicated to directly characterize equilibrium with a continuum of qualities. I find tractable conditions that are necessary and sufficient.

The following lemma shows that I can restrict my attention to the expected payoffs of the boundary sellers in each submarket.

Lemma 1 *(Single Crossing Property) Suppose* $[q_1, q_2]$ *and* $[q_2, q_3]$ *form two separate submarkets with the ratio of buyers to sellers,* λ *and* λ'*, respectively, and buyers are indifferent between the two submarkets. Sellers whose qualities are below (above)* q_2 *prefer the submarket with* $[q_1, q_2]$ *(*$[q_2, q_3]$*) to the submarket with* $[q_2, q_3]$ *(*$[q_1, q_2]$*), if and only if*

(1) q_2 *is indifferent between the two submarkets, and*

(2) $\lambda > \lambda'$.

[8] More generally, $V\left(q;\underline{q},\overline{q},\lambda\right) = V\left(q-\underline{q};0,\overline{q}-\underline{q},\lambda\right)$.

Proof. See Appendix. ■

The intuition behind this result is similar to that of the common single crossing property that is based on the trade-off between trading probability and deterministic transaction price. The smaller the cost is, the more willingly a seller is to trade the good. Therefore, if q_2-quality seller is indifferent between the two submarkets that have different levels of buyer competition (trading probability) and different bidding behaviors of buyers, sellers whose qualities are lower (higher) than q_2 prefer the submarket with relatively more buyers (with relatively higher transaction prices).

Corollary 1 $\{q_0 = 0, q_1, ..., q_n = 1\}$ *and* $\{\lambda_1, ..., \lambda_n\}$ *constitute an equilibrium with* n *submarkets if and only if*

(1) (Boundary Sellers' Indifference) q_k*-quality seller is indifferent between the* k*-th submarket and the* $(k+1)$*-th submarket,* $k = 1, ..., n-1$*,*

(2) (Buyers' Indifference) buyers are indifferent over all active submarkets ($\lambda_k > 0$)*, and weakly prefer active submarkets to inactive submarkets* ($\lambda_k = 0$)*,*

(3) (Monotone Market Arrangement) $\lambda_k > \lambda_{k+1}$ *if* $\lambda_k > 0$*, and* $\lambda_{k+1} = 0$*, if* $\lambda_k = 0, k = 1, ..., n-1$*, and*

(4) (Market Clearing Condition) $\beta = \sum_{k=1}^{n} (q_k - q_{k-1}) \lambda_k$.

(3) is the generalization of the result in the two-quality case that relatively more buyers join submarket L than submarket H. (2) and (3) imply the following monotone market arrangement for sellers. If buyers are indifferent over all submarkets, then

$$q_1 - q_0 \leq q_2 - q_1 \leq ... \leq q_n - q_{n-1}.$$

That is, higher-quality submarkets entail more quality uncertainty. This is the analogue to the result in the two-quality case that there is no quality uncertainty in submarket L but a positive amount of quality uncertainty remains in submarket H.

4.4 Partial Equilibrium Analysis

The analysis from now on proceeds as follows. First, in this subsection, I suppose buyers' equilibrium utility $u \in (0, \Delta)$ is known, and find $\{q_0 = 0, q_1, ..., q_n = 1\}$ and $\{\lambda_1, ..., \lambda_n\}$ that are consistent with u. In other words, I find a partition of sellers and the corresponding ratios of buyers to sellers with which buyers get the same utility u in every submarket.[9] Second, in the next subsection, I endogenize u by imposing the market clearing condition, $\beta = \sum_{k=1}^{n} (q_k - q_{k-1}) \lambda_k$. Subsequently, let $z_k = q_k - q_{k-1}, k = 1, ..., n$.

[9] Buyers may strictly prefer some submarkets to others. I call such equilibrium "partially indifferent equilibrium" and characterize the set of such equilibria in Appendix A.

Preliminaries

To facilitate the analysis, I introduce some functions. Let $\lambda(z,u)$ be the value such that $u = U(0, z, \lambda(z,u))$. $\lambda(z,u)$ is the tightness (the ratio of buyers to sellers) that is required to guarantee buyers utility u when quality uncertainty is z in a submarket. $\lambda(z,u)$ is well-defined for $z \leq \Delta^2/2u$ if $u < \Delta/2$, and for $z \leq 2(\Delta - u)$ if $u \geq \Delta/2$. For later use, let $\overline{z}(u)$ be the maximum z such that $\lambda(z,u)$ is well-defined. $\lambda(z,u)$ is strictly decreasing in both z and u, because U is strictly decreasing in z and λ. Intuitively, as quality uncertainty increases buyer competition must reduce to ensure buyers a constant utility u. Similarly, for a fixed amount of quality uncertainty, buyer competition must be smaller to deliver greater utility to buyers.

Next, let $W_L(z,u) = V(0; 0, z, \lambda(z,u))$ and $W_U(z,u) = V(z; 0, z, \lambda(z,u))$. $W_L(z,u)$ ($W_U(z,u)$) is the expected payoff of the lower (upper) boundary seller when buyers get utility u in a submarket with quality uncertainty z. After arranging terms,

$$W_L(z,u) = \begin{cases} \Delta + \frac{z}{2} - u - u\frac{2z}{(2\Delta - z)} + u\ln\frac{2u}{(2\Delta - z)}, & \text{if } z \leq \Delta, \\ \frac{z}{2} + \Delta - \frac{2zu}{\Delta} - u - \frac{zu}{\Delta}\ln\frac{z}{(2\Delta - z)} + u\ln\frac{2u}{(2\Delta - z)}, & \text{if } z \in (\Delta, 2(\Delta - u)), \\ \Delta\left(1 - \frac{2zu}{\Delta^2} + \sqrt{1 - \frac{2zu}{\Delta^2}}\right) - \frac{zu}{\Delta}\ln\frac{\left(1+\sqrt{1-2zu/\Delta^2}\right)^2}{2zu/\Delta^2} & \text{if } z \geq 2(\Delta - u), \end{cases}$$

and

$$W_U(z,u) = \begin{cases} \Delta - \frac{z}{2} - u + u\ln\frac{2u}{(2\Delta - z)}, & \text{if } z \leq 2(\Delta - u), \\ 0, & \text{if } z \geq 2(\Delta - u). \end{cases}$$

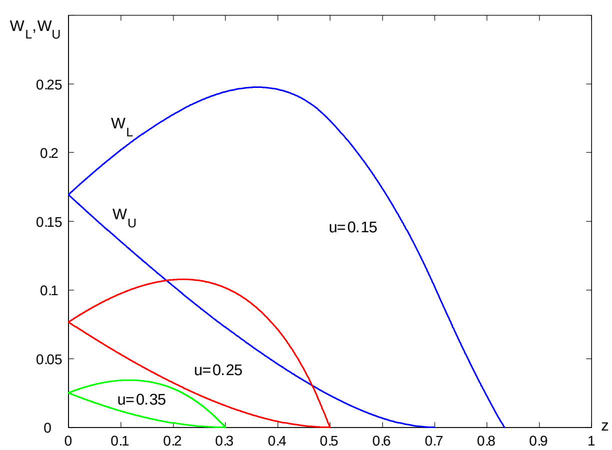

Figure 5: W_L and W_U as functions of z for different values of u.

Figure 5 shows sample paths of $W_L(\cdot, u)$ and $W_U(\cdot, u)$. First, consider $W_U(\cdot, u)$. As quality uncertainty increases buyer competition must decrease to guarantee buyers utility u. In addition, from the upper boundary seller's perspective, higher z implies lower $\underline{q}$ for fixed $\overline{q}$, resulting in lower average quality. This makes buyers bid lower. Both of these effects lower the expected payoff of the upper boundary seller.

Now consider $W_L(\cdot, u)$. As quality uncertainty increases $\lambda(z, u)$ decreases, which reduces the lower boundary seller's expected payoff. However, from the lower boundary seller's perspective, higher z implies higher $\overline{q}$ for fixed $\underline{q}$ and so greater average quality. Since buyers bid higher, this offsets the first effect. For z small, the second effect dominates, while the first effect does for z sufficiently large. Overall, $W_L(\cdot, u)$ is increasing first and decreasing later.

Both W_L and W_U are decreasing in u. For a fixed amount of quality uncertainty, buyer competition must be smaller to provide buyers with higher utility. This effect lowers both W_L and W_U.

