NSF PIRE Summer School July 2009

<u>X-ray Clusters Problem Set</u>

1. From the equation of hydrostatic equilibrium $\nabla P = -\rho_g \nabla\phi(r)$ and the ideal gas law, derive the following relationship for the total gravitating mass of a galaxy cluster:

$$M(< R) = -\frac{kTR}{G\mu m_p}\left(\frac{d\log\rho_g}{d\log R} + \frac{d\log T}{d\log R}\right).$$

   Here ρ_g is the mass density of the X-ray–emitting plasma, T is its temperature, R is the (3D) distance from the center of the cluster, G is the gravitational constant and m_p is the proton mass. What is μ? Numerically evaluate μ for a fully ionized plasma containing only H and He in their typical cosmic abundance ratio (for simplicity you can use 10 H to 1 He by number). How would the value of μ change for a plasma with the typical cosmic composition of elements? (You do not need to give a number, just state whether it would be more or less than the pure H/He case.)

2. The gas density ρ in many clusters of galaxies has been described by the isothermal-β law:

$$\rho_g(R) = \rho_{g0}[1 + (R/R_c)^2]^{-3\beta/2},$$

   where R is the (three-dimensional) radius in the cluster and R_c and β can be considered fitting parameters. Evaluate the eqn. for $M(< R)$ from problem 1 using this density profile and the assumption that the cluster temperature is isothermal. How does the mass of the cluster (under this model) grow at large radii?

3. By definition, the virial radius of a cluster is the radius within which virial equilibrium holds. One can obtain a fairly good approximation to the virial radius in practice by using the radius within which the mass density of the cluster is a factor of $\sim$200 times the critical density $\rho_{\rm crit} = 3H^2/8\pi G$. This radius is referred to as R_{200} and the corresponding mass as M_{200}. Using the gravitating mass profile for the isothermal-β model (result from problem 2), derive an expression for the virial radius (use R_{200}) in terms of T, β, and R_c.

4. Let's now assume that clusters are not isothermal, but that their X-ray–emitting plasma follows a polytropic equation of state, viz., $p \propto \rho^\gamma$. Using the isothermal-β law profile for ρ_g (given in problem 2) and hydrostatic equilibrium, determine how the total cluster mass varies with radius for this polytropic model. Verify that the limit for an isothermal cluster agrees with the result from problem 2. Something odd happens to the mass profile for values of $\gamma > 1 + 1/3\beta$. What is this?