Advanced Macroeconomics I
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Yale University

Guillermo L. Ordoñez

Week 4
Introduction

- Credit frictions → amplification & persistence of shocks

- Two roles for capital
  - Factor of production
  - Collateral for loans

- Negative productivity shock
  - Reduces output; reduces value of collateral
  - Reduces borrowing, which reduces output further
  - "Multiplier" effects amplifies losses
Mechanism Summary

Credit cycles

Fig. 1

The constrained firms suffer a capital loss on their landholdings, which, because of the high leverage, causes their net worth to drop considerably. As a result, the firms have to make yet deeper cuts in their investment in land. There is an intertemporal multiplier process: the shock to the constrained firms' net worth in period $t$ causes them to cut their demand for land in period $t$ and in subsequent periods; for market equilibrium to be restored, the unconstrained firms' user cost of land is thus anticipated to fall in each of these periods, which leads to a fall in the land price in period $t$; and this reduces the constrained firms' net worth in period $t$ still further. Persistence and amplification reinforce each other. The process is summarized in figure 1.

In fact, two kinds of multiplier process are exhibited in figure 1, and it is useful to distinguish between them. One is a within-period, or static, multiplier. Consider the left-hand column of figure 1, marked “date $t$” (ignore any arrows to and from the future). The productivity shock reduces the net worth of the constrained firms, and forces them to cut back their demand for land; the user cost falls to clear the market; and the land price drops by the same amount (keeping the future constant), which lowers the value of the firms' existing landholdings, and reduces their net worth still further. But this simple intuition misses the much more powerful intertemporal, or dynamic, multiplier. The future is not constant. As the arrows to the right of the date $t$ column in figure 1 indicate, the overall drop in the land price is the cumulative fall in present and future user costs, stemming from the persistent reductions in the
Agents

- Farmers. measure 1
  \[ E_t \sum_{s=0}^{\infty} \beta^s x_{t+s} \]
- Gathers, measure m
  \[ E_t \sum_{s=0}^{\infty} \beta'^s x'_{t+s} \]
- Farmers *more* impatient \((\beta < \beta')\)
  (will imply that Farmers are the borrowers in equilibrium)
- Both use land \(k_t\) to produce fruit
- Value of land \(k_t q_t\) used as collateral
Farmers

- Farmers’ production function for fruit

\[ y_{t+1} = (a + c)k_t \]

\[ ak_t = \text{sellable fruit} \]

\[ ck_t = \text{"bruised fruit" which must be consumed} \]

- Assume \( c > a(\frac{1}{\beta} - 1) \)

(in equilibrium farmer wants to consume \( ck_t \) and sell \( ak_t \))
Farmers (constrained)

- Can borrow $b_t$ at rate $R$

- Borrowing Constraint (from inalienability of farmers' human capital)

$$Rb_t \leq q_{t+1}k_t$$

- Farmers' "flow of funds" constraint

$$(a + c)k_{t-1} + b_t + q_t k_{t-1} = x_t + Rb_{t-1} + q_t k_t$$

$x_t$ is consumption of fruit
Gatherers (unconstrained)

- They do not have specific skills to threat not paying.
- Gatherers’ production function for fruit

\[ y'_{t+1} = G(k'_t) \]

\( G(\cdot) \) has decreasing returns to scale

- Gatherers’ budget constraint

\[ G(k'_{t-1}) + b'_t + q_t k'_{t-1} = x'_t + Rb'_{t-1} + q_t k'_t \]

\( x'_t \) is consumption of fruit
Equilibrium

- Sequences of land prices, allocations of land, debt, consumption for farmers and gatherers

\[ \{q_t, k_t, k'_t, b_t, b'_t, x_t, x'_t\} \]

such that everyone’s optimizing and markets clearing.

- No uncertainty: perfect foresight
Equilibrium Results: Farmers

- Farmers always borrow the maximum and invest in land

\[ b_t = q_{t+1}k_t/R \quad \text{and} \quad x_t = ck_{t-1} \]

- Implied optimal land holdings

\[ k_t = \frac{1}{q_t - q_{t+1}/R} \left[ (a + q_t)k_{t-1} - Rb_{t-1} \right] \]

\[ u_t = q_t - q_{t+1}/R = \text{”down payment”} \]

- Farmers spend entire net worth on difference between price of new land \( q_t \) and amount against which they can borrow against each unit of land \( q_{t+1}/R \)
Equilibrium Results: Gatherers

- Gatherer’s demand for land

\[ G'(k'_t)/R = u_t = q_t - \left(\frac{q_{t+1}}{R}\right) \]

user cost
Farmers in the Aggregate

- Farmer aggregate landholding & borrowing

\[ K_t = \frac{1}{u_t} [(a + q_t)K_{t-1} - RB_{t-1}] \]

\[ B_t = \frac{1}{R} q_{t+1} K_t \]
Market Clearing

- Land market resource constraint
  \[ mk'_t + K_t = \bar{K} \]