Recursive method

I apply the following recursive method in finding the set of partial equilibria. Suppose z_1 (the amount of quality uncertainty in the first submarket) is given. Since the upper boundary seller in the first submarket is the lower boundary seller in the second submarket, z_2 is determined so that $W_U(z_1, u) = W_L(z_2, u)$. In the same way, I can find $z_3, z_4, ...$ This process stops once $z_1 + ... + z_n \geq 1$ for some n. If $z_1 + ... + z_n = 1$, then the sequences $\{q_0, ..., q_n\}$ and $\{\lambda_1, ..., \lambda_n\}$ such that $q_k - q_{k-1} = z_k$ and $\lambda_k = \lambda(z_k, u)$ constitute a partial equilibrium.

For more systematic analysis, I define the following function. Define $\gamma_+(\cdot, u) : [0, \overline{z}(u)] \to (0, \overline{z}(u)]$ so that $W_U(z, u) = W_L(\gamma_+(z, u), u)$. $\gamma_+(z, u)$ is the amount of quality uncertainty in the next submarket, when quality uncertainty is z in some submarket. By an immediate extension, let $\gamma_+^k(z, u) = \gamma_+\left(\gamma_+^{k-1}(z, u), u\right)$ for $k \geq 1$ where $\gamma_+^0(z, u) = z$. I use the following results later.

Lemma 2 *(1) $\gamma_+^k(\cdot, u)$ is continuous.*

(2) $\gamma_+^k(\cdot, u)$ is strictly increasing on $[0, 2(\Delta - u))$, and constant on $[2(\Delta - u), \overline{z}(u)]$.

(3) $\gamma_+^k(z, \cdot)$ is continuous and strictly decreasing.

Proof. See Appendix. ■

Partial equilibrium with one submarket

A partial equilibrium with one submarket (in which a positive measure of buyers participate in the market) exists if and only if $\overline{z}(u) > 1$. Figure 6 shows such equilibria for different u's. In the left panel, one-submarket equilibrium is the unique equilibrium. In the right panel, there exists an equilibrium with two submarkets. To see this, consider $z + \gamma_+(z, u)$. In the left panel, $z + \gamma_+(z, u)$ is greater than 1 for all z, and therefore, there does not exist an equilibrium with two submarkets. In the right panel, $z + \gamma_+(z, u)$ is smaller than 1 if z is close 0, while it is greater than 1 if z is large (for example, when $\gamma_+(z, u) = 1$). Since $z + \gamma_+(z, u)$ is continuous in z, there exists z^* such that

$z^* + \gamma_+ (z^*, u) = 1$. In general, one-submarket equilibrium is a unique partial equilibrium if and only if $\gamma_+ (0, u) \geq 1$, which holds when u is sufficiently small.

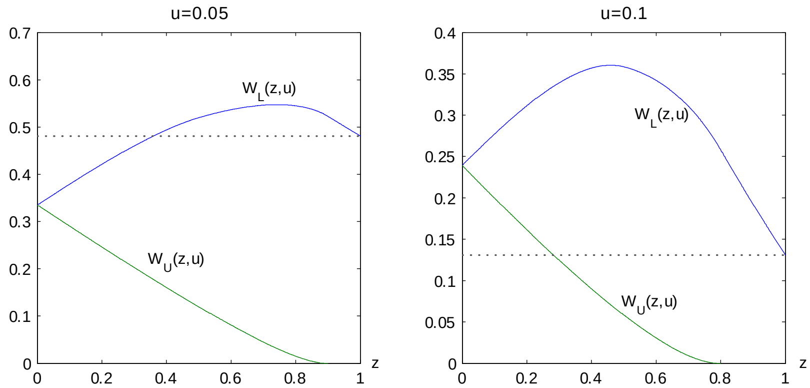

Figure 6: One-submarket partial equilibrium for $\Delta = 0.5$.

Partial equilibrium with n submarkets

Consider an equilibrium with n submarkets, $\{z_1, ..., z_n\}$. By the incentive compatibilities of the boundary sellers in each submarket, $z_k = \gamma_+^{k-1} (z_1, u)$ for all k. Since $\sum_{k=1}^{n} \gamma_+^{k-1} (z, u)$ is strictly increasing and $z_1 + ... + z_n = 1$, a necessary and sufficient condition for an n-submarket partial equilibrium to exist is

$$\sum_{k=1}^{n} \gamma_+^{k-1} (0, u) < 1 < \sum_{k=1}^{n} \gamma_+^{k-1} (\overline{z} (u), u) = n \cdot \overline{z} (u) .$$

Lemma 3 *A partial equilibrium with* n *submarkets exists if and only if* $u \in (\underline{u}_n, \overline{u}_n)$ *where* $\underline{u}_n$ *and* $\overline{u}_n$ *are the values such that* $\sum_{k=1}^{n} \gamma_+^{k-1} (0, \underline{u}_n) = 1$ *and* $\overline{z} (\overline{u}_n) = 1/n$*, respectively.*

Proof. This follows from the fact that both $\sum_{k=1}^{n} \gamma_+^{k-1} (0, u)$ and $n \cdot \overline{z} (u)$ are strictly decreasing in u (See (3) in Lemma 2 and the definition of $\overline{z} (u)$). ■

The set of partial equilibria

Given u, let $\overline{N} (u)$ be the smallest integer such that $\sum_{k=1}^{\overline{N}(u)+1} \gamma_+^{k-1} (0, u) \geq 1$, and let $\underline{N} (u)$ be the smallest integer that is strictly greater than $1/\overline{z} (u)$.

Proposition 2 *(The set of partial equilibria) Given* u*,* $\underline{N} (u) \leq \overline{N} (u)$*, and there exists a (unique)* n*-submarket partial equilibrium if and only if* $\underline{N} (u) \leq n \leq \overline{N} (u)$*.*

Proof. Since $\gamma_+(\cdot, u)$ is increasing and $\gamma_+(\overline{z}(u)) = \overline{z}(u)$, $\gamma_+(z) \leq \overline{z}(u), \forall z \in [0, \overline{z}(u)]$. Therefore,

$$1 \leq \sum_{k=1}^{\overline{N}(u)+1} \gamma_+^{k-1}(0,u) = 0 + \sum_{k=2}^{\overline{N}(u)+1} \gamma_+^{k-1}(0,u) < \sum_{k=2}^{\overline{N}(u)+1} \overline{z}(u) = \overline{N}(u) \cdot \overline{z}(u).$$

The strict inequality is due to the fact that $W_U(0,u) > 0 = W_L(\overline{z}(u), u)$. By the definition of $\underline{N}(u)$, $\overline{N}(u) \geq \underline{N}(u)$.

Now suppose $\underline{N}(u) \leq n \leq \overline{N}(u)$. Then

$$\sum_{k=1}^{n} \gamma_+^{k-1}(0,u) \leq \sum_{k=1}^{\overline{N}(u)} \gamma_+^{k-1}(0,u) < 1 < \sum_{k=1}^{\underline{N}(u)} \overline{z}(u) \leq \sum_{k=1}^{n} \gamma_+^{k-1}(\overline{z}(u), u).$$

Since $\sum_{k=1}^{n} \gamma_+^{k-1}(\cdot, u)$ is continuous and strictly increasing, there exists a unique z^n such that $\sum_{k=1}^{n} \gamma_+^{k-1}(z^n, u) = 1$. ■

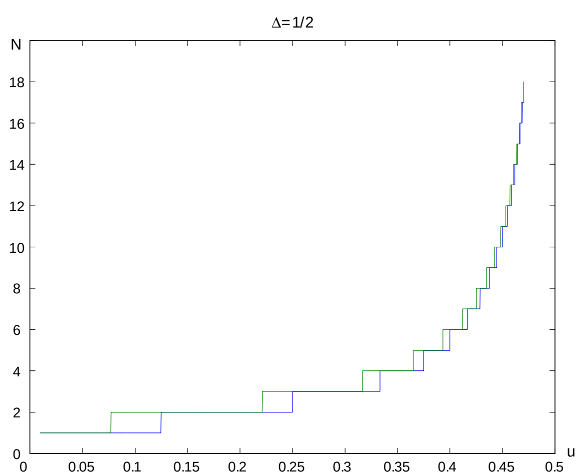

Figure 7: The upper and lower bounds of the possible number of submarkets.

Figure 7 shows how $\underline{N}(u)$ and $\overline{N}(u)$ vary as u changes. Both are step functions whose jump sizes are always equal to 1 and increase without bound as u approaches Δ. Intuitively, buyers get a high utility only when quality uncertainty in each submarket is sufficiently small. Therefore, for u close to Δ, a market must be segmented into many submarkets.

