PIRE Weak Lensing Problems

1. Show that the unweighted quadrupole moments of a galaxy image convolved with a PSF are the sum of the moments of the PSF and the original image. [Extra: show the relation between original and convolved moments if PSF and galaxy are both Gaussian]

2. Assume you are trying to measure the velocity dispersion of a singular isothermal sphere (galaxy cluster) located at redshift $z = 0.3$ by measuring the lensing shear induced on galaxies located at $z = 1$.

   (a) What is the lensing critical density at the cluster redshift? Assume LCDM Universe with $H_0 = 70$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.

   (b) An isothermal sphere with velocity dispersion sigma has

   $$\rho(r) = \frac{\sigma^2}{2\pi G r^2}. \quad (1)$$

   If the cluster has $\sigma = 1000$ km/s, what is the shear $\gamma$ vs radius? What is the radius where an Einstein ring could appear?

   (c) There are 20 resolved background galaxies per square arcminute in the background, with $\sigma_\gamma = 0.3$. What is the uncertainty in the mean shear in an annulus at radius $\theta$ and width $\Delta\theta$? What is the S/N ratio of the shear measurement in this annulus?

   (d) What is the accuracy of the WL estimation of $\sigma$ using the shear data from these galaxies? Assume that you cannot use regions with $\gamma > 0.3$ and your measurement extends as far as 0.5 degrees from the cluster center.

3. If you have time left: How far away would the Sun need to be in order to produce multiple image of a background star?