- Land market clearing
  \[ u_t = q_t - q_{t+1}/R = G' \left( \frac{1}{m} \left( \bar{K} - K_t \right) \right) / R \]

- No bubbles in land price: \( \lim_{s \to \infty} E_t(R^{-s}q_{t+s}) = 0 \)
Steady State

\[ u^* = (1 - 1/R)q^* = a \]

\[ u^* = G' \left( \frac{1}{m}(\bar{K} - K^*) \right) / R \]

\[ (R - 1)B^* = aK^* \]
Steady State

We are now in a position to compare consumption paths (8a), (8b), and (8c). In the steady state, the user cost equals $a$; and so, given the farmer’s discount factor $\beta$, investment gives him discounted utility $\beta c / (1 - 2\beta)$, saving gives $\beta 2 c / (1 - 2\beta)$, and consumption gives one. By assumption 1, investment strictly dominates saving; and by assumption 2, investment strictly dominates consumption. This completes the proof of our earlier claim about farmers’ optimal behavior in the neighborhood of the steady state.

Figure 2 provides a useful summary of the economy. On the horizon...
One-time Productivity Shock with Credit Constraints

- Say $y_{t+1} = (1 + \Delta)(a + c)k_t$
- Period of shock (period $t$)
  \[ u(K_t)K_t = (a + \Delta a + q_t - q^*)K^* \]
- Subsequent periods (periods $t + s$, $s = 1, 2, ...$)
  \[ u(K_t)K_t = (a + \Delta a + q_t - q^*)K^* \]
One-time Productivity Shock with Credit Constraints

- Log-linearize around steady state

- Define for variable $X_t$ the proportional change from steady state

\[ \hat{X}_t = \frac{X_t - X^*}{X^*} \]

- Period of shock (period $t$)

\[ (1 + 1/\eta)\hat{K}_t = \Delta + \frac{R}{R - 1}\hat{q}_t \]

- Subsequent periods (periods $t + s$, $s = 1, 2, ...$)

\[ (1 + 1/\eta)\hat{K}_{t+s} = \hat{K}_{t+s-1} \]

where $\eta$ denotes elasticity of land supply of gatherers to user cost
Response of Land Price & Land Holdings

- **Land price response**
  \[
  \hat{q}_t = \frac{1}{\eta} \Delta 
  \]

- **Overall land holding response**
  \[
  \hat{K}_t = \frac{1}{1 + \frac{1}{\eta}} \left(1 + \frac{R}{R - 1 - \frac{1}{\eta}}\right) \Delta > 1
  \]
Response of Land Price & Land Holdings

- Land price response
  \[ \hat{q}_t = \frac{1}{\eta} \Delta \]

- Overall land holding response
  \[ \hat{K}_t = \frac{1}{1 + \frac{1}{\eta} \left(1 + \frac{R}{R - 1} \frac{1}{\eta}\right)} \Delta \]

  \[ > 1 \]

- Say \( \eta = 1, R = 1.05 \)
  \[ \hat{K}_t \approx 11\Delta \]
Static Response of Land Price & Land Holdings

- Land price response

\[ \hat{q}_t | q_{t+1} = q^* = \frac{1}{\eta} \frac{R - 1}{R} \Delta \]

- Overall land holding response

\[ \hat{K}_t | q_{t+1} = q^* = \Delta \]
Response of Output & Productivity

\[ \hat{Y}_{t+s} = \frac{a + c - Ra}{a + c} \left( \frac{(a + c)K^*}{Y^*} \right) \hat{K}_{t+s-1} \]

Productivity diff. Farmers’ share
Net Worth Shock

- One time reduction in debt obligations
- Increases net worth
- Farmer increases leverage, production
- Another view of Bernanke-Paulson policies?
One-time Productivity Shock at First-Best Steady State

- Say $y_{t+1} = (1 + \Delta)(a + c)k_t$
- Output rises by $\Delta$
- Net worth rises
- But prices $q^0$ unaffected; land $k^0$ unaffected
- No change to future variables
Conclusions

- Firms’ productive capital also used as collateral
- Amplification of real shocks through lower collateral value of capital
- Real effects of lower asset values
Critiques/Comments

- Kocherlakota (QR, 2000): Quantitative importance likely to be small if land & capital share less than 0.4
- Andres Arias (WP, 2005): Calibrated RBC model with KM credit constraints deliver small amplification effects
- Does this work through "investment wedge?" or TFP, or both?
- Real effects of housing/stock bubbles