Lemma 4 *The jump sizes of $\underline{N}(u)$ and $\overline{N}(u)$ are always equal to 1. For $u < \Delta$, both $\underline{N}(u)$ and $\overline{N}(u)$ are finite, but as u tends to Δ they approach infinity.*

Proof. The jump size of $\underline{N}(u)$ is 1 by the definition of $\underline{N}(u)$. The result for $\overline{N}(u)$ comes from (3) in Lemma 2. For $u < \Delta$, $\gamma_+(0, u) < z_k$ for all $k > 1$, and so $N(u) < 1/\gamma_+(0) + 1$. The last result follows from the construction of $\underline{N}(u)$ and the fact that $\overline{N}(u) \geq \underline{N}(u)$. ■

4.5 General Equilibrium Analysis

The partial equilibrium analysis showed that there exists a unique n-submarket partial equilibrium, $\{z_1, ..., z_n\}$ and $\{\lambda_1, ..., \lambda_n\}$, if and only if $u \in (\underline{u}_n, \overline{u}_n)$. For $u \in (\underline{u}_n, \overline{u}_n)$, let $\beta_n(u) = \sum_{k=1}^{n} z_k \lambda_k$.

The following proposition shows that if $u \in \left(\underline{u}_{n+1}, \overline{u}_n\right)$ then $\beta_{n+1}(u) > \beta_n(u)$.

Proposition 3 *Given u, if there exist two partial equilibria with different numbers of submarkets, the total measure of buyers is greater in the equilibrium with more submarkets than in the other equilibrium.*

Proof. See Appendix. ■

The overall quality uncertainty in a market is smaller when the market is segmented into more submarkets. Therefore, for buyers to get the same utility there must be relatively fewer buyers with less submarkets. This implies the following result (See Figure 8).

Corollary 2 *For fixed $\beta > 0$, buyers are better off when a market is segmented into more submarkets.*

Now let $\overline{\beta}_n$ be the value such that

$$\overline{\beta}_n = \sup_{u \in (\underline{u}_n, \overline{u}_n)} \beta_n(u).$$

It is immediate that $\overline{\beta}_1 = \infty$ because one-submarket (trivial) equilibrium always exists.

Proposition 4 *(1) For $n > 1$, there exists $\overline{\beta}_n < \infty$ such that an equilibrium with n submarkets exists if and only if $\beta < \overline{\beta}_n$ ($\beta \leq \overline{\beta}_n$ if the supremum is achieved). Therefore, for endogenous market segmentation, β must be small.*

(2) As n tends to infinity, $\overline{\beta}_n$ converges to 0. Therefore, a market can be segmented into many submarkets if and only if β is sufficiently small.

Proof. I use the following fact.

$$\beta_n(u) = \sum_{k=1}^{n} z_k \cdot \lambda(z_k, u) \leq \ln\left(\frac{\Delta}{u}\right) \sum_{k=1}^{n} z_k = \ln\left(\frac{\Delta}{u}\right).$$

$\overline{\beta}_n < \infty$ because multiple submarkets are sustainable only when $W_U(0,u) > W_L(1,u)$ and so u is bounded away from 0. $(z_1,...,z_n)$ is continuous in u, and $\beta_n(u)$ converges to 0 as u tends to $\overline{u}_n$. This establishes the first result.

The second result is due to the fact that n is large only when u is sufficiently close to Δ (Lemma 4). ■

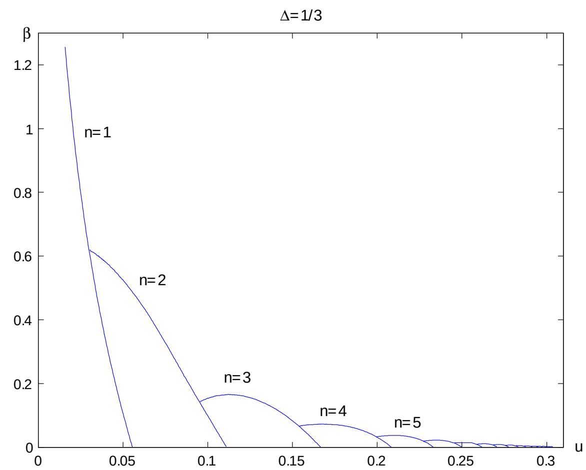

Figure 8: The set of general equilibria.

Figure 8 shows the possible numbers of submarkets and buyers' equilibrium expected utilities for each β. The smaller β is, the more submarkets a market can be segmented into. Intuitively, when β is smaller, sellers have stronger incentive to attract more buyers by separating from higher-quality sellers. This allows more submarkets to be sustainable in equilibrium. In the limit as β tends to 0, quality uncertainty can be eliminated through market segmentation.

For some β there exist two equilibria with the same number of submarkets. Those equilibria differ in how evenly distributed quality uncertainty is across submarkets. Buyers' expected utility is higher when quality uncertainty is more evenly distributed.

Remark 3 The numerical example suggests a much sharper result regarding the behavior of $\overline{\beta}_n$: $\overline{\beta}_n$ is strictly decreasing in n. This implies that for any β there exists $N(\beta)$ such that there exists an equilibrium with n submarkets if and only if $n \leq N(\beta)$. This is a consistent finding in numerical analyses I have performed. Unfortunately, I cannot establish this result analytically. There are two prominent approaches to this problem. One is to consider the difference equation derived from the game. The other is to apply the Lagrangian method and compare $\overline{\beta}_n$'s. The difficulty in the first approach is that my problem generates a complicated two-dimensional difference equation. The difficulty in the second approach is that, while it is possible to derive conditions for $\overline{\beta}_n$ for fixed n, it is quite involved to compare $\overline{\beta}_n$ and $\overline{\beta}_{n+1}$.

5 A Continuum Quality: Varying Surplus Cases

This section supplements the previous section by studying varying surplus cases. I first solve for submarket outcomes for the general continuum quality case. Then I consider a linear example and provide a necessary and sufficient condition of the relationship between buyers' values and sellers' costs for endogenous market segmentation. Last, I analyze another extreme case where buyers' values are independent of sellers' costs.

5.1 Environment

As in the previous section, there is a continuum of qualities uniformly distributed over $[0, 1]$. A unit of q-quality good costs $c(q)$ to a seller and yields utility $v(q)$ to a buyer. Unlike in the previous section, I do not impose parametric assumptions on c and v, but use the following regularity conditions.

Assumption 1 *c and v are continuous. c is strictly increasing and v is increasing.*

Assumption 2 *There exists $\Delta > 0$ such that $v(q) - c(q) \geq \Delta$ for all $q \in [0, 1]$.*

Assumption 3 *For any q',*

$$\int_{q'}^{q''} \left(v(q) - c\left(q''\right)\right) dq \text{ is strictly quasi-concave in } q''.$$

Assumption 3 ensures that buyers' symmetric mixed bidding strategy is unique and has a convex support. For $q' \in [0, 1]$, let $r(q')$ be the value such that

$$r\left(q'\right) = \arg \max_{q'' \in [0,1]} \int_{q'}^{q''} \left(v(q) - c\left(q''\right)\right) dq.$$

$r(q')$ is the highest quality the monopsonist is willing to trade when the seller's quality is known to be greater than q'. $r(q')$ corresponds to $q' + \Delta$ in the previous section.

5.2 Submarket Analysis

The submarket analysis proceeds as in the previous section. I use the same notations.

First, I find buyers' expected payoff in a submarket in the same fashion as before.

$$\begin{aligned} U\left(\underline{q}, \overline{q}, \lambda\right) &= \pi_0 M\left(\underline{q}, \overline{q}\right) \\ &= \pi_0 \frac{\min\left\{r\left(\underline{q}\right), \overline{q}\right\} - \underline{q}}{\overline{q} - \underline{q}} \left(E_{\underline{q}, \min\left\{r\left(\underline{q}\right), \overline{q}\right\}}\left[v(q)\right] - c\left(\min\left\{r\left(\underline{q}\right), \overline{q}\right\}\right)\right). \end{aligned}$$

To interpret this expression, fix $\underline{q}$ and λ. When $\overline{q}$ is close to $\underline{q}$, quality uncertainty is small, and therefore, all qualities fully trade $\left(\underline{b} = c\left(\overline{q}\right) < c\left(r\left(\underline{q}\right)\right)\right)$. As $\overline{q}$ increases, buyers' minimum bid $\left(c\left(\overline{q}\right)\right)$

increases, which lowers their payoff. On the other hand, the average quality of goods ($E_{\underline{q},\overline{q}}[v(q)]$) improves, which increases buyers' payoff. Whether $U(\underline{q},\overline{q},\lambda)$ is increasing in $\overline{q}$ or not depends on the relative importance of these two effects. In the constant surplus case, the former effect always dominates the latter. In general, if v increases sufficiently faster than c, $U(\underline{q},\overline{q},\lambda)$ is increasing in $\overline{q}$. For example, when $v(q)=\sigma q+\Delta$ and $c(q)=q$, $U(\underline{q},\overline{q},\lambda)$ is increasing in $\overline{q}$ if and only if $\sigma\geq 2$.

If quality uncertainty is sufficiently large and v does not increase sufficiently faster than c, then $\underline{b}=c(r(\underline{q}))<c(\overline{q})$. In this case, further increase of $\overline{q}$ always lowers buyers' expected payoff. This is because the probability of buyers' meeting sellers whose qualities are lower than $r(\underline{q})$ decreases. The fractional term in $U(\underline{q},\overline{q},\lambda)$ reflects this effect.

Sellers' expected payoffs are also calculated in the same way as in the previous section. Let $s(\underline{q},\lambda)$ be the value such that

$$c(s(\underline{q},\lambda))=E_{\underline{q},s(\underline{q},\lambda)}[v(q)]-\pi_0\frac{r(\underline{q})-\underline{q}}{s(\underline{q},\lambda)-\underline{q}}\left(E_{\underline{q},r(\underline{q})}[v(q)]-c(r(\underline{q}))\right).$$

$s(\underline{q},\lambda)$ is the generalization of $\underline{q}+\Delta\left(1+\sqrt{1-\pi_0}\right)$ in the previous section. Again, there are three cases.

(1) $\overline{q}\leq r(\underline{q})$.

$$\begin{aligned}
V(\underline{q};\underline{q},\overline{q},\lambda) &= E_{\underline{q},\overline{q}}[v(q)]-c(\underline{q})-(1+\lambda)U(\underline{q},\overline{q},\lambda)-\pi_0\left(c(\overline{q})-c(\underline{q})\right),\\
V(\overline{q};\underline{q},\overline{q},\lambda) &= E_{\underline{q},\overline{q}}[v(q)]-c(\overline{q})-(1+\lambda)U(\underline{q},\overline{q},\lambda).
\end{aligned}$$

(2) $r(\underline{q})<\overline{q}<s(\underline{q},\lambda)$.

$$\begin{aligned}
V(\underline{q};\underline{q},\overline{q},\lambda) &= E_{\underline{q},r(\underline{q})}[v(q)]-c(\underline{q})\\
&\quad-\left(1+\ln\frac{E_{\underline{q},\overline{q}}[v(q)]-c(\overline{q})}{U(\underline{q},\overline{q},\lambda)}\right)U(\underline{q},\overline{q},\lambda)-\pi_0\left(c(r(\underline{q}))-c(\underline{q})\right)\\
&\quad-U(\underline{q},\overline{q},\lambda)\int_{c(\underline{q})}^{c(\overline{q})}\left(\frac{\overline{q}-\underline{q}}{c^{-1}(b)-\underline{q}}\frac{1}{E_{\underline{q},c^{-1}(b)}[v(q)]-b}\right)db,
\end{aligned}$$

$$V(\overline{q};\underline{q},\overline{q},\lambda)=E_{\underline{q},r(\underline{q})}[v(q)]-c(\overline{q})-U(\underline{q},\overline{q},\lambda)\left(1+\ln\frac{E_{\underline{q},\overline{q}}[v(q)]-c(\overline{q})}{U(\underline{q},\overline{q},\lambda)}\right).$$

(3) $\overline{q}\geq s(\underline{q},\lambda)$.

$$\begin{aligned}
V(\underline{q};\underline{q},\overline{q},\lambda) &= E_{\underline{q},s(\underline{q},\lambda)}[v(q)]-c(\underline{q})-U(\underline{q},\overline{q},\lambda)\frac{\overline{q}-\underline{q}}{s(\underline{q},\lambda)-\underline{q}}\\
&\quad-U(\underline{q},\overline{q},\lambda)\int_{c(\underline{q})}^{\overline{b}}\left(\frac{\overline{q}-\underline{q}}{c^{-1}(b)-\underline{q}}\frac{1}{E_{\underline{q},c^{-1}(b)}[v(q)]-b}\right)db,\\
V(\overline{q};\underline{q},\overline{q},\lambda) &= 0.
\end{aligned}$$

5.3 Linear Example

As a concrete example that departs from the constant surplus assumption, I consider the case in which $c(q) = q$ and $v(q) = \sigma q + \Delta$ where $\Delta > 1$ and $1 \leq \sigma < 2$.[10] For simplicity, I restrict attention to equilibria with two submarkets, submarket L and submarket H. Let λ_L and λ_H be the ratios of buyers to sellers in submarket L and in submarket H, respectively. In addition, let $\pi_{L,0} = 1/e^{\lambda_L}$ and $\pi_{H,0} = 1/e^{\lambda_H}$.

Buyers' expected payoffs in each submarket are

$$\begin{aligned} U(0, q, \lambda_L) &= \pi_{L,0}\left(\sigma\frac{q}{2} + \Delta - q\right), \\ U(q, 1, \lambda_H) &= \pi_{H,0}\left(\sigma\frac{q+1}{2} + \Delta - 1\right). \end{aligned}$$

The expected payoffs of the boundary seller in each submarket are

$$V(q; 0, q, \lambda_L) = \sigma\frac{q}{2} + \Delta - q - (1 + \lambda_L) U(0, q, \lambda_L),$$

and

$$V(q; q, 1, \lambda_H) = \sigma\frac{q+1}{2} + \Delta - q - (1 + \lambda_H) U(q, 1, \lambda_H) - \pi_{H,0}(1 - q).$$

In equilibrium, buyers and the boundary seller are indifferent between the two submarkets. An equilibrium with two submarkets is characterized by (q^*, α^*) such that

$$U\left(0, q^*, \frac{\alpha^*}{q^*}\right) = U\left(q^*, 1, \frac{\beta - \alpha^*}{1 - q^*}\right)$$

and

$$V\left(q^*; 0, q^*, \frac{\alpha^*}{q^*}\right) = V\left(q^*; q^*, 1, \frac{\beta - \alpha^*}{1 - q^*}\right).$$

Figure 9 shows q^* as a function of σ for different values of β. As σ increases, q^* decreases. The intuition behind this pattern is as follows. As σ increases, for fixed q^*, submarket H becomes more attractive to buyers than submarket L. The only way to recover buyers' indifference between the two submarkets is to lower q^* so that quality uncertainty in submarket H increases, while that of submarket L decreases.

If σ is sufficiently large then buyer are much more willing to trade with higher-quality sellers. Sellers have a strong incentive to join submarket H, and thus, there cannot exist an equilibrium with two submarkets. The cutoff value of σ depends on β because it affects the importance of trading probability relative to transaction price, and therefore, sellers' incentive to join submarket L.

[10] $\Delta > 1$ ensures full trade in every submarket.

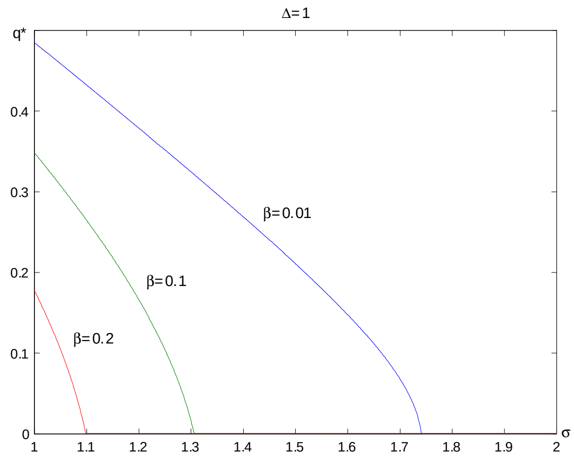

Figure 9: The boundary quality, q^*, between the two submarkets as a function of σ.

5.4 Necessary and Sufficient Condition for Endogenous Market Segmentation

The previous example suggests the possibility that a market cannot be segmented if v increases sufficiently faster than c with respect to q. The following result shows that it is indeed the case in general.

Theorem 1 *A market can be segmented for some β if and only if*

$$M(0,q) > M(q,1) \text{ for some } q \in (0,1).$$

In other words, endogenous market segmentation is impossible for any β if and only if $M(0,q) \leq M(q,1)$ for all $q \in (0,1)$.

Proof. See Appendix. ■

For an easy interpretation of this result, suppose full trade occurs in every submarket. Then the condition for no market segmentation is equivalent to

$$E_{0,q}\left[v\left(q'\right)\right] - c(q) \leq E_{q,1}\left[v\left(q'\right)\right] - c(1), \forall q \in (0,1).$$

Since the inequality obviously holds for q close to 1, in the regular case (for instance, when v and c are linear), the condition shrinks to

$$v(0) - c(0) \leq E_{0,1}\left[v\left(q'\right)\right] - c(1).$$

Roughly, the left-hand side is buyers' expected payoff when quality uncertainty is minimized subject to sellers' incentive compatibility, while the right-hand side is buyers' expected payoff when the potential trading surplus is maximized subject to information constraint. Then the condition states that endogenous market segmentation depends on the relative importance between the amount of quality uncertainty and the amount of trading surplus. When trading surplus increases sufficiently fast with respect to quality, a market cannot be segmented. Intuitively, when social surpluses from trading higher-quality goods are sufficiently larger than those from lower-quality goods, buyers are much more willing to trade with higher-quality sellers. Then sellers have a strong incentive to mimic higher-quality sellers, which prohibits the coexistence of multiple submarkets. In the previous linear example, the inequality holds if and only if $\sigma \geq 2$.

5.5 The Constant Value Case

This subsection considers the constant value case, that is, $v(q) = v$ for all $q \in [0, 1]$. For simplicity, I assume that $r(0) = 1$ so that full trade occurs in every submarket.

I first characterize the partial equilibrium in which buyers get utility $u \in (0, v - c(0))$. In a submarket with $\left[\underline{q}, \overline{q}\right]$, for buyers to get u,

$$u = \frac{1}{e^{\lambda}} \left(v - c(\overline{q})\right).$$

Notice that λ is determined only by u and $\overline{q}$. Let $\lambda(q, u)$ be the tightness that is required to ensure buyers utility u in a submarket whose highest quality is q ($\lambda(q, u) = \ln((v - c(q))/u)$). For fixed u, $\lambda(q, u)$ is well-defined only when $c(q) \leq v - u$. To simplify notations, I assume that for fixed u, if $c(q) > v - u$ then q-quality seller fully reveals her quality. This is without loss of generality because such qualities do not trade in equilibrium, whether they are revealed or not.

Buyers' bidding strategy F in a submarket is given by

$$F(b) = \frac{1}{\lambda(\overline{q}, u)} \ln \frac{v - c(\overline{q})}{v - b}, b \in \left[c(\overline{q}), v - u\right].$$

From F, I can calculate sellers' expected payoffs.

Lemma 5 *Given u, if $c(q) \leq v - u$ then the expected payoff of q-quality seller in a submarket whose highest quality is q' is*

$$W\left(q; q', u\right) = v - c(q) - (1 + \lambda(q, u))\, u, \text{ if } q \geq q',$$

and

$$W\left(q; q', u\right) = v - c(q) - \left(1 + \lambda\left(q', u\right)\right) u - \pi_0\left(q', u\right)\left(c\left(q'\right) - c(q)\right), \text{ if } q < q',$$

where $\pi_0(q, u) = 1/e^{\lambda(q,u)}$.

Each seller is indifferent over all submarkets whose highest qualities are lower than her own quality. Among submarkets whose highest qualities are higher than her own quality, each seller prefers the submarket whose highest quality is lowest.[11] This finding leads to the following result.

Proposition 5 *Every partial equilibrium is characterized by a cutoff quality* $q^* \in [0,1]$ *such that all qualities above* q^* *are fully revealed and all qualities below* q^* *form one submarket.*

Proof. By Lemma 5 and the subsequent discussion, it is an equilibrium that all qualities above q^* are fully revealed and all qualities below q^* form one submarket.

To show that this is the only equilibrium structure, suppose there exists $\left[\underline{q},\overline{q}\right]$ such that $0<\underline{q}<\overline{q}$, qualities in $\left[\underline{q},\overline{q}\right]$ form a submarket, and trade occurs in the submarket. The expected payoff of $\underline{q}$-quality seller, $V\left(\underline{q};\overline{q},u\right)$, is strictly smaller than $V\left(\underline{q};q',u\right)$ for all $q'\leq\underline{q}$. Therefore sellers whose qualities are close to q_1 deviate. ■

To find the set of general equilibria, fix u and q^*. Let

$$\begin{aligned}
\beta\left(u,q^*\right) &= q^*\lambda\left(q^*,u\right)+\int_{q^*}^{1} q\lambda\left(q,u\right)dq \\
&= q^*\ln\left(\frac{v-c\left(q^*\right)}{u}\right)+\int_{q^*}^{1}\max\left\{\ln\left(\frac{v-c\left(q\right)}{u}\right),0\right\}dq \\
&= \int_{0}^{1}\max\left\{\ln\frac{v-\max\left\{c\left(q^*\right),c\left(q\right)\right\}}{u},0\right\}dq.
\end{aligned}$$

$\beta\left(u,q^*\right)$ is decreasing in both u and q^*. Intuitively, buyers receive higher utility when the level of competition among buyers is smaller. Also, higher q^* implies less resolution of uncertainty. Therefore, to ensure buyers the same utility, the level of competition among buyers must be smaller.

Proposition 6 *For fixed* $\beta>0$*, there exists a continuum of equilibria. Each equilibrium differs in the cutoff quality* $q^*\in[0,1]$ *such that all qualities above* q^* *are fully revealed and all qualities below* q^* *form one submarket. Buyers' expected payoff,* u^**, is determined so that*

$$\beta=\int_{0}^{1}\max\left\{\ln\frac{v-\max\left\{c\left(q^*\right),c\left(q\right)\right\}}{u^*},0\right\}dq.$$

6 Discussion

6.1 Sensitivity to Submarket Exchange Process

My results are robust to perturbations of submarket exchange process. The only requirement is that the expected payoffs of sellers and buyers in a submarket exhibit the same qualitative

[11] To see this, observe that

$$V\left(q;q',u\right)-V\left(q;q,u\right)=-\left(\frac{c\left(q'\right)-c\left(q\right)}{v-c\left(q'\right)}-\ln\left(1+\frac{c\left(q'\right)-c\left(q\right)}{v-c\left(q'\right)}\right)\right)u.$$

Since $x-\ln\left(1+x\right)$ is increasing for $x>0$, $V\left(q;q',u\right)$ is strictly decreasing in q'.

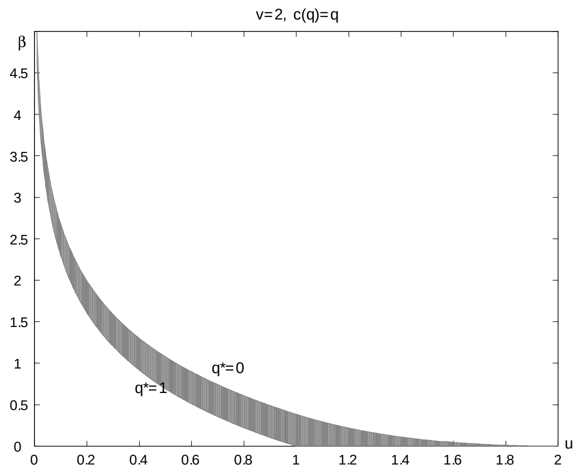

Figure 10: The set of general equilibria in the constant value case.

properties: (1) buyers' expected payoff is continuous, and decreases, in both quality uncertainty and buyer competition (the ratio of buyers to sellers), and (2) sellers' expected payoffs are continuous, and increase, in both the average quality of goods and buyer competition. For example, a small perturbation of matching technology does not affect any qualitative result. When buyers observe the number of competitors, no qualitative result (but some quantitative results) changes.[12]

6.2 Comparison to Menzio (2007)

Menzio studied the constant value case in the labor market context. In his setup, with the urn-ball matching technology, an equilibrium with two submarkets exists if and only if β is neither too large nor too small, and there cannot exist more than two submarkets.[13]

The crucial difference is submarket exchange process. In his model, a seller (firm) selects a buyer (worker) if she is matched with more than one buyer, and then they engage in an alternating offer bargaining game. The consequence of this is that ex post payoffs of sellers are independent of λ (the ratio of buyers to sellers in a submarket). λ affects only sellers' matching probability. In my model λ affects not only sellers' matching probability but also buyers' bidding behavior and, consequently, ex post payoffs of sellers. This translates into sellers' having stronger incentive to separate themselves from higher-quality sellers than in Menzio's.

It is not feasible to compare my setup to Menzio's beyond the constant value case. It is not known how one can generalize the alternating offer bargaining solution for the interdependent value case. The best known results in bargaining with interdependent values are for the case where the

[12] If full trade occurs in every submarket, the expected payoffs of agents do not change whether buyers observe the number of competitors or not.

[13] He considered more general class of matching technologies. He showed that if the inverse of job-finding probability is concave then there can exist more than two distinct submarkets.

uninformed player makes all the offers (Deneckere and Liang (2006)). But that case yields a trivial result - no market segmentation- whether values are interdependent with costs or not.

6.3 Efficiency

Does endogenous market segmentation improve efficiency? In general, the result is ambiguous. On the one hand, less quality uncertainty allows more qualities to trade. On the other hand, there is an accompanying distortion in the matching process: relatively more buyers trade with lower-quality sellers. The latter has a negative effect on efficiency, unless social surpluses from trading lower-quality goods are sufficiently larger than those from higher-quality goods. The welfare result depends on the relative importance of these two. For example, in the constant surplus case, if the lemons problem is very severe in the underlying environment, that is, only some lowest qualities trade without market segmentation (Δ is small), then it is more efficient that a market is segmented. To the contrary, if all qualities fully trade even without market segmentation (that is, $\Delta \geq 1$), social welfare is lower when a market is segmented.

Appendix A: Partially Indifferent Equilibrium

Appendix A studies the equilibrium in which buyers strictly prefer some submarkets to others in the constant surplus case.

Before I proceed, I define the inverse of $\gamma_+(\cdot, u)$. For $z \in [0, \overline{z}(u)]$, let $\gamma_-(z, u)$ be the value such that $W_U(\gamma_-(z,u), u) = W_L(z, u)$. Also, recursively, let $\gamma_-^k(z,u) = \gamma_-\left(\gamma_-^{k-1}(z,u), u\right), k = 1, 2, ...$ where $\gamma_-^0(z,u) = z$.

I first consider equilibria with two submarkets. Suppose z_1 and z_2 are the measures of sellers in each submarket. Since buyers strictly prefer the first submarket to the second submarket, $z_2 > \overline{z}(u)$. Then u must be smaller $\Delta/2$. Otherwise, by the incentive compatibility, $z_1 \geq \overline{z}(u)$, in which case a market is empty of buyers. In addition, $\gamma_-(\overline{z}(u), u) \leq z_1 < \overline{z}(u)$. The first inequality guarantees that the lower boundary seller in the second submarket is indifferent between the two submarkets. The second inequality is for the existence of a positive measure of buyers in the market. Since $z_1 + z_2 = 1$, this type of equilibrium exists if and only if $\gamma_-(\overline{z}(u), u) + \overline{z}(u) < 1$. There is a continuum of such equilibria because $(z_1, 1 - z_1)$ is an equilibrium for any $z_1 \in \left[\gamma_-(\overline{z}(u), u), 1 - \overline{z}(u)\right]$.

In general, a partially indifferent equilibrium exists if and only if $u < \Delta/2$ and $\gamma_-(\overline{z}(u), u) + \overline{z}(u) < 1$. For $u \geq \Delta/2$, such equilibrium does not exist because if $z_n > \overline{z}(u)$ then, by the incentive compatibilities of the boundary sellers, $z_k \geq \overline{z}(u)$ and so all submarkets are empty of buyers. For $2(\Delta - u) + \Delta^2/(2u) \geq 1$, it is because buyers must be indifferent over all submarkets. Subsequently, I assume that $u < \Delta/2$ and $2(\Delta - u) + \Delta^2/(2u) < 1$.

Assumption 4 *Whenever $\{z_1, ..., z_n\}$ is an equilibrium, $z_{k+1} \geq z_k, k = 1, ..., n-1$.*

This assumption requires the monotone arrangement even for inactive submarkets. This is without loss of generality because whenever there exists an equilibrium that does not satisfy this assumption there exists another equilibrium that has the same number of submarkets, yields the same outcomes, and satisfies this assumption.

Let $\overline{n}(u)$ be the largest integer such that $\gamma_-^{\overline{n}(u)-1}(\overline{z}(u), u)$ is well-defined and $\sum_{k=0}^{\overline{n}(u)-1} \gamma_-^k(\overline{z}(u), u) < 1$.

Proposition 7 *Suppose $u < \Delta/2$ and $\overline{z}(u) + \gamma_-(\overline{z}(u), u) < 1$. There exists a partially indifferent equilibrium with n submarkets in which buyers are indifferent over the first $n-1$ submarkets and strictly prefer those submarkets to the last submarket if and only if $2 \leq n \leq \overline{n}(u)$. If exists, there is a continuum of such equilibria.*

Proof. Fix n and consider z such that $\gamma_-(\overline{z}(u), u) \leq z \leq \overline{z}(u)$ and $\sum_{k=1}^{n-1} \gamma_-^{k-1}(z, u) + \overline{z}(u) < 1$. Such n exists if and only if $2 \leq n \leq \overline{n}(u)$. In addition, if exists, the set of such z is a subinterval of $[2(\Delta - u), \overline{z}(u)]$. Now consider a sequence $\left\{\gamma_-^{n-2}(z, u), \gamma_-^{n-3}(z, u), ..., \gamma_-^1(z, u), z, 1 - \sum_{k=1}^{n-1} \gamma_-^{k-1}(z, u)\right\}$. By construction, this is an equilibrium. ■

If the measure of sellers in the last submarket is greater than $2\overline{z}(u)$ then there exists a partial equilibrium with more than $\overline{n}(u)$ submarkets. The following result shows that, even considering all the possibilities, the number of submarkets cannot be greater than $\overline{N}(u)$. An important implication is that the results on general equilibrium do not change by the existence of partially indifferent equilibria.

Proposition 8 *There does not exist a partial equilibrium with more than $\overline{N}(u)$ submarkets, whether buyers are indifferent over all submarkets or not.*

Proof. Suppose there exists a partially indifferent equilibrium with n submarkets. Because of Assumption 4, there exists $m < n$ such that $z_k > \overline{z}(u)$ if and only if $k > m$. By the incentive compatibilities of the lower boundary sellers, $z_m \geq \gamma_-(\overline{z}(u), u)$ and $z_k = \gamma_-^{m-k}(z_m, u), k = 1, ..., m-1$. Since $0 < z_1$, $\gamma_+^{k-1}(0, u) < \gamma_+^{k-1}(z_1, u) = z_k$ and

$$\sum_{k=1}^{m} \gamma_+^{k-1}(0, u) < \sum_{k=1}^{m} z_k < 1 - (n-m)\overline{z}(u).$$

Now notice that

$$\sum_{k=1}^{n} \gamma_+^{k-1}(0, u) < \sum_{k=1}^{m} \gamma_+^{k-1}(0, u) + (n-m)\overline{z}(u) < 1.$$

By the definition of $\overline{N}(u)$, $n \leq \overline{N}(u)$.
■

Appendix B: Omitted Proofs

Proof of Lemma 1: I prove the result for the general continuum quality case.

Let F_1 and F_2 be buyers' bidding strategies in each submarket. Also, let $[\underline{b}_1, \overline{b}_1]$ and $[\underline{b}_2, \overline{b}_2]$ be the supports of F_1 and F_2 respectively. The result is obvious if $\overline{b}_1 \leq c(q_2)$. From now on, suppose $\overline{b}_1 > c(q_2)$.

Observe that $\overline{b}_1 \leq \overline{b}_2$. This is because $U(q_1, q_2, \lambda) = U(q_2, q_3, \lambda')$ and so

$$\overline{b}_1 = E_{q_1,q_2}[v(q)] - U(q_1, q_2, \lambda) \leq v(q_2) - U(q_2, q_3, \lambda') \leq \overline{b}_2.$$

($\Rightarrow$) It is straightforward that q_2-quality seller is indifferent between the two submarkets. I first show that if $\lambda = \lambda'$ then F' first-order stochastically dominates F, which implies that sellers strictly prefer the submarket with $[q_2, q_3]$. To show this, fix $b \in \left[\underline{b}_2, \bar{b}_1\right]$. If $b \geq c(q_3)$ then

$$e^{\lambda F(b)} = \frac{M(q_1, q_2)}{E_{q_1,q_2}[v(q)] - b}, \text{ and } e^{\lambda' F'(b)} = \frac{M(q_2, q_3)}{E_{q_2,q_3}[v(q)] - b}.$$

Since $E_{q_1,q_2}[v(q)] \leq E_{q_2,q_3}[v(q)]$ and $M(q_1, q_2) = M(q_2, q_3)$ (because $U(q_1, q_2, \lambda) = U\left(q_2, q_3, \lambda'\right)$ and $\lambda = \lambda'$), $F(b) \geq F'(b)$. If $b < c(q_3)$ then

$$e^{\lambda F(b)} = \frac{M(q_1, q_2)}{E_{q_1,q_2}[v(q)] - b}, \text{ and } e^{\lambda' F''(b)} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{M(q_2, q_3)}{E_{q_2,c^{-1}(b)}[v(q)] - b}.$$

$F(b) > F'(b)$ follows if

$$\frac{c^{-1}(b) - q_2}{q_3 - q_2}\left(E_{q_2,c^{-1}(b)}[v(q)] - b\right) > E_{q_1,q_2}[v(q)] - b.$$

If $\underline{b}_2 \geq c(q_3)$ then the condition is vacuously satisfied. Suppose $\underline{b}_2 = c(r(q_2)) < c(q_3)$. When $b = \underline{b}_2$,

$$\frac{c^{-1}(b) - q_2}{q_3 - q_2}\left(E_{q_2,c^{-1}(b)}[v(q)] - b\right) = M(q_2, q_3) = M(q_1, q_2) > E_{q_1,q_2}[v(q)] - c(r(q_2)).$$

Observe that as b increases from $\underline{b}_2 = c(r(q_2))$ to $c(q_3)$, the left-hand side decreases more slowly than the right-hand side. Therefore the inequality holds for any $b \in [\underline{b}_2, c(q_3))$.

Now suppose $\lambda < \lambda'$. Since q_2-type can mimic q_1-type,

$$V(q_2; q_1, q_2, \lambda) \geq V(q_1; q_1, q_2, \lambda) - (1 - \pi_0)(c(q_2) - c(q_1)).$$

Since $c(q_1) < c(q_2) < \underline{b}_2$,

$$V(q_2; q_2, q_3, \lambda) = V(q_1; q_2, q_3, \lambda) - (c(q_2) - c(q_1))\left(1 - \pi_0'\right).$$

Then

$$V(q_1; q_2, q_3, \lambda) - V(q_1; q_1, q_2, \lambda) \geq (c(q_2) - c(q_1))\left(\pi_0 - \pi_0'\right) > 0.$$

This contradicts the supposition that no seller deviates.

($\Leftarrow$) I first show that $\pi_0' > \pi_0 e^{\lambda F(c(q_2))}$, that is, the probability of trading of q_2-quality seller in the $[q_1, q_2]$ submarket, $1 - \pi_0 e^{\lambda F(c(q_2))}$, is greater than that in the $[q_2, q_3]$ submarket, $1 - \pi_0'$. Suppose not. Then

$$\pi_0' e^{\lambda' F'(b)} = \pi_0' \leq \pi_0 e^{\lambda F(b)} \text{ for } b \leq \underline{b}_2,$$

and

$$\begin{aligned} \pi_0' e^{\lambda' F'(b)} &= \frac{\pi_0' M(q_2, q_3)}{E_{q_2,q_3}[v(q)] - b} = \frac{U\left(q_2, q_3, \lambda'\right)}{E_{q_2,q_3}[v(q)] - b} \\ &\leq \pi_0 e^{\lambda F(b)} = \frac{U\left(q_1, q_2, \lambda'\right)}{E_{q_1,q_2}[v(q)] - b}, \forall b \in \left[c(q_3), \bar{b}_1\right]. \end{aligned}$$

In addition, for $b \in (\underline{b}_2, c(q_3))$,

$$\pi_0' e^{\lambda' F'(b)} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{\pi_0' M(q_2, q_3)}{E_{q_2, c^{-1}(b)}[v(q)] - b} = \frac{q_3 - q_2}{c^{-1}(b) - q_2} \frac{U(q_2, q_3, \lambda')}{E_{q_2, c^{-1}(b)}[v(q)] - b},$$

and

$$\pi_0 e^{\lambda F(b)} = \frac{U(q_1, q_2, \lambda)}{E_{q_1, q_2}[v(q)] - b}.$$

$\pi_0' e^{\lambda' F'(b)} < \pi_0 e^{\lambda F(b)}$ follows from the fact that $\pi_0' e^{\lambda' F'(\underline{b}_2)} = \pi_0' \leq \pi_0 e^{\lambda F(c(q_2))}$, and as b increases, $\pi_0' e^{\lambda' F'(b)}$ increases more slowly than $\pi_0 e^{\lambda F(b)}$. Therefore, $\pi_0' e^{\lambda' F(b)}$ first-order stochastically dominates $\pi_0 e^{\lambda F(b)}$. This implies that

$$\begin{aligned} V(q_2; q_2, q_3, \lambda) &= \int_{\underline{b}'}^{\bar{b}'} \max\{b - c(q_2), 0\} \, d\left(\pi_0' e^{\lambda' F'(b)}\right) \\ &> \int_{\underline{b}}^{\bar{b}} \max\{b - c(q_2), 0\} \, d\left(\pi_0 e^{\lambda F(b)}\right) = V(q_2; q_1, q_2, \lambda), \end{aligned}$$

which is a contradiction.

(i) $q < q_2$

Since q-type seller can mimic q_2-type seller,

$$V(q; q_1, q_2, \lambda) \geq V(q_2; q_1, q_2, \lambda) + (c(q_2) - c(q)) \left(1 - \pi_0 e^{\lambda F(c(q_2))}\right).$$

In addition, since $\underline{b}_2 > c(q_2)$, both q-type and q_2-type seller trade in the $[q_2, q_3]$ submarket whenever there are matched buyers. Hence,

$$V(q; q_2, q_3, \lambda) = V(q_2; q_2, q_3, \lambda) + (c(q_2) - c(q)) \left(1 - \pi_0'\right).$$

Then

$$V(q; q_1, q_2, \lambda) - V(q; q_2, q_3, \lambda) \geq (c(q_2) - c(q)) \left(\pi_0' - \pi_0 e^{\lambda F(q_2)}\right) > 0.$$

(ii) $q > q_2$

The result is obvious if $q \geq \bar{b}_1$. Consider the case where $q < \bar{b}_1$.

(ii-1) $\pi_0' > \pi_0 e^{\lambda F(c(q))}$

Since q-type seller can mimic q_2-type seller,

$$V(q; q_2, q_3, \lambda) \geq V(q_2; q_2, q_3, \lambda) - (c(q) - c(q_2)) \left(1 - \pi_0'\right).$$

In addition,

$$\begin{aligned}
V(q;q_1,q_2,\lambda) &= \sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(b-c(q))\,dF^k(b)\\
&= \sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(b-c(q_2))\,dF^k(b)-\sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(c(q)-c(q_2))\,dF^k(b)\\
&= \sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{\bar b_1}(b-c(q_2))\,dF^k(b)-\sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{c(q)}(b-c(q_2))\,dF^k(b)\\
&\quad -\sum_{k=1}^{\infty}\pi_k\int_{c(q)}^{\bar b_1}(c(q)-c(q_2))\,dF^k(b)\\
&= V(q_2;q_1,q_2,\lambda)-\sum_{k=1}^{\infty}\pi_k\int_{c(q_2)}^{\bar b}\min\{b-c(q_2),c(q)-c(q_2)\}\,dF^k(b)\\
&\le V(q_2;q_1,q_2,\lambda)-(c(q)-c(q_2))\left(1-\pi_0e^{\lambda F(c(q))}\right).
\end{aligned}$$

Hence

$$V(q;q_2,q_3,\lambda)-V(q;q_1,q_2,\lambda)\ge(c(q)-c(q_2))\left(\pi_0'-\pi_0e^{\lambda F(c(q))}\right)>0.$$

(ii-2) $\pi_0'\le\pi_0e^{\lambda F(c(q))}$

In this case, similarly to the argument for $\pi_0'>\pi_0e^{\lambda F(c(q_2))}$, one can show that $\pi_0'e^{\lambda' F(b)}$ first-order stochastically dominates $\pi_0e^{\lambda F(b)}$ for $b\ge c(q)$. Therefore, $V(q;q_2,q_3,\lambda)>V(q;q_1,q_2,\lambda)$. **Q.E.D.**

Proof of Lemma 2: (1), (2), and the continuity of $\gamma_+^k(z,\cdot)$ come from the properties of $W_L(\cdot,u)$ and $W_U(\cdot,u)$.

To show that $\gamma_+^k(z,\cdot)$ is strictly decreasing, first consider $k=1$.

$$\frac{\partial\gamma_+(z,u)}{\partial u}=-\frac{\partial W_U(z,u)/\partial u-\partial W_L(\gamma_+(z,u),u)/\partial u}{-\partial W_L(\gamma_+(z,u),u)/\partial z}.$$

The denominator is negative because $W_L(\cdot,u)$ is strictly decreasing at $\gamma_+(0,u)$ and so at any $z>\gamma_+(0,u)$. Applying the implicit function theorem, I also find that

$$\frac{\partial W_U(z,u)}{\partial u}-\frac{\partial W_L(\gamma_+(z,u),u)}{\partial u}>0,\ z\in(0,\bar z(u)).$$

Therefore, $\gamma_+^k(z,\cdot)$ is strictly decreasing if $k=1$.

Now suppose $\gamma_+^m(z,\cdot)$ is strictly decreasing in u for $m=1,...,k-1$. Then

$$\begin{aligned}
\frac{\partial\gamma_+^k(z,u)}{\partial u} &= -\frac{\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial u-\partial W_L\left(\gamma_+^k(z,u),u\right)/\partial u}{-\partial W_L(\gamma_+(z,u),u)/\partial z}\\
&\quad-\frac{\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial z\cdot\partial\gamma_+^{k-1}(z,u)/\partial u}{-\partial W_L(\gamma_+(z,u),u)/\partial z}.
\end{aligned}$$

The first line is negative by the same argument as before. The second one is also negative because $\partial W_U\left(\gamma_+^{k-1}(z,u),u\right)/\partial z, \partial\gamma_+^{k-1}(z,u)/\partial u, \partial W_L\left(\gamma_+(z,u),u\right)/\partial z < 0$. **Q.E.D.**

Proof of Proposition 3: Given u, suppose $\{z_1,...,z_m\}$ and $\{z_1',...,z_{m-1}'\}$ are partial equilibria. Let $f(z,u) = z\cdot\lambda(z,u)$. Then $f(\cdot,u)$ is strictly concave.

First notice that $z_2 < z_1'$, and $z_{k+1} \le z_k', k = 2,...,m-1$ because $z_1 > 0$, and $z_1 + ... + z_m = z_1' + ... + z_{m-1}' = 1$. From the strict concavity of f, $f(z_k') \le f(z_{k+1}) + f'(z_{k+1})(z_k' - z_{k+1}), k = 1,...,m-1$, with strict inequality for $k=1$. Then

$$\begin{aligned}\sum_{k=1}^{m-1} f\left(z_k'\right) &< \sum_{k=1}^{m-1}\left[f\left(z_{k+1}\right) + f'\left(z_{k+1}\right)\left(z_k' - z_{k+1}\right)\right] \\ &= \sum_{k=2}^{m} f\left(z_k\right) + \sum_{k=1}^{m-1} f'\left(z_{k+1}\right)\left(z_k' - z_{k+1}\right) \\ &< \sum_{k=2}^{m} f\left(z_k\right) + \sum_{k=1}^{m-1} f'\left(z_1\right)\left(z_k' - z_{k+1}\right) = \sum_{k=2}^{m} f\left(z_k\right) + z_1 f'\left(z_1\right) < \sum_{k=1}^{m} f\left(z_k\right).\end{aligned}$$

The last inequality is due to the fact that $f(0) = 0$ and f is strictly concave. This establishes the result because the total measure of buyers is given by

$$\beta_n = z_1\lambda(z_1,u) + ... + z_n\lambda(z_n,u) = \sum_{k=1}^{n} f(z_k,u).$$

Q.E.D.

Proof of Theorem 1: $(\Rightarrow)$ Suppose $M(0,q) \le M(q,1)$ for all $q \in (0,1)$. I first show that there cannot exist an equilibrium with two submarkets. Suppose a two-submarket equilibrium exists and let q^* be the boundary seller between the two submarket. Then for some $\lambda, \lambda' \ge 0$, $U(0,q^*,\lambda) \ge U\left(q^*,1,\lambda'\right)$.[14] Since $M(0,q^*) \le M(q^*,1)$ and $U(0,q^*,\lambda) = M(0,q^*)/e^{\lambda} \ge U\left(q^*,1,\lambda'\right)/e^{\lambda'}$, for $U(0,q^*,\lambda) \ge U\left(q^*,1,\lambda'\right)$, $\lambda \le \lambda'$. That is, there must be less buyer competition in the second submarket. In this case, sellers in the first submarket prefer the second submarket (see the sufficiency proof of Lemma 1).

Now consider an equilibrium with $n\,(>2)$ submarkets, $\{q_1,...,q_n\}$ and $\{\lambda_1,...,\lambda_n\}$. By the same argument as before, $M(q_{k-1},q_k) > M(q_k,q_{k+1})$ for all k such that $\lambda_k > 0$. It is enough to show that if $M(0,q_1) \le M(q_1,1)$ then there exists at least one $k > 1$ such that $M(0,q_1) \le M(q_k,q_{k+1})$. I get the result by applying the following claim inductively.

Claim: For $q_1 < q_2 < q_3$, $M(q_1,q_3)$ cannot be strictly greater than both $M(q_1,q_2)$ and $M(q_2,q_3)$.

Proof: There are three cases. (1) $r(q_1) \le q_2$.

$$\begin{aligned}M(q_1,q_3) &= \frac{r(q_1)-q_1}{q_3-q_1}\left(E_{q_1,r(q_1)}\left[v\left(q'\right)\right] - c\left(r\left(q_1\right)\right)\right) \\ &\le \frac{r(q_1)-q_1}{q_2-q_1}\left(E_{q_1,r(q_1)}\left[v\left(q'\right)\right] - c\left(r\left(q_1\right)\right)\right) = M(q_1,q_2).\end{aligned}$$

[14] Weak inequality is used because there may be no trade in the second submarket.

(2) $r(q_1) \geq q_3$.

$$M(q_1, q_3) = E_{q_1,q_3}\left[v\left(q'\right)\right] - c(q_3) \leq E_{q_2,q_3}\left[v\left(q'\right)\right] - c(q_3) = M(q_2, q_3).$$

(3) $r(q_1) \in (q_2, q_3)$

$$\begin{aligned} & (q_2 - q_1) M(q_1, q_2) + (q_3 - q_2) M(q_2, q_3) \\ = & \int_{q_1}^{q_2} (v(q) - c(q_2))\, dq + \max_{q' \in [q_2, q_3]} \int_{q_2}^{q'} \left(v(q) - c\left(q'\right)\right) dq \\ \geq & \int_{q_1}^{q_2} (v(q) - c(q_2))\, dq + \int_{q_2}^{r(q_1)} \left(v(q) - c\left(q'\right)\right) dq \\ = & \int_{q_1}^{r(q_1)} (v(q) - c(q_2))\, dq = (q_3 - q_1) M(q_1, q_3). \end{aligned}$$

Since $M(q_1, q_3)$ is less than or equal to a weighted average of $M(q_1, q_2)$ and $M(q_2, q_3)$, it cannot be the case that $M(q_1, q_3) \geq M(q_1, q_2), M(q_2, q_3)$. **Q.E.D.**

($\Leftarrow$) Suppose for some $q^* \in (0, 1)$, $M(0, q^*) > M(q^*, 1)$. Let $\underline{\lambda}$ be the value such that

$$U(0, q^*, \underline{\lambda}) = \frac{1}{e^{\underline{\lambda}}} M(0, q^*) = M(q^*, 1).$$

By the strict inequality, $\underline{\lambda} > 0$ and then $V(q^*; 0, q^*, \underline{\lambda}) \geq V(q^*; q^*, 1, 0) = 0$. Now given $\lambda \geq \underline{\lambda}$, let $\lambda' (< \lambda)$ be the value such that $U(0, q^*, \lambda) = U\left(q^*, 1, \lambda'\right)$. If λ is sufficiently large, then λ' is also sufficiently large. For λ and λ' sufficiently large,

$$V(q^*; 0, q^*, \lambda) < v(q^*) - c(q^*) \leq E_{q^*, \min\{r(q^*), 1\}}\left[v\left(q'\right)\right] - c(q^*) \approx V\left(q^*; q^*, 1, \lambda'\right).$$

Since $V(q^*; 0, q^*, \cdot)$ and $V(q^*; q^*, 1, \cdot)$ are continuous, there exists $\lambda^* > 0$ such that $V(q^*; 0, q^*, \lambda^*) = V\left(q^*; q^*, 1, \lambda'(\lambda^*)\right)$. This establishes an equilibrium with two submarkets for $\beta = q^* \lambda^* + (1 - q^*) \lambda'(\lambda^*) > 0$. **Q.E.D.**